SYNOPSIS ON SERIES OF LECTURE NOTES ON JAGADGURU SANKARACHARYA SRI BHARATHI KRISHNA TIRTHAJI MAHARAJA'S WORK VEDIC MATHEMATICS

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ON
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VEDIC MATHEMATICS

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SIXTEEN SUTRAS AND THEIR COROLLARIES

Sūtras

1. एकाधिकेन पूर्वेण
   Ekādhikena Pūrveṇa (also a corollary)
2. निखिलं नवतश्चरमं दशतः
   Nikhilaṃ Navataścaramaṃ Daśataḥ
3. ऊर्ध्वतिर्यग्भ्याम्
   Ūrdhva-tiryagbhyām
4. परावर्त्य योजयेत्
   Parāvartya Yojayet
5. शून्यं साम्यसमुच्चये
   Śūnyaṃ Sāmyasamuccaye
6. (आनुरूप्ये) शून्यमन्यत्
   (Ānurūpye) Śūnyamanyat
7. संकलनव्यवकलनाभ्याम्
   Saṅkalana-vyavakalanābhyam (also a corollary)
8. पूरणापूरणाभ्याम्
   Pūraṇāpūraṇābhyām
9. चलनकलनाभ्याम्
   Calana-Kalanābhyām
10. यावदूनम्
    Yāvadūnam
11. व्यष्टिसमष्टिः
    Vyaṣṭisamaṣṭiḥ
12. शेषाण्यङ्केन चरमेण
    Śeṣāṇyaṅkena Carameṇa
13. सोपान्त्यद्वयमन्त्यम्
    Sopāntyadvayamantyam
14. एकन्यूनेन पूर्वेण
    Ekanyūnena Pūrveṇa
15. गुणितसमुच्चयः
    Guṇitasamuccayaḥ
16. गुणकसमुच्चयः
    Guṇakasamuccayaḥ

Sub-Sūtras or Corollaries

1. आनुरूप्येण
   Ānurūpyeṇa
2. शिष्यते शेषसंज्ञः
   Śiṣyate Śeṣasaṃjñaḥ
3. आद्यमाद्येनान्त्यमन्त्येन
   Ādyamādyenāntya-mantyena
4. केवलैः सप्तकं गुण्यात्
   Kevalaiḥ Saptakaṃ Guṇyāt
5. वेष्टनम्
   Veṣṭanam
6. यावदूनं तावदूनम्
   Yāvadūnaṃ Tāvadūnam
7. यावदूनं तावदूनीकृत्य वर्गं च योजयेत्
   Yāvadūnaṃ Tāvadūnīkṛtya Vargañca Yojayet
8. अन्त्ययोर्दशकेऽपि
   Antyayordaśake'pi
9. अन्त्ययोरेव
   Antyayoreva
10. समुच्चयगुणितः
    Samuccayaguṇitaḥ
11. लोपनस्थापनाभ्याम्
    Lopanasthāpanābhyām
12. विलोकनम्
    Vilokanam
13. गुणितसमुच्चयः समुच्चयगुणितः
    Guṇitasamuccayaḥ
    Samuccayaguṇitaḥ

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INTRODUCTION

His Holiness Jagad Guru Shankaracharya Sri Bharathi Krishna Tirthaji Maharaj of Govardhan Mutt, Puri, who had his Post Graduation in Sanskrit, Philosophy, English, Mathematics, History and Science, had initiated several mathematical formulations through the application of sixteen main sutras and thirteen Upasutras, stated to have been found in the Appendix of Adharvana Veda. The modus operandi of these sutras was explained and the application of them in different topics, in working of mathematics, which include not only the general algebraical working but also higher and advanced working in differentials, integrals, polynomials, trigonometry, partial differentiation, solution of equations (simple, quadratic, cubic, biquadratic, etc), problems concerned with coordinate geometry, logarithms, exponential, summation of series, solid geometry, etc. A list of all the sixteen main sutras and thirteen upasutras is given. However in each part of the lecture notes the concerned and relevant sutras and upasutras are explained elaborately together with applications.

The methods are surprisingly simple, at the same time elegant and even able to solve in much less time than the current methods. The philosophy behind these methods is a total faith in the dictum that :

- The solution of any problem is in the problem itself.
- A symmetry of some order exists always in the problem which needs to be tapped. (Working is based also on such existing symmetries).
- A totally and a comprehensive thought of working is clearly depicted.
- Aim is always towards a mental working, preservation and lesser complication in working details.

These features are clearly established in operational techniques as explained by the great seer Sri Bharathi Krishna Tirthaji.

We have taken up a project to work out the details and also to explore the possibilities of advancing these methods to arrive-at important deductions which are found difficult or which are being worked out elaborately using current methods. The entire work is divided into a number of topics such as multiplication, division, equations, calculus, coordinate geometry, logarithms, trigonometry, squares, square-roots, cube-roots, and higher roots and the like. We intend to bring out lecture notes to enable not only mathematics to go through these methods and spread this message to the general public.

SYNOPSIS ON LECTURE NOTES – I – MULTIPLICATION

This volume deals with different methods on multiplication of numbers using different sutras, the modus operandi of which are explained. The Sutras used are mainly:

(1) Urdhva Tiryagbhyam (2) The concept of Vinculum (3) Nikhilam Navatascaramam Dasatah and (4) (a) Ekanunena (b) Upasutram Antyor Dasakepi (two special cases) under which multiplications, are carried from right to left, left to right, the series multiplication, combined operations, moving multiplier method multiplication using groups, a stepwise method of obtaining the multiplication of numbers and multiplication of any numbers using theoretical base, working base etc., leading finally to most general multiplication of many numbers. All the methods are explained in detail with the help of different numbers. Polynomial multiplication provides also a general proof (by converting the number into a polynomial)

CHAPTER 1 URDHVA TIRYAGBHYAM:

The main Sutram Urdhva Tiryagbhyam is considered as the most general one for the multiplication of two numbers.

Application of Urdhva Tiryagbhyam Sutram (General Multiplication Method) both from Right to Left and Left to Right are worked out and compared with the conventional method. The details of the multiplication are shown.

The following are the various applications of the above Sutram.

- Multiplication with step diagrams is worked out. The result from the steps is derived.

If the multiplicand and the multiplier do not have same number of digits, the non-existing digits are considered as zeros.

- Multiplication is worked out upto 9 (nine) digits. The steps are shown by means of diagrams. Procedure is worked out for both the Right Hand and Left Hand Multiplication.

- Two different working details for the multiplication are shown as VM1 and VM2

- Moving multiplier Method is detailed and in this method ; one can start from any position, which is also illustrated. This method has advantage in multiplication where both multiplicand and multiplier do not have same number of digits.

- Attempts are made to locate the value of a particular position in the final answer without working the entire multiplication. This is worked out for Right to Left Multiplication.

- A comparision is made among the Right to Left Method, the Left to Right Method and the Conventional method.

- Method of Combined Operations

    a. Sum of Products is detailed

    b. Product of numbers at a single stretch of working is detailed (Series Multiplication)

Multiplication of numbers having any number of digits can be worked out with the help of symmetry and on the basis of the expected position of Units, Tens, Hundreds, etc, in the answer.

The diagrams are very clearly shown in a number of multiplications having more than two in number and extended to more than two digits in general. This seems to be a novel type of working which can be easily understood and worked out. The example of Series Multiplication of five numbers having three digits each is worked out from symmetry diagram.

The above method of multiplication of many numbers can be applied using Vinculum wherever it is necessary.

