Vedic Mathematics

Lecture Notes – 2

Division

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VEDIC MATHEMATICS
OR SIXTEEN SIMPLE MATHEMATICAL FORMULAE

SIXTEEN SUTRAS AND THEIR COROLLARIES

Sūtras	*Sub-Sūtras or Corollaries*
1. एकाधिकेन पूर्वेण *Ekādhikena Pūrveṇa* (also a corollary)	1. आनुरूप्येण *Ānurūpyeṇa*
2. निखिलं नवतश्चरमं दशतः *Nikhilaṃ Navataścaramaṃ Daśataḥ*	2. शिष्यते शेषसंज्ञः *Śiṣyate Śeṣasaṃjñaḥ*
3. ऊर्ध्वतिर्यग्भ्याम् *Ūrdhva-tiryagbhyāṃ*	3. आद्यमाद्येनान्त्यमन्त्येन *Ādyamādyenāntya-mantyena*
4. परावर्त्य योजयेत् *Parāvartya Yojayet*	4. केवलैः सप्तकं गुण्यात् *Kevalaiḥ Saptakaṃ Guṇyāt*
5. शून्यं साम्यसमुच्चये *Śūnyaṃ Sāmyasamuccaye*	5. वेष्टनम् *Veṣṭanam*
6. (आनुरूप्ये) शून्यमन्यत् *(Ānurūpye) Śūnyamanyat*	6. यावदूनं तावदूनम् *Yāvadūnaṃ Tāvadūnaṃ*
7. संकलनव्यवकलनाभ्याम् *Saṅkalana-vyavakalanābhyam* (also a corollary)	7. यावदूनं तावदूनीकृत्य वर्गं च योजयेत् *Yāvadūnaṃ Tāvadūnīkṛtya Vargañca Yojayet*
8. पूरणापूरणाभ्याम् *Pūraṇāpūraṇābhyām*	8. अन्त्ययोर्दशकेऽपि *Antyayordaśake'pi*
9. चलनकलनाभ्याम् *Calana-Kalanābhyām*	9. अन्त्ययोरेव *Antyayoreva*
10. यावदूनम् *Yāvadūnam*	10. समुच्चयगुणितः *Samuccayaguṇitaḥ*
11. व्यष्टिसमष्टिः *Vyaṣṭisamaṣṭiḥ*	11. लोपनस्थापनाभ्याम् *Lopanasthāpanābhyām*
12. शेषाण्यङ्केन चरमेण *Śeṣāṇyaṅkena Carameṇa*	12. विलोकनम् *Vilokanam*
13. सोपान्त्यद्वयमन्त्यम् *Sopāntyadvayamantyam*	13. गुणितसमुच्चयः समुच्चयगुणितः *Guṇitasamuccayaḥ* *Samuccayaguṇitaḥ*
14. एकन्यूनेन पूर्वेण *Ekanyūnena Pūrveṇa*	
15. गुणितसमुच्चयः *Guṇitasamuccayaḥ*	
16. गुणकसमुच्चयः *Guṇakasamuccayaḥ*	

(Editor of the original book on Vedic Mathematics)

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DIVISION

Chapter I

I. By Nikhilam Rule:

(a) Special cases of dividing with 9, 8, 7 and 6 are dealt with here

Special Case 1: (Divisor is 9)

Vedic Method Steps are as follows

1) Partition the given number (dividend) into two parts. The second part is to be provided one digit place. This represents the remainder part whereas the first part gives the quotient. The first part may contain more than one digit. First / Second
(Quotient) / (Remainder)
2) The second step is to put down the first digit in the first part as it is as a part of the answer
3) Then it is carried out to the next digit lying either in the quotient part or the remainder part as the case may be. After this carrying out, an addition takes place The process is continued till the addition finally takes place in the remainder column

Examples clearly show the above method.

I) Remainder is less than the divisor, 9.