- **POLYNOMIAL MULTIPLICATION:** Vedic method of multiplication is applied in polynomial, which is definitely much simpler than the conventional method. Worked out examples are given. This is extended to multiplication of any number of polynomials using the symmetry diagrams.

- **MULTIPLICATION BY GROUPS:** Nicholas Group of Mathematicians (UK) extends the multiplication by considering a multiplicand 124 as (12) 4 or 1 (24) and a multiplier 113 as (11) 3 or 1(13). The multiplication is considered with the group as one entity in Units / Tens / hundreds / etc, followed by Urdhva Tiryak Sutram.

We extend these groups to different numbers of grouping. Both multiplicand and multiplier, in general, need not have same type of grouping.

In general, this multiplication could be elegant when we convert this into a polynomial multiplication, taking each group to represent as Units or Tens or Hundreds or Thousands, etc as the case may be. This is clearly demonstrated by considering different groups in multiplicand and multiplier. This method is also applicable by considering the Vinculum.

The grouping definitely reduces the number of steps in multiplication and is hence, definitely simpler than non-grouping multiplication. These groupings can be expressed in polynomials with base 10. We tried for different bases as well, which could be successfully worked out.

CHAPTER II VINCULUM:

In the concept of Vinculum, the value of any digits in the given number, if generally more than five, could be expressed in terms of smaller digits, to facilitate this oral multiplication. The Vinculum method also simplified the division as explained under division. All the methods described under Urdhva Tiryagbhyam are also applicable to Vinculum concept of the numbers. These are illustrated..

CHAPTER III NIKhilam Navatascaramam Dasatah :

In working out the multiplication, the Sutram, **"Nikhilam Navatascaramam Dasatah"** is also applied by considering a base for the multiplicand and multiplier. Base may be same or different for the two – multiplicand and multiplier. Base can be distinguished between the theoretical base and working base.

The final result can be converted from working base of the simultaneous multiplication of many numbers. If the numbers are expressed in different bases, one needs to convert to the common base for all. In such case the procedure is also demonstrated. A general procedure is described. The multiplication is worked out using the concept of a base and the deficiencies or excesses with reference to the considered base. The theoretical base in general is considered in powers of 10. One can use a working base different from theoretical base so that it is nearer the number. Multiplies or sub-multiplies of a base can be considered, if necessary.

The different methods are explained by considering this concept of base in working out multiplication of numbers.

In the book on Vedic Mathematics of Swamiji, multiplication between two numbers using a common base for multiplicand and multiplier is explained. Usage of multiples or sub multiples of the base of either of the two numbers is also explained.

The British Authors in their work on Vedic Mathematics, 1982 with reference to Swamiji's work have extended the above method to three number and four numbers multiplication using a different base at least for one of the numbers. They gave a proof for the above using algebraic expressions for multiplication of the numbers considering the same theoretical base for all the numbers.

In the present work, the authors have extended the multiplication in a most general way by introducing the concept of a theoretical base for each number, and different working bases for different numbers of the multiplication.

The authors have explained the method of obtaining the results of any number multiplication with the help of (1) a theoretical base for each number (2) a working base for each number (3) Deficiencies or excesses with reference to working base of the number (4) Equalization to the same base for all the numbers

(5) multiplication factor $\left(MF = \dfrac{\text{Equalizing Base}}{\text{Working Base of each number}}\right.$

(6) modified numbers and (7) modified deficiencies or excesses, consequent on equalization of the working base. These are tabulated and made use of in the working of the problem as given in the text :

The method of arriving at the result of multiplication consists of numbers of parts equal to as many as numbers to be multiplieo. The working of each part, together with the allowed placement of the result of that part, and also the digits which are to be carried out to the next part are differentiated with reference to any of the theoretical base of the given numbers. This is well detailed using different examples.

The details of obtaining the values of different parts together with the reduction, consequent on equalization of the working base are explained more elaborately with the help of different examples.

It is interesting to note that the same result of the multiplication could be obtained (a) for different working base equalization (b) for different placements of the values of different parts on the basis of theoretical base of any number, as exemplified in the text with examples.

The final answer is read out by computing the values in a sequential order of the various parts, as shown in the examples (This is the Method II).

The authors have finally given a general proof for the multiplication of many numbers in terms of only the working bases, which is a valuable contribution.

This method is workable for any multiplication of numbers through a derivation of a general algebraic expansion and the same can also be used to get the result of the multiplication by substituting the values in the derived expansion. This provides also a proof and serves as the method I.

Through all the above efforts it could be finally achieved that a more simplified method of obtaining the value of the multiplication which makes use a common multiplication factor (α) for all the parts other than the first part of the answer. The common multiplication factor (α) is the product of all the working bases of the numbers other than the equalized working base as the numerator. This method does not require (a) the placement details for each part and also (b) the theoretical base for each number. This method is workable only with the knowledge of working base of each number, equalization base, the modified numbers and modified deficiencies or excesses. This is also workable for any equalization base.

All the methods are applicable to any numbered multiplications.

The examples are worked out under different methods (1) using the derived formula for the purpose of proof, we called it as method I, (2) using the method of writing down the answer into different parts with the help of the placement, equalization to the common working base, as given in the tabular form which are prepared earlier, we called it as method II. (3) The much simplified method derived from the proof has a common multiplication factor as the numerator called alpha (α) for all the parts other than the first part of the answer. This is the Alpha method.

Another simplified method called the based method wherein multiplication of many numbers can be carried out using different working bases, deficiencies by excesses and couple with series method of multiplications.

Upasutram – Applications :

Ekanyunena Purvena and Antyayor Dasakepi.

As corollaries under Nikhilam Sutram, (a) Ekanyunena Purvena and one Upa Sutram (b) Antyayor Dasakepi are applied as special case of multiplication, the details of which are elaborated.

CHPATER IV

All the methods are programmed using C++ language and the out put of the programs are tested for the same result.

The answer shown in parts led to an algorithm for the programming of any number multiplication consisting of many digited numbers using different working bases, different equalization of working base and different placements as well.

The authors claim that a most general method for getting the result could be well-programmed.

SYNOPSIS ON LECTURE NOTES – II – DIVISION

This volume deals with different methods on division of numbers and also polynomials using different sutras, the modus operandi of which are explained.

The sutras used are mainly:

1. Nikhilam 2. Urdhva Tiryagbhyam

3. Paravartya yojayet 4. Vinculum and

5. Vilokanam.

The different methods applied are

1. Nikhilam Method for numbers. 2. Straight Division 3. Paravarthya Method 4. Argumental Division for both Polynomials and numbers.

A number of examples are worked out in each method to facilitate one to work with ease.

Chapter I deals with the division method by Nikhilam Sutram both for a single digit divisor and multi digited divisor. Proofs are given for the method used.

Chapter II deals with the application of Urdhva Tiryag sutram for the division wherein the divisor is partitioned into two parts (Straight Division). a. Dhwajanka b. Part Divisor.

As such the problem is reduced to be simple in the sense that throughout the working the divisor can be only one digit, the part divisor if one chooses one can have more than one digit. The working details of the application of the straight division for many examples are given in the lecture notes.

In this chapter the division of non – decimal numbers wherein.

a) the dividend only is having a decimal

b) the divisor only is having a decimal

c) more generally both the divisor and the dividend having decimal points, are separately dealt with.

It is interesting to note that the working, using the method described by Swamiji is simpler than the current method.

Many examples could be worked out wherein the quotient and remainder could be shown. The calculation of the absolute remainder is also explained. A Comparison of the existing method and Swamiji's method very clearly shows the ease and elegance in arriving at the result. Proofs are deduced for a number of cases where the Dhwajanka has more than one digit.

In the straight division method, in the first instance, a negative quotient or remainder is avoided through the process of reduction which is explained separately. As a second attempt the negative concept is considered as it is (without reduction) i.e., representing it in the form of Vinculum. It is noticed that the second procedure is much easier than the first. A number of examples are given using both the methods.

In addition to the above a more simplified method of reduction process is also well explained which makes use of a reduction in the previous value of the quotient followed by addition of the remainder by the part divisor for each reduction in the quotient which is due to negativity of the obtained quotient.

Chapter III deals with the methods explained Swamiji's on combined operation on

(a) Division and Multiplication (b) Division and Addition. A few examples are given.

In Chapter IV the division by Paravartya Method for polynomials is given with a number of examples. The Paravartya method of division for Polynomials in case the divisor is having higher power than the dividend and when the co-efficient of the highest power in the divisor is not unity the division is explained by two different methods. This is further applied to numbers wherein the divisor, if necessary, can be brought to the condition suitable for Paravartya form using one or the combination of the following methods.

a) Multiples or sub-multiples b) Vinculum
c) Nikhilam.

If the reminder is greater than divisor, by Vilokanam one can subtract from it "n" times the divisor to enable the resulting remainder to be less than the divisor. This is necessarily to be followed by addition of "n" to the previous quotient value. If the remainder shows negative value (by Vinculum Method) then an addition of "n" times the divisor will enable the negative remainder to become positive (n is the required minimum integer). This is followed by a subtraction of "n" from the previous quotient.

Chapter V deals with the method of Argumental division in case of polynomials. A general formula for arriving at the number of variables that are to be considered in the argumental division is arrived at as equal to (m-n+1) where m is the highest degree of the variable in the dividend, n stands for the highest degree of the variable in the divisor.

The Argumental division is extended to numbers. The Argumental in general, either polynomials or numbers makes use of left to right multiplication division.

The same formula is applicable but with m as number of digits in dividend and "n" as number of digits in divisor.

The simplified argumental method as explained by Swamiji is slightly different method which makes use of a combination of straight division and argumental division methods. This is worked out for the problems given by Swamiji in his text. It is interesting to note that a good number of examples from Hall and Knight's Algebra text are worked out using the above principles. These steps are also clearly explained in this chapter.

In Chapter VI the polynomial division in one variable, using straight division is explained. An interesting concept is introduced in working the division - for different values of x, and the details are worked out. These are verified by using the division algorithm.

In this chapter, details are worked for the division with zero remainder at every step. The quotient and remainder are worked out. As a second case, the division is carried out under non-zero remainder process which also gives its quotient and its remainder, a relation between these two could be worked out for different values of x.

A deduction of quotients obtained in zero remainder division method from the corresponding quotients in the non – zero remainder division method is brought out. This is particularly attempted as division with non – zero remainders doesn't involve fractions in quotients.

In the final chapter, the division of Bipolynomials is exemplified by the straight division method both by partition method and without partition method. It is noticed that this method can easily be extended to polynomial in three variables and also with any number of variables.

Examples of two variable and three variable polynomial are given in this chapter.

A polynomial results from 1)Its reciprocal, 2) By multiplication of two polynomials, 3) A root of polynomial, 4) Log of polynomial, 5) Exponential of polynomial, 6) Trigonometric functions, squaring / cubing, $(p)^n$

i.e., $P(x)\,Q(x)$, $P^2(x)$, $P(x) \div Q(x)$, $\dfrac{1}{P(x)}$, $\sqrt{P(x)}$, $\sqrt[3]{P(x)}$, $\sqrt[5]{P(x)}$, $P^m(x)$, Log P(x), exp P(x), Trigonometric P(x) e.t.c.. These will be dealt with separately extensively in lecture notes on power series.

The Straight Division method as applied to single variable is applicable to Bipolynomial where two Variables are considered The Authors have extended St. division method where the Bipolynomial (x,y) is written in a particular order of x, x^2,x^3,........,y, xy, x^2y, x^3y,..........., y^2,y^2x, y^2x^2, y^3x^3,........by y^3, y^3x, y^3x^2, y^3x^3,..... in a line.

Both the dividend and the divisor are written in this order.

This is explained in chapter VI on Bipolynomials.

In the result of the division it is considered that.

- Any term whose form is not in the dividend is counted as a remainder.

- Any term which is obtained in the process of division in the corresponding dividend is added to the like term in the dividend. thus modifying the given dividend term. This modified dividend is made use of in the final calculation

- Making use of these two basics the division is carried out.

This method is considered to be most general as it is extendable to polynomial with any number of variables which are also attempted in the present working with three variables (x,y and z).

These methods have been computerized and the computerized results have been included in the lecture notes.

The present investigation is more general method for a polynomial division having any number of variables, where as the method given by Nicholas in their book on vertical – crosswise. is confined only to two variables. A novelty is introduced in the present lecture notes in the sense that the remainders could be further divided successively to obtain the quotient and further remainders accordingly. This is a continuous process and the limit is as per the choice or need of the investigation. It is this part that the authors have exemplified.

In the division of Bipolynomials British authors have used an array system of placement both for dividend and divisor and used the concept of Dhwajanka and part divisor method of Swamiji. They gave worded out quotients term by term and row by row. They worked out the remainders pertaining to each row. Such as 1st row remainders, 2nd row remainders and so on. But this method may not be workable as if is a polynomial having more than two variables. Such it is attempted in this Lecture notes to workout a more general case, the straight division as conceived by Swamiji . Hence the given dividend and divisor of Bipolynomials in general any number of Variables is written in a line and the working is carried out both by Partition method and non – partition method. In the first one, one has the knowledge of the remainder region.

A comparison of the results of method and that by British author's method is also brought out. We feel that the method adopted by us in this lecture notes is most general one having any number of variables either in the dividend or the divisor.

It is interesting to note that the authors have extended the division successively on the remainders considering them as dividends. This is well explained in division of Bipolynomials and method is well computerised.

In the Argumental Division the array method of the Bipolynomials is carried out for the determination of quotients. The details of this procedure and the results are compared in the form of a table with the straight division method. It is suggestive that one can extend the division to obtain the quotient of any order of one's choice.

In chapter VII deals with the computer programming for the following methods. Most of the Computer Programming have been carried out by Mr. I. Hanumanthu under the supervision of Prof. D.S. Murthy. The programming document and execution of results for a few problems in each chapter are given.

SYNOPSIS ON LECTURE NOTES – III(a) – EQUATIONS

This Lecture Notes is confined mainly

I. To solve.

- Simple equations
- Simultaneous simple equations
- Multiple simultaneous equations including special types of equations.
- Quadratic Equations
- Simultaneous Quadratic Equations
- Cubic Equations
- Biquadratics
- Higher degree Equations

II. To explain different methods of Factorization.

The sutras that are used are

- Paravartya Yojayet (Paravartya)
- Sunyam Samya Samuccaye
- Sopantyadwayamantyam
- Antyayoreva (Upasutram)
- Anurupyena Sunyam Anyat
- Sankalana Vyavakalanabhyam

- Adyamadyenantyamanthyena. (Upasutram followed by Anurupyena)
- Lopanasthapanabhyam (Upasutram)
- Purana Apuranabyam ..
- Gunita Samuccayah Samuccaya Gunitah
- Gunaka Samuccayah

CHAPTER I

By using Paravartya Yojayet Sutram, simple equations which are classified into the following different standard forms are deduced, the solution for which on identifying the given problem, can be simply written down . The Standard result is also given.