Examples:

i) 32 ÷ 9

Current Method

9) 32 (3
27
5

Vedic Method

9) 3 / 2
/ 3
3 / 5

Quotient = 3
Remainder = 5

Example:

(i) 27 ÷ 9

Current Method

9) 27 (3
27
0

Quotient = 3
Remainder = 0

Vedic Method

9) 2 / 7
/ 2
2 / 9
1 / 0

(Vilokanam)

Quotient = 2 + 1 = 3
Remainder = 0

1) In case the remainder is more than the divisor and has two or more digits, then it has to be treated as new dividend
2) The first process of partitioning the new dividend into the quotient and remainder parts is continued which is followed by division until finally the remainder comes out as a value less than the divisor (Refer example ii below)
3) All the additional quotients thus obtained in series are to be added to the original quotient (Refer example iii page 4)
4) If we get two digits as a single unit in the answer, then the first digit is added to the previous one. This is clearly shown in examples below (Whenever two or more than two digits are obtained as a single unit in Vedic Mathematical operations, retaining only the last digit, all the remaining digits are transferred to the immediate left hand position by addition) Refer example iii page 4.

(ii) 368 ÷ 9

Current Method

9) 368 (40
360
8

Vedic Method

9) 36 / 8
3 / 9
39 / 1 / 7
/ 1
1 / 8

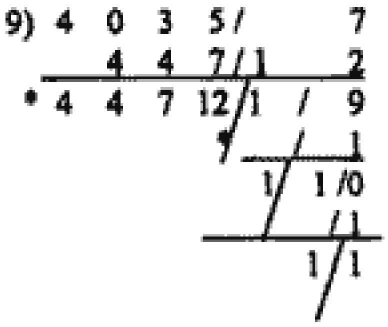
Quotient = 39 + 1 = 40, Remainder = 8

(iii) 40357 + 9

Current Method

```
9) 40357 ( 4484
   36
    43
    36
     75
     72
      37
      36
       1
```

Vedic Method



Quotient = 4482 + 1 + 1 = 4484,
Remainder = 1

* If in the quotient one gets more than one digit then one has to carry to the previous digit all the digits excepting the last digit.

* i.e., 4 4 7 12 = 4482

Special Case 2: (Divisor is 8)

(1) First two steps : (a) concerned with the partition of the dividend and (b) for obtaining the first digit in the quotient are common as for the divisor 9

(2) The third step is to carry out twice the first quotient digit to the next digit either in the quotient place or the remainder place as the case may be.

(3) Then the process is continued as explained in the first case (divisor 9). Examples are given below

(i) 31 + 8

Current Method

```
8) 31 (3
   24
    7
```

Quotient = 3
Remainder = 7

Vedic Method

```
8) 3 / 1
     / 6
   3 / 7
```

Quotient = 3
Remainder = 7

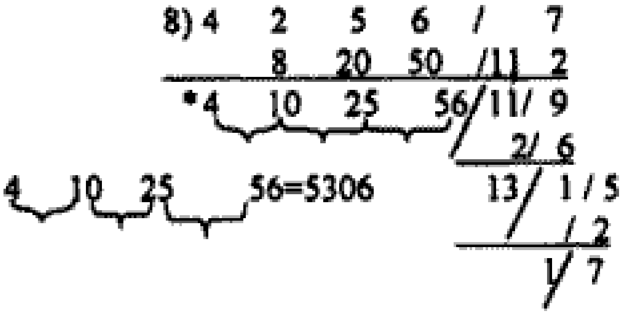
(ii) 42567 ÷ 8

Current Method

```
8) 42567 (5320
   40
    25
    24
     16
     16
      07
```

Quotient = 5320
Remainder = 7

Vedic Method



Quotient = 5306 + 13 + 1 = 5320
Remainder = 7

Special Case 3: (Divisor is 7)

(1) Also in the case of divisor 7, the first two steps . (a) concerned with the partition and (b) for obtaining the first digit in the quotient are common as in the case of divisor 9.