(i) $ax + b = cx + d$, the solution is $x = \frac{d-b}{a-c}$

(ii) $(x + a)(x + b) = (x + c)(x + d)$, the solution is $x = \frac{cd-ab}{a+b-c-d}$ if $cd = ab$, $x = 0$

(iii) $\frac{ax+b}{cx+d} = \frac{p}{q}$, the solution is $x = \frac{pd-bq}{aq-cp}$

(iv) $\frac{m}{x+a} + \frac{n}{x+b} = 0$, $m + n = 0$ the solution is $x = \frac{-mb-na}{m+n}$ which can be extended to a general equation of the type $\frac{m}{x+a} + \frac{n}{x+b} + \frac{p}{x+c} = 0$

$$\text{if } m+n+p=0$$

the solution is $x = \dfrac{-mbc-nac-pab}{m(b+c)+n(a+c)+p(a+b)}$

if $m+n+p \neq 0$ it becomes Quadratic

This also can be further generalised to any number of terms, with different specific conditions.

$$\frac{m}{x+a}+\frac{n}{x+b}+\frac{p}{x+c}+\frac{q}{x+d}=0,$$

If (a) $\{m+n+p+q\}=0$, (coeff of x^3)

(b) $\{m(b+c+d)+n(a+c+d)+p(a+b+d)+q(a+c+b)\}=0$, then (coeff of x^2)

$$X=\frac{-mbcd-nacd-pabd-qabc}{m(bc+bd+cd)+n(ac+ad+cd)+p(ab+ad+bd)+q(ac+bc+ab)}$$

In addition to the standard form through the vedic mathematical principles (sutras) certain relations and symmetries existing among the components of the equations, could be unearthed leading to the deduction of the solution by a simpler method.

1) The first among them is the Sutram Sunyam Samya Samuccaye, which states that in a total expression, if there is certain similarity (Samyam), then the samyam of the expression is zero leading to the solution of the equations.

The definitions given for such Samya Samuccaye are clearly described in different disguises. In this Lecture notes–III(a), the meaning of these, the application and modus operandi of them together with the deduction of the result, is clearly explained for all the disguises. These cover practically almost all forms of simple equations including seeming cubics or seeming biquadratics of special types and extensions.

Certain equations, though of higher degree such as cubics or Biquadratics, finally result in Simple equations, as such they are seeming Cubics and seeming Biquadratics, but actually turn out to be simple equations.

(i) Under seeming Cubics we can give one general form as $(x-2a)^3+(x-2b)^3=2(x-a-b)^3$

In such a case the solution is $x=a+b$ and is obtained by Sunyam Samya Samuccaye Sutram. This can be claimed as writing the answer at sight, if only the relationship

$(x-2a+x-2b)$ of LHS (without cubes) $=2(x-a-b)$ of the RHS (without cube) is verified. Then the solution is $x-a-b=0$ (samyam = sunyam) $\therefore\ x=a+b$

(ii) In case of seeming Biquadratics of the type

(a) $\dfrac{(x+a+b)^3}{(x+a+2b)^3}=\dfrac{x+a}{x+a+3b}$, $N_1+D_1=N_2+D_2$ (under the cubics)

Where the numerators and denominators are in Arithmetical progression as per a specific degree, (N_1, N_2, D_1, D_2 are in A.P.) can be solved by Sunyam Samya Samuccaye Sutram using the principle of Cross

– addition and the relations existing between numerators and denominators. The value of x is $\frac{-1}{2}$ (2a +3b)

The result can be simply written after a few checks only.

(a) $(x + a)(x + b)(x + c)(x + d) = (x + e)(x + f)(x + g)(x + h)$

or

$$\frac{(x+a)(x+b)}{(x+e)(x+f)} = \frac{(x+g)(x+h)}{(x+c)(x+d)}$$

After certain specific conditions among the constants are satisfied, the answer can be brought out by simple cross addition and by the application of Sunyam Samya Sumuccaye as

$(x + a) + (x + b) + (x + c) + (x + d) = (x + e) + (x + f) + (x + g) + (x + h) = 0$

The conditions to be sastified are clearly worked out in this Lecture Notes–III(a). The solution of such forms can be simply written down after verifying the conditions.

Special type of extension of the Sunyam Samya Sumuccaye Sutram such as

(i) $\frac{x+a}{b+c} + \frac{x+b}{c+a} + \frac{x+c}{a+b} = -3$ $\quad x = -(a + b + c)$

(ii)

$$\frac{x+a^2}{(a+b)(a+c)}+\frac{x+b^2}{(b+c)(b+a)}+\frac{x+c^2}{(c+a)(c+b)}=\frac{x-bc}{a(b+c)}+\frac{(x-ca)}{b(c+a)}+\frac{x-ab}{c(a+b)}$$

$x = (ab + bc + ca)$

$$\frac{x+ab}{b}+\frac{x+bc}{c}+\frac{x+ca}{a}=a+b+c$$

$\therefore$ 0 or ab + bc + ca

Where one can solve the equations by a suitable distribution of the terms, re–orientations, splitting of the terms, by the application of Adyamadyena, addition or subtraction of equal values, completing a cyclic form and suitable compounding of the terms and the like.

Different disguises of the 'Samyam':

These are worked out with the help of certain specific operations like relations existing among the numerators and denominators or among the various components of the expressions in the given equations. Application of paravartya, cross – multiplication, cross – addition, cross – subtraction and identifying special relations like Arithmetical progression, application of Adyamadyena Antyamantyena sutram, merging of one or two terms into other terms of the equations, application of paravartya division, sometimes by mere observation (Vilokanam) etc., to enable one to arrive at further working details which lead to the solution.

Sunyam Samya Samuccaye Sutram

- If there is a 'common' in totality then that is Samyam and is zero.
- In the equation $(x + a)(x + b) = (x + c)(x + d)$, if $ab = cd$, then $x = 0$
- In the equation $\frac{N_1}{D_1} + \frac{N_2}{D_2} = 0$

Where N_1 and N_2 are numbers and D_1 and D_2 are expressions.

If numerators are equal then $D_1 + D_2$ is the Samyam and is zero.

- $\frac{N_1}{D_1} = \frac{N_2}{D_2}$

where N_1, N_2, D_1, D_2 are expressions.

The following relations define Samyam

a) $N_1 + N_2 = D_1 + D_2$

b) $N_1 + D_1 = N_2 + D_2$

c) $N_1 \sim D_1 = N_2 \sim D_2$

d) $N_1 \sim N_2 = D_1 \sim D_2$

Then that equality is Samyam and is zero.

$$\frac{N_1}{D_1}+\frac{N_2}{D_2}=\frac{N_3}{D_3}+\frac{N_4}{D_4}$$

where N_1, N_2, N_3, N_4 are numbers and D_1, D_2, D_3, D_4 are expressions.

If numerators are equal and also if $D_1 + D_2 = D_3 + D_4$, then that is Samyam and is equal to zero.

If numerators are different, then first equate them by L.C.M and then apply the sutram.

If LHS and RHS do not have same number of terms then try for the merging method.

➢ $$\frac{N_1}{D_1}-\frac{N_2}{D_2}=\frac{N_3}{D_3}-\frac{N_4}{D_4}$$

Transpose to get the standard form (5).

(a) $$\frac{x+a}{x+b}+\frac{x+c}{x+d}=\frac{x+e}{x+f}+\frac{x+g}{x+h}$$

Coefficients of x should be same both in the numerator and denominator in each term such that the sum of ratios ($\frac{N}{D}$) of the coefficients of x on the LHS = sum of similar ratios on the RHS. After this test, one has to proceed to the division by Paravartya to convert it into the form (5). Then the sutram Samyam is worked out accordingly.

(b) Even if the coefficients of x are different in any term then also test if the sum of the ratios ($\frac{N}{D}$) of the coefficients of x on the LHS = the sum of the ratios in the RHS. If this condition is satisfied then Paravartya Division is applied to convert it into the form (5). The solution is worked out accordingly.

➢ $$\frac{N_1}{D_1}+\frac{N_2}{D_2}=\frac{N_3}{D_3}+\frac{N_4}{D_4}$$

Where numerators are only numbers. If numerators are not equal then LCM can be considered to make numerators equal. Which will modify the equation.

In the modified equation if $D_1 + D_2 = D_3 + D_4$ then Samyam is applied. Sometime the LCM method may be a bit cumbersome and the relation $D_1 + D_2 = D_3 + D_4$ may not satisfy with the modified denominators which is noticed only at the end. Hence another method is suggested which makes use of certain preliminary test to see that the sum of the ratios ($\frac{N}{D}$) of the coefficients of x on the LHS = that of the RHS. After this test, one has to proceed to another relation namely to see if $N_1D_2 + N_2D_1 = N_3D_4 + N_4D_3$ which is the Samyam and hence zero.

➢ $$\frac{N_1}{D_1}+\frac{N_2}{D_2}=\frac{N_3}{D_3}+\frac{N_4}{D_4}$$

If the numerators also conta:: x, then the test of sum of ratios ($\frac{N}{D}$) of x coefficients on both sides should be equal in order to apply Paravartya Division. The equation has to be converted to the standard form given in (8) and then the sutram is worked out accordingly.

Or

One can even in the beginning test for $N_1 D_2 + N_2 D_1 = N_3 D_4 + N_4 D_3$ with the unmodified equation which is Samyam and is hence zero.

The working details of various disguises are clearly exemplified in this

Lecture Notes–III(a).

Different Merging Operations:

(1) $\frac{m}{x+a}+\frac{n}{x+b}=\frac{m+n}{x+c}$

Merging of the right side fraction into the left is carried out so that only two terms will remain.

After merging $\frac{m(a-c)}{x+a}+\frac{n(b-c)}{x+b}=0$ from this x can be derived

(2) Multiple merger of the type

$$\frac{m}{x+a}+\frac{n}{x+b}+\frac{p}{x+c}+\frac{q}{x+d}+\frac{r}{x+e}+\ldots\ldots = \frac{m+n+p+q+r+\ldots\ldots}{x+w}$$

The multiple merging, should result finally in two terms from which x can be obtained.

$$\frac{m(a-w)\ldots.(a-e)(a-d)(a-c)}{x+a}+\frac{n(b-w)\ldots(b-e)(b-d)(b-c)}{x+b}=0$$

$$\therefore x = \frac{-bm(a-w)\ldots(a-e)(a-d)(a-c)-an(b-w)\ldots(b-e)(b-d)(b-c)}{m(a-w)\ldots(a-e)(a-d)(a-c)+n(b-w)\ldots(b-e)(b-d)(b-c)}$$

(3) Complex Merger

In complex merger, three different forms are considered here and they are

- $\dfrac{m}{ax+b}+\dfrac{n}{cx+d}=\dfrac{p}{ex+f}+\dfrac{q}{rx+s}$
- $\dfrac{ax+b}{cx+d}+\dfrac{ex+f}{gx+h}=\dfrac{px+q}{rx+s}+\dfrac{jx+m}{nx+p}$
- $\dfrac{ax^2+bx+c}{dx+e}+\dfrac{fx^2+gx+h}{ix+j}=\dfrac{kx^2+vx+m}{nx+p}+\dfrac{qx^2+rx+s}{tx+u}$

These are, though harder, by the applications of (a) paravartya sutram, (b) paravartya division in certain cases, (c) LCM, (d) cross multiplication, (e) Sunyam Samya Samuccaye Sutram (f) also using the relations existing among numerators and denominators, (g) Viloknam etc., solvable very easily but after verifying certain specific tests which exist among various relations of the components as described in disguises. In addition a few specific examples

are also worked out coming under each category elaborately in this Lecture Notes–III(a).

- Certain special types of simple equations of several forms are also dealt with

as give below:

(i) $\frac{p}{(x+a)(x+b)}+\frac{q}{(x+b)(x+c)}+\frac{r}{(x+c)(x+a)}=0$

(Cyclic degree)

The solution of this is

x = <u>Sum of each numerator multiplied by the absent number of its denominator reversed</u>

Sum of Numerators

$$x=\frac{-(pc+qa+rb)}{p+q+r}$$

ii) $\frac{1}{AB}+\frac{1}{AC}=\frac{1}{AD}+\frac{1}{BD}$, where A, B, C, D are the binomials in Arithmetical

progression, is solved by the application of sopantyadvayam antyam (2P+ L) sutram.

Arithmetical progression in A,B,C,D, may be either in independent terms or coefficients of x or in both.

One Example:

$$\frac{1}{(x+3)(x+5)}+\frac{1}{(x+3)(x+7)}=\frac{1}{(x+3)(x+9)}+\frac{1}{(x+5)(x+7)}$$

A B A C A D B C

By sopantyadvayam antyam 2(x+7)+ (x+9) = 0 $\Rightarrow x = \frac{-23}{3}$

(P = penultimate, here it is x+7 L = ultimate, here it is x+9)

(iii) The following special type having terms with the specific relation in an equation where A,B, C are binomials, D and E are numbers.

$$\frac{AC+D}{BC+E}=\frac{A}{B} \text{ can be}$$

solved by Antyayoreva sutram $\frac{A}{B}=\frac{D}{E}$

Example: $\frac{2x^2+3x+6}{x^2+5x+8}=\frac{2x+3}{x+5}$;

$$\frac{x(2x+3)+6}{x(x+5)+8}=\frac{2x+3}{x+5}$$

$$\frac{2x+3}{x+5}=\frac{6}{8}\Rightarrow x=\frac{3}{5}$$

(iv) Consideration of Special summation of series in case of fraction – additions where denominators are in Arithmetical progression, (which can be among independent terms or coefficients of x or both) for writing down the summation of any number of terms using Antyayorevasutram in different context is explained.

Example:

Find the Sum S_3 of

$$\frac{1}{(x+3)(x+5)}+\frac{1}{(x+5)(x+7)}+\frac{1}{(x+7)(x+9)}+---$$

By Antyayoreva Sutram $S_3 = \dfrac{3}{(x+3)(x+9)}$

(v) Summation of another special type of series, where the difference between the two binomial factors of the denominators is the numerator, Summation of this series can be also dealt with Antyayoreva Sutram.

Example:

Find the Sum of

$$\frac{1}{(x+2)(x+3)}+\frac{3}{(x+3)(x+6)}+\frac{4}{(x+6)(x+10)}$$

By Antyayoreva Sutram $S_3 = \dfrac{8}{(x+2)(x+10)}$

All these come under special type of simple equations where different types of Sutras and subsutras could be applied to solve the equations practically at a glance. The algebrical proofs for the sanctity of the application of these sutras are also clearly brought out, in Lecture Notes. A few more examples are worked out in the notes.

Simultaneous simple equations:

In case of simultaneous simple equations, solving by Vedic method is attempted in three ways.

- ✓ paravartya rule following cyclic degree is considered to be very elegant and is explained in this Lecture Notes–III (a) clearly.

- ✓ The Sutram Sunyamanyat is applicable only when the ratio of one of the variables is equal to the ratio of independent terms. The result of this application is that the values of the other variables are invariably zero. This is extendable for any number of variables

- ✓ By using the Upasutram Sankalanavyavakalanabhyam, one can obtain the solution.