(2) In the third step the first quotient is multiplied by 3 Then the process of carrying over the result of multiplication to the quotient part / reminder part is continued
(3) Then the process is continued as given in the first case (divisor 9). This is clearly shown
in examples below

Examples:

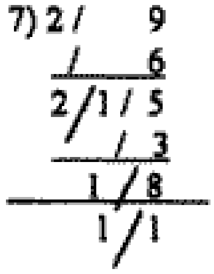
(i) 29 ÷ 7

Current Method

```
7) 29 (4
   28
    1
```

Quotient = 4
Remainder = 1

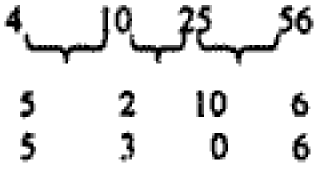
Vedic Method



(Vilokanam)

Quotient = 2 + 1 + 1 = 4
Remainder = 1

* Simplification



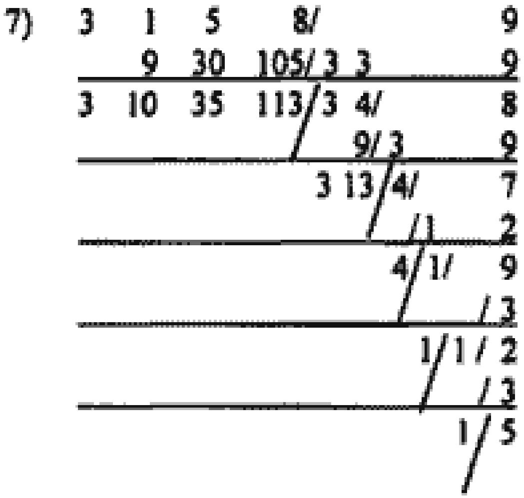
(ii) 31589 ÷ 7

Current Method

7) 31589 (4512
28
35
35
08
7
19
14
5

Quotient = 4512
Remainder = 5

Vedic Method



Quotient = 4463 + 43 + 4 + 1 + 1 = 4512
Remainder = 5

Special case 4: (Divisor is 6)

(1) Also in the case of divisor 6, the first two steps are same as in the case of divisor 9

(2) But the corresponding multiplier in the third step is 4

(3) This is again followed by the same procedure as in the case of the divisor 9 The following examples are self-explanatory

Examples:

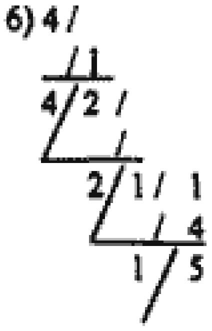
(i) 47 ÷ 6

Current Method

6) 47 (7
42
5

Quotient = 7
Remainder = 5

Vedic Method



Quotient = 4 + 2 + 1 = 7
Remainder = 5

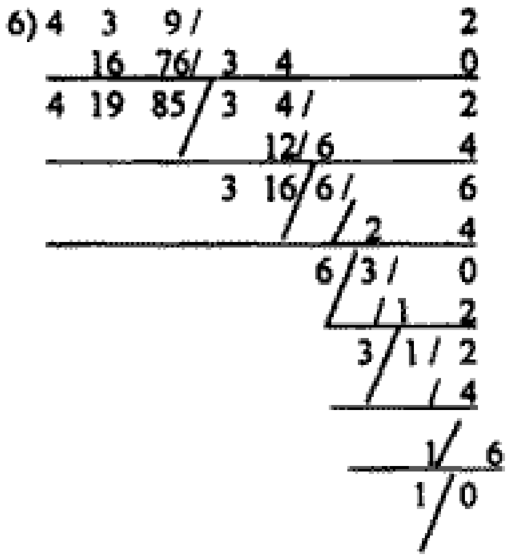
ii) 4392 ÷ 6

Current Method

```
6) 4392 (732
   42
    19
    18
     12
     12
      0
```

Quotient = 732
Remainder = 0

Vedic Method



Quotient = 675 + 46 + 6 + 3 + 1 + 1 = 732
Remainder = 0

The proofs for these divisions are worked out by following the polynomial form in x (x = 10)