Multiple simultaneous equations (Equations containing three or more variables):.

These Equations are solved by Lopanasthapanabhyam Sutram, Anurupyena, Paravartya, cross–multiplication.

Multiple simultaneous equations are classified into two types.

- ✓ In the first type two or more of the equations have zero on the RHS, where as only one equation has significant number. In this type one can obtain the values by Lopanasthapanabhyam or by paravartya.

- ✓ In the second type all the equations have significant figures on the RHS. In this case three methods are applied to get the solution and these are clearly explained in this Lecture Notes–III(a).

Example:

$$x - 2y + 3z = 2 \quad \text{——— A}$$

$$2x - 3y + z = 1 \quad \text{——— B}$$

$$3x - y + 2z = 9 \quad \text{——— C}$$

(a) To derive two zero equations using the above three equations by way of combinations i.e., suitable subtraction/ addition. After this, it is treated as the 1st type, by clubbing them with any of the given equations.

$$\left.\begin{array}{ll} \text{From } 2B - A & 3x - 4y - z = 0 \\ \text{From } 9B - C & 15x - 26y + 7z = 0 \end{array}\right\} \text{are obtained}$$

(b) From any two of the given equations, two more equations are derived with two unknowns on the LHS, with the remaining unknown and the constant on the RHS. Then the RHS is treated as the final constant.

The two Simultaneous equations are solved by the paravartya method

The third variable is obtained by Substitution.

Paravartya Method (Direct):

$$\left.\begin{array}{ll} \text{From A} & x - 2y = 2 - 3z \\ \text{From B} & 2x - 3y = 1 - z \end{array}\right\} \text{are derived}$$

Applying paravartya to the two equations.

x y constant and z terms are considered as one unit.

$$\begin{array}{ccccc} 1 & \times & -2 & \times & (2-3z) \\ 2 & & -3 & & (1-z) \end{array}$$

$$x = \frac{(-2)(1-z)-(-3)(2-3z)}{(-2)2-1(-3)} = 7z-4$$

$$y = \frac{(2-3z)2-(1-z)1}{(-2)2-1(-3)} = 5z-3$$

substituting x, y in c, $z = 1$

$x = 7z - 4 = 3$; $y = 5z - 3 = 2$

$x = 3$, $y = 2$, $z = 1$

(c) Lopanasthapanabhyam i.e., elimination of variables one after another successively to culminate into two *simultaneous equations in two unknowns* which are subjected to paravartya.

Further extension of these methods can be clearly worked out based on these above principles and is extendable to any number of unknowns.

Example:

$2x - y + 3z + 4w = 25$ ——A

$x + 2y - z + 2w = 10$ ——B

$5x - 3y + 3z - 3w = -7$ ——C

$-x + 4y - 4z + w = -1$ ——D

Paravartya Method: (Direct Application) by

Considering any two of the given equations.

From A $\quad 2x - y = 25 - 3z - 4w$ ———— O

From B $\quad x + 2y = 10 + z - 2w$ ———— S

Applying paravartya to O and S

$$\begin{array}{ccc} x & y & \text{constant term} \\ 2 & -1 & 25 - 3z - 4w \\ 1 & 2 & 10 + z - 2w \end{array}$$

$$x = \frac{(-1)(10 + z - 2w) - 2(25 - 3z - 4w)}{(-1)1 - 2(2)} = 12 - z - 2w$$

$$y = \frac{(25 - 3z - 4w)1 - (10 + z - 2w)2}{(-1)1 - 2(2)} = z - 1$$

Substituting x,y in C $\quad 5z + 13w = 70$ ———— T

Substituting x,y in D $\quad z + 3w = 15$ ———— U

Applying paravartya to T and U

$$\begin{array}{ccc} z & w & \text{constant term} \\ 5 & 13 & 70 \\ 1 & 3 & 15 \end{array}$$

$$z = \frac{(13)(15) - 3(70)}{(13)1 - 5(3)} = \frac{15}{2}$$

$$w = \frac{70(1) - 15(5)}{13(1) - 5(3)} = \frac{5}{2}$$

$$y = z - 1 = \frac{13}{2}\ ; \qquad x = 12 - z - 2w = -\frac{1}{2}$$

$\therefore\ x = -\frac{1}{2}, y = \frac{13}{2}, z = \frac{15}{2}, w = \frac{5}{2}$

All the above details are exemplified very clearly in this Lecture Notes.

It is felt that there is definitely an ease in solving the equations with the help of the sutras. The simplicity of the working is striking.

The full working details of all the above are elaborately described in this Lecture Notes–III(a).

An attempt is made to formulate the equations in many unknowns belonging to different classes such those, which give raise to an unique solution, or by infinite number of solutions, in consistent having no solution, trivial and non–trivial solution.

It is also attempted, a conversion of one set of equations belonging to one class to another set of equations belonging to another class and vice – versa. For solving the equations Vedic Sutras are applied and the necessary conditions for conversion are derived form determinants full details are given in the Section 3.

CHAPTER II

Quadratic Equations :

The Quadratic Equations are solved mainly by using two relations 1) By Adyamadyena and Antyamantyena and 2) The most general is by using the relation between the first differential and the discriminant as $D_1 = \pm \sqrt{Discriminant}$. In the first relation by using Adyamadyena Sutram, the second term is split into 2 pats in such a manner that the ratio of the first term of Quadratic Equation nd the split term (Adyamadyena) is equal to the ratio of the ıst term of the split to the last term of Quadratic Equation. /hile this gives the first factor and the second factor is ɔtained by using the same Adyamadyena and ntyamantyena Sutram by the first factor and the Quadratic. ıe working details are explained fully under the Quadratic ection.

‹ample : $x^2 + 7x + 12$

$$x^2 + 3x + 4x + 12 = 0$$

$$\frac{x^2}{3x} = \frac{4x}{12} \qquad \therefore\ x + 3 \text{ is first factor}$$

$$\frac{x^2}{x} = \frac{12}{3} = x + 4 \qquad \text{is second factor}$$

The second method is considered to be most neral in deriving the first derivative and then equate it to › root of discriminant. By the 2[nd] method one can simply ite down the two values immediately.

ample : $x^2 + 7x + 12 = 0$

$$2x + 7 = \pm\sqrt{49 - 48} = \pm 1$$

$$\therefore\ x = -3, -4$$

There are special types which make use of the ciprocal relations, the various relations between the merator and denominator are already explained under ınyam Samya Samuccaye as applied to simple equations, › merger type already explained under simple equations.

Example : $\dfrac{x+3}{2x-7}=\dfrac{2x-1}{x-3}$

$N_1 + D_1 = 3x - 4$

$N_2 + D_2 = 3x - 4$

$\therefore$ By Sunyam Samya Samuccaye Sutram

$3x - 4 = 0 \Rightarrow x = \dfrac{4}{3}$ one solution

$N_1 \sim N_2 = x - 4$

$D_1 \sim D_2 = x - 4$

$\therefore$ By Sunyam Samya Samuccaye Sutra, x = 4 another solution.

$\therefore\ x = 4, \dfrac{4}{3}$

Simultaneous Quadratic Equations :

This have been solved by using the different sutras Vilokanam, methods adopted for simple Quadratic Equation's, substitution method, methods converting into standard equation form, factorisation, Sunyamanyat, Paravartya and the like.

Swamiji has illustrated a number of examples of different forms of Simultaneous Quadratic Equations. The Authors have given working details of different examples.

The reversal phenomenon of obtaining the solutions and identifying one of the variables as zero in certain cases where the same ratio exist between the constants and one of the variables of the Simultaneous Quadratic Equations, are considered to be Surprisingly simple.