(A) $(x - a)\big)\, bx^3 + cx^2 + dx + e\big(\,(bx^2 + x(c + ab) + d + a(c + ab)$

$\underline{bx^3 - abx^2}$

$(c + ab)x^2 + dx$

$(c + ab)\underline{x^2 - ax(c + ab)}$

$dx + ax\,(c + ab) + e$

$\underline{dx - ad}$

$ax(c+ab) + e + ad$

$\underline{ax(c+ab) - a^2(c+ab)}$

$e + ad + a^2(c + ab)$

$= e + a[d + a(c + ab)]$

(B) $(x - a)\big)\, bx^2 + cx + d\,\big((bx + (c + ab)$

$\underline{bx^2 - abx}$

$(c + ab)x + d$

$\underline{(c + ab)x - (c + ab)a}$

$d + (c + ab)a$

(C) $x\text{-}a)\big)bx+c\big((b$

$\underline{bx-ab}$

$\underline{c+ab}$

Depending on the remainder further division takes place.

Proof:

where x is base, i.e , 10
In case of 9, 'a' becomes 1, i.e., the value obtained on application of *Nikhilam Sutram to 9
(* refer to Lecture notes I Vedic Mathematics on Multiplication).
In case of 8, 'a' becomes 2, i.e., the value obtained on application of Nikhilam Sutram to 8
In case of 7, 'a' becomes 3, i e , the value obtained on application of Nikhilam Sutram to 7
In case of 6, 'a' becomes 4, i e., the value obtained on application of Nikhilam Sutram to 6

So we are multiplying in the third step the first quotient digit by 1, 2, 3 and 4 respectively
Considering example (ii) in the special case 4 when divisor is 6 (page No 7).
Applying equation (A) a=4, b=4, c=3, d=9, e=2
The quotient is 675
The remainder is 342
This is $3x^2+4x+2$, and Applying equation B. (b = 3, c = 4, d = 2)
Quotient is 46 remainder is 66.

This is again written as 6x+6 and applying equation (C).
Dividing by x-a, we get 30 as the remainder and 6 as the quotient

The remainder is 3x+0
When divided by x-a The quotient is 3 remainder is 3a = 12 = x+2 Applying equation (C)

When divided by x-a the quotient is 1 and the remainder is 6
When 6 is divided by 6 the quotient is 1 and the remainder is zero

∴ 675+46+6+3+1+1 is explained

(b) General Method of division by applying Nikhilam Rule:

Step 1:
First partition the dividend into two parts from right end such that the remainder part consists of as many digits as the divisor has.

Step 2:
Apply Nikhilam Sutram to the divisor to get the new divisor. Division is now carried out by the new divisor value so obtained After this, the procedure is as follows.

This is shown by a specific example.

Consider one example 223 ÷ 78.

Partition 223 as 2/23 (Divisor has two digits)

The value obtained by applying the Nikhilam Sutram to the divisor 78 is 22, which is the new divisor in operation i.e , we are dividing by a lesser number.

1) 223 ÷ 78

Current Method

78) 223 (2
$\underline{156}$
67

Quotient = 2
Remainder = 67

Vedic Method

First Part (Quotient
Original
Divisor → $\underline{78}$) 2 / 23 ← Second Part (Remainder
New → 22 $\underline{/ 44}$
Divisor 2 / 67 ← Answer

Quotient = 2
Remainder = 67

Step 3: Bring down the first digit of the first part (quotient part) of the dividend as it is, to the answer.

Step 4: Then multiply this digit with the new divisor, digit by digit and put down the result from the next digit onwards and below the dividend.(it may enter into the remainder part)

Step 5: Then addition is performed between this Multiplication result and the corresponding dividend as shown in the example.

Step 6 : If the result of this addition is to be placed in the quotient, then we have to repeat the process of multiplication of that value (pertaining to the quotient part) with the new divisor.