It is also seen that the working details are found to be simple by considering a set of factors for xy by Vilokanam and then they are tested for their combination explaining the given equation. This approach is considered to be simpler.

Example : $x + y = 28$

$$xy = 187$$

The set of factors of 187 are (1, 187), (11, 17) out of these if we select (11, 17) it will satisfy the given simple equation by mere Vilokanam. Hence one set of values of x and y are 11 and 17.

Other set can be simply written down by reversal operation as $x = 17$ and $y = 11$.

The method of cross-multiplication together with the cross addition and cross subtraction enables us to write down two equations into one equation which can be solved for. The working details are fully explained under the simultaneous Quadratic Equations Section. Different procedures adopted by Swamiji are detailed in this section.

CHAPTER III

Factorization of simple Quadratics, harder quadratics, Simple Cubics higher degree equations using Several Sutras and upa Sutras and specific relations are explained with specific examples.

- ✓ Vilokanam
- ✓ Adyamadyena Antyamantyena

- ✓ Anurupyena
- ✓ Argumentation
- ✓ Gunita Samuccayah Samuccaya Gunitah
- ✓ Paravartya yojayet
- ✓ Lopana Sthapanabhyam
- ✓ Purana Apuranabhyam
- ✓ Differential Relations
- ✓ Successive Differentiation

The ease with which Cubics and higher degree equations could be solved using Vedic Methods is significant.

Existing methods such as Cardon's Method, Descarte's Method are considered to be Laborious where as the methods adopted by Swamiji are very elegant and simple. This can be seen even in the section (12) by way of comparison between the two methods.

In solving the Quadratic equations, a general formula such as first differential of Quadratic expression, $D_1 = \pm\sqrt{Discriminant}$ is extremely helpful in writing down the values of x.

Example 1 : $x^2 - 4x + 2 = 0$

$$D_1 = \quad 2x - 4 = \pm\sqrt{16-8}$$

$$2x - 4 = \pm\sqrt{8} \Rightarrow x = 2 \pm \sqrt{2}$$

Example 2 : $x^2 + x + 3 = 0$

$$D_1 = 2x + 1 = \pm\sqrt{1-12} \Rightarrow x = \frac{-1 \pm \sqrt{11}\, i}{2}$$

Solving Quadratic Equation using Adyamadyena Antyamantyena followed by Anurupyena is really appreciable. The method is simple. One should split the x coefficient into two parts such that the ratio of Adyamadyena which is the ratio of x^2 term to the first split x term should be equal to the ratio of Antyamantyena which is the ratio of the second split part of x term to the last (constant term) term. In such a case after simplification, the sum of the numerator + denominator is the first factor of the Quadratic Equation.

This factor is used in determining the second factor by applying Adyamadyena Sutram in the sense, that the ratio of the first term of the Quadratic Equation to the first term of the derived factor + the ratio of last term of the Quadratic Equation to the last term of the derived factor.

Example $x^2 + 9x + 20 = 0$

$\Rightarrow x^2 + 5x + 4x + 20$

$\frac{x^2}{5x} = \frac{4x}{20}$ (which is Anurupyena)

$\Rightarrow$ (x + 5) is a factor.

$\frac{x^2}{x} + \frac{20}{5} = (x + 4)$ is another factor

Coming to the Cubic Equations, several Sutras are found helpful in solving them. The first one is Vilokanam i.e. either by inspection or by trial. By inspection, if the Sum of the coefficients S_c of the Cubic equation vanishes then $(x - 1)$ is a factor of the equation. If the sum of the coefficients of odd powers S_0 = Sum of the coefficients of even powers S_e then $(x + 1)$ is a factor. We can get the information using the above relations. Then (1) we can divide, by Paravartya Method, the Cubic Equation by the above to get the Quadratic Equation. But Swamiji has extended the reduction of the highest degree into the product of two or more lower degree expressions i.e. more simply (2) We can use the method of Argumentation followed by Gunita Samuccayah, Samuccaya Gunitah Sutram to arrive at the Quadratic equation. or (3) We can also directly factorise based on the highest degree of the equation.

The general Cubic equation considered is $(x + a)(x + b)(x + c) = x^3 + x^2(a + b + c) + x(ab + bc + ca) + abc$ where $-a, -b, -c$ are the roots of the Cubic equation and the factors are $(x + a)(x + b)(x + c)$. Factorization is attempted by finding out the probable combination of three factors (a, b, c) of the constant term which explains the coefficients of x^2 and x of the above cubic equation. Full details are described in the Section on Factorization with Examples

or

More elegantly we can aim at solving the Equations by using a very important relation which is called as 'Gunita Samuccayah Samuccaya Gunitah' which states that, "the sum of the coefficients of all the terms of the given equation is equal to the product of the sum of the coefficients in each factor of the equation." For both the second and third methods, one can arrive at first and last terms of quadratic Equation by using Adyamadyena Sutram as explained in all problems. The Quadratic Equation thus derived is further solved for its roots. These methods are much simpler than the current methods as clearly shown in the section (12).

The application of Adyamadyena Sutram is extended to any higher degree equation also.

It is clearly understandable that these methods are applicable to higher degree equations as well (Refer section11).

It is noted that these methods are easily workable when the solutions are integers, + ve or – ve. But when the solutions happen to be of decimal values, such problems are separately dealt with in the next Lecture Notes III (b) elaborately. However the complex roots can be deducable at the stage of solving Quadratic Equations which are also illustrated.

Lopana Sthapanabhyam is the sutram, wherein one or many unknowns are to be eliminated to establish the others. The method is clearly given in solving the simple simultaneous equations and also multiple equations with many unknowns.

The cubic and higher degree equations can also be solved by an application of the Sutram 'Purana Apuranabhyam'. This is by considering a suitable standard form of $(x + a)^3$ and rewriting the expression with the help of Paravartya sutram and Purana sub form, which process eliminates the Square term in the cubic equation. Similarly by such process, in solving the fourth degree equation, the cubic term can be eliminated and so on. One can apply this principle to higher degree equations also.

The full working details for Cubics, biquadratics are given in the section (10) of the text.

The sutram by Vilokanam is of special importance in the sense that mere inspection and trial, one may be able to arrive at solutions of the equations, particularly when they are integers. It is also possible that in certain cases one can arrive at the regions of solutions. In such case one has to specifically work out the exact value of the solutions to a particular range of decimal using different Technique, which is separately dealt with in the next Lecture Notes III (b)

Attempt by Vilokanam method is detailed in section (6) of the text. However it cannot reveal the repetition of factors, if any, of the equation.

Factorization can also be carried out using differential calculus (calana – kalana).

In the general cubic equation, where (x + a) (x + b) (x + c) are factors, it can be shown that the first differential D_1 = Sum of the product of the factors taking two at a time.

Second differential D_2 = 2! (sum of the factors)

In a Quadratic Equation, first differential = Sum of the factors.

This concept is easily extendable to the equations of any higher degree. Successive differentials and their relation with the factors of the equation are worked out. Swamiji, with the help of successive Differentiation, worked out for the repetition of the factors. The method of finding out the repeated factors are explained clearly in the section(9) of the text..

In the Lecture Notes III (b) concentration is aimed at

- The methods for square roots, cube roots and higher degree roots as a general procedure for numbers and polynomials are described.
- The cubic and higher degree equations could be solved wherein the roots are irrational, a decimal range is also worked out.

- Roots of polynomials in the ascending (or) descending degree could be worked out.