Step 7 : Placement of this result followed by addition is similar to the one already explained.

Step 8 : If in the partition, the quotient part of the dividend consists as more than one digit (eg. 2 onwards), then all the digits are to be first exhausted

Step 9 : The multiplication with the new divisor stops with the last quotient digit of the answer.

Step 10: If the remainder is more than the original divisor (eg. 4), then a fresh division is carried out with this remainder as the new dividend. This process is continued until the remainder is less than the original divisor

Step 11: While in addition more than one digit is obtained as a single unit (eg. 2 and 4), then the usual carrying over of all digits (excepting the right hand most) to the immediate previous digit(s) is applied to obtain the answer

Examples are given below

2) 31242 ÷ 898

Current Method

$$\begin{array}{l}
898)\ 31242\ (34 \\
\qquad\ \underline{2694} \\
\qquad\ \ 4302 \\
\qquad\ \ \underline{3592} \\
\qquad\quad 710
\end{array}$$

Quotient = 34
Remainder = 710

Vedic Method

$$\begin{array}{lrcccc}
\underline{898}) & 31 & / & 2 & 4 & 2 \\
102 & 3 & / & 0 & 6 & \\
 & & / & 4 & 0 & 8 \\
\cline{2-6}
 & 34 & / & 6 & 10 & 10 \\
\cline{2-6}
 & 34 & / & 7 & 1 & 0
\end{array}$$

Quotient = 34
Remainder = 710

1203423 + 98789

Current Method

98789) 1203423 (12
98789
215533
197578
17955

Quotient =12
Remainder = 17955

Vedic Method

98789) 12 / 0 3 4 2 3
01211 0 / 1 2 1 1
/ 0 2 4 2 2
12 /1 7 9 5 5

Quotient =12
Remainder = 17955

45679 + 99

Current Method

99) 45679 (461
396
607
594
139
99
40

Quotient = 461
Remainder = 40

Vedic Method

99) 4 5 6 / 7 9
01 0 4 /
0 / 5
/ 0 10
4 5 10 / 12 19
4 6 0 / 1 /3 9
/0 1
1 /3 10

Quotient = 460+1 = 461
Remainder = 40

(1) If the value obtained by applying Nikhilam Sutram to the given divisor is greater than the divisor (eg. 5), one should first go in for a computed divisor which can be a multiple or sub multiple of the original divisor

(2) From this a new divisor, (less than the original divisor) is arrived by applying Nikhilam Sutram to the computed divisor.

(3) The division is carried out with the new divisor until one gets a remainder which is less than the computed divisor.

(4) At this end one has to multiply only the quotient by ratio of computed divisor to the original divisor to bring the result equivalent to working with original divisor

(5) If the remainder is greater than the original given divisor, this has to be divided again by the original divisor to get the final result.

(6) The quotient values so obtained are to be added to the previous quotient

(7) It can also be achieved by subtracting n times (n is positive integer) the original divisor from the remainder, so that the result of subtraction gives a (positive) value less than the divisor In such a case, to get the final quotient one has to add the value n to the quotient obtained so far.

5) 11121 ÷ 21

Current Method

```
21) 11121 (529
     105
       62
       42
       201
       189
        12
```

Quotient = 529
Remainder = 12

* Nikhilam is applied to the computed Divisor and the result is used as new divisor The division is continued until the remainder is found to be less than the computed divisor at which stage, the corresponding quotient is to be multiplied or divided by the number which is used as multiple or sub-multiple to get the computed value. If thus obtained remainder is greater than the original divisor, then one has to continue the division with original divisor which gives the corresponding quotient and the final remainder. * The quotient so obtained is added to the other quotient part, resulting the final quotient.