The methods given by Swamiji are found to be extended to any higher degree equations and to any decimal point. These form the major part of the next Lecture Notes III (b)

SYNOPSIS ON LECTURE NOTES – III (B)* – SQUARES, CUBES, EXPANSIONS, ROOTS (NUMBERS AND POLYNOMIALS) AND EQUATIONS (Contd.)

The Lecture Notes is divided into 4 parts and each part has its Sections

Part I consists of two Sections A and B

Section A deals with Squares and Square roots of numbers and Polynomials. The Sutras, that are used for Squares are Yavadunam, Anurupyena, Duplex (Dwandvayoga) concept. The Straight Division method is used for the determination of Square roots of both the Numbers and Polynomials. A comparison of existing method with the method developed by Swamiji is also clearly brought out.

The ease and elegance is significantly noticed in Swamiji's method of determining the Squares and Square Roots.

Section B deals with two units. The first unit is confined to Cubes and General Expansions and the Second, to Cube Roots and Higher Roots of Numbers and Polynomials in one Variable.

Starting with a General Introduction to the topic for the Section B, the methods adopted for obtaining cubes using two sutras Yavadunam and Anurupyena are explained.

* Volume - IV

The method of expansion, in general-having, a number of elements a,b,c and raised to a positive Integer Power is detailed following Swamiji's Symmetry Principle and Homogeneity. This is worked out on the basis of groups of terms having different coefficient for each group, but the same coefficient for all the terms coming under one group. The method of determination of coefficients is clearly worked out. An attempt is made for the placement of expansion terms based on their original placements in the identified form in the number for example (...c + b + a) or (a + b + c...).

A number of tables required for the working of problems in roots are detailed in the text. Some of the tables are shown in the text and the remaining in the Part IV. It is interesting to note that these tables can be used for determination of roots, decimal working and also elemental expansions wherein each element can be represented / replaced by a function. In general, from these tables, one can directly read out the subtraction terms necessary for working out the roots including higher order. These are shown in an exhaustive manner with an intention that, one can adopt such procedures easily to other higher orders as well. It is also serves a good Algorithm for computer programming.

The second unit of Section B deals with Cube Roots and Higher Roots for both the Numbers and Polynomials. This is again divided into two parts, the first dealing with Swamiji's methods and the second with Taylor's Expansion Method as described by British Authors.

- Swamiji's Methods for determining the Cube Roots are (1) Argumentation method (... JKL Method) (2) Straight Division Method.

The 1st Method is applicable only for Perfect Cubes, whereas the 2nd Method (Straight Division Method) can be applied to perfect and imperfect cubes as well. The working details of Argumentation (...JKL Method) are given in the text elaborately.

These methods are also applicable to the higher order Roots. All these are exemplified in the Part I. A brief description of Swamiji's Method for Determination of Cube Root is given in the Annexure in Part IV.

The subtraction terms in working out the roots are taken from the concerned expansion tables (Tables A to L are in the text and Tables M to U are in Part IV)

- Taylor's Expansion Method described by British Authors is described in the text along with the working details. A comparison of both the methods is clearly brought out in a number of examples.

- The problems are worked to about 8 decimal range noticing clearly the usefulness of increasing the decimal range particularly in the accuracy of the result. These are encouragingly worked out specifically from the point of view of algorithm for the purpose of Computer Programming.

  The same methods are applied in determining the Roots of Polynomials written in any order (ascending (or) descending powers). But the methods make use of an important principle that the carrying out of the remainder does not exist as the working details are chosen for zero remainders.

Part II is an extension of the work presented in the Lecture Notes III(a) under Equations.

It consists of extension of solving of cubic and higher order equations using two different methods.

Section C deals with Application of Swamiji's method to solve all the solutions of the cubic equations.

Section D describes the Taylor's expansion as envisaged by British Authors coupled with Swamiji's concepts of Duplex and Triplex terms to solve one of the solutions.

Section E deals with all the solutions of 4^{th}, 5^{th} and 8^{th} Degree Equations.

i. A combination of Swamiji's method using Vilokanam, Argumentation and Taylor's expansion method for the refinement value for 4^{th} Degree equation.

ii. For the 5^{th} Degree Equations one solution is arrived using both the methods. The refined value for one solution is finally considered for reduction to 4^{th} Degree Equation, which is solved completely by Swamiji's method Purana Apuranabhyam and Argumentation.

iii. It is noticed that the refinement of solutions by substitutions method can also be worked successfully and accurately using Swamiji's Method. For the 8^{th} Degree Equation using only Swamiji's Method through Vilokanam and other sutras all the solutions could be arrived. The solutions are compared with the results of Taylor's Expansion Method.

Sutras that are used are

(1) Vilokanam (2) Adyamadyena Antyamantyena (3) Argumentation (4) Gunita Samuccaya Samuccaya Gunitah (5) Purana Apuranabhyam (6) Duplex concept (7) Expansion Tables using Swamiji's method.

Part III

A novel additional feature is, the extension of determination of roots of polynomials consisting of any numbers of variables using Swamiji's Method.

Part IV

This part consists of two Annexures. Annexure I deals with the description of Swamiji's Straight Division Method. Annexure II deals with the tables from M to U.

SYNOPSIS ON LECTURE NOTES – V – RECURRING DECIMALS, AUXILIARY FRACTIONS, DIVISIBILITY AND THEORY OF OSCULATION, POWER SERIES, PARTIAL FRACTIONS, DIFFERENTIAL EQUATIONS INCLUDING PARTIAL, LOGARITHMS & EXPONENTIALS, TRIGONOMETRIC FUNCTIONS INCLUDING INVERSE AND HYPERBOLIC, HCF

Section –1 deals with Recurring Decimals using several Vedic formulae. Full working details are clearly shown for each formula applied. The Vedic code language as explained by Swamiji is also enumerated with examples.

In Section –2 a new concept of Auxiliary fractions (called as Sahayaks) as introduced by Swamiji is explained. The method is simpler and the introduction of Auxiliary fractions, vulgar fractions and the modus operanda of the method are clearly explained. The working when carried out in comparison with the current method is strikingly significant which makes use of the sutras such as Ekadhikas etc. Vedic code method is clearly demonstrated.

Section –3 is another innovative, introduced by Swamiji, of the concept of osculation (theory of osculators, simple, complex, multiplex osculators), which are termed as Vestanas. Swamiji has explained is very interesting question as to how one can determine whether a certain given number, (it may be any long number) is divisible by the given number using Vedic method. The concept of theory of osculation, positive osculator, negative osculator, complex osculator, multiplex osculator has been well defined and applied to determine the divisibility. Quite a good number of problems are worked out.

Section-4 deals with a few problems worked out using both the current method and Vedic method. (Integrations by partial fractions).

Section-5 deals with powers of polynomials. Quite a good number of problems are worked out. The method making use of logarithms and differentiation appears to be elegant.

Section-6 deals with evaluation of logarithms and exponentials. The method is a continuation of the one described in power series.

Section-7 deals with the evaluation of Trigonometric, Hyperbolic and Inverse functions, which could be considered for every small angles as an approximation. The details of a Simpler method are worked out in appendix for the inverse sin, cos and tan functions. This method is also very approximate, particularly for small angles.

Section-8 deals with Differential equations. An attempt is made to solve the particular integral in a few types of differential equations. The method is considered to be novel in the sense that a comparison with current method gives a very surprisingly simple at work. We have extended to study the partial differential equations; the working details of which elegant suggested by the British author. It is felt that a more comprehensive and extensive work on the Differential equations is worthy of attempt. The Director proposes to take it up at a later stages.

Section –9 deals with HCF. It is interesting in the sense that the working by using Vedic methods is worthwhile practicing.