Vedic Method

New Divisor applying Nikhilam Sutram to 21 is 79. 79 > 21. Hence, we consider multiple of 21.
21 × 4 = 84 (computed divisor)
New divisor by applying Nikhilam Sutram to the computed divisor 84 is 16 which is less than the original divisor 21

```
4×21 = 84) 111 /   2       1
       16   16 /
              2 /  12
             __/___9______54
             129 / 23     55
             129/  2 / 8   5
               /       / 2 12
             129 / 2 / 10  17

             129   2 /1 / 1  7
                    /   / 1  6
             131       1 / 2 13
             132         / 3  3
              x 4
            * 528          33  (Vilokanam)
              529          12
```

Quotient = 529
Remainder = 12

33 Original *
-21 Division(1x21
12 Remainder

Chapter – II

Straight Division:

(a) Application of Urdhva Tiryak Sutram for numbers and also by Vinculum method :

Vedic Method of straight Division:

The following steps are to be considered.

1. Partition of the divisor:

Partition the divisor into two parts, such that one part is called Dhwajanka (flag), which takes place in the multiplication in the problem, and the other part, representing as part divisor is active in dividing the dividend. The part divisor can have one digit, two digits, three digits, four digits, etc., so also the Dhwajanka can have one or more digits. The partition of the divisor is such that the division and multiplication can be carried out with ease as much as possible. However, a general method is also workable.

2. In case of single digit divisor, in order to apply this method, one has to convert it necessarily into vinculum to enable the partition into Dhwajanka and part divisor

3. Relation between Dividend partition and Divisor partition:

Partition the given dividend into two parts. The left most part is the quotient region and the other is the remainder region. The remainder region should have number of digits equal to the Dhwajanka concerned with the divisor Keeping this in view the partition is drawn by counting the digits from the right extreme, towards left which defines the remainder region. This is diagrammatically represented as follows.

Divisor		**Dividend**	
Dhwajanka (Flag D)	First Part (Quotient region)	: Second Part (Remainder Region)	
Part Divisor (PD)	Working Details	: Working Details	
	Quotient	: Remainder	(answer line)

':' represents partition in the dividend

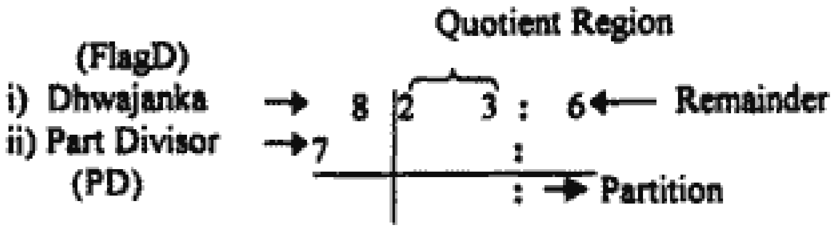
4. In the partition it is to be noticed that the position of the partition (:) represents invariably the decimal point The examples clearly show the types of partition of the Dividend consequent on the partition of the divisor. (In doing so an important point is to be taken into consideration). For example

(1) When the number of digits in the divisor is equal or less than that in the dividend, the problem is simpler (when there are no decimals in the divisor and dividend) in partitioning the dividend, following the usual rules of the partition.

Some examples are given below for the partition of the divisors and dividends.

Eg. (1) 236 + 78

Divisor	Dividend
78	236



Eg. (2) 3689 +123

i) One way of representation

23	3	6	:	8	9
1			:		
			:		

ii) Another way of representation

3	3	6	8	:	9
12				:	
				:	

(3) 98645 + 34567

Number of ways of representations

Divisor	Dividend
34567	98645

(i)

4567	9	:	8	6	4	5
3		:				
		:				

(ii)

567	9	8	:	6	4	5
34			:			
			:			

(iii)

67	9	8	6	:	4	5
345				:		

(iv)

$$\begin{array}{r|ccccc c}
7 & 9 & 8 & 6 & 4 & : & 5\\
3450 & & & & & : & \\ \hline
 & & & & & : &
\end{array}$$

(2) When the number of digits in the Dhwajanka is greater than that of the dividend <u>showing a deficiency</u>, the partition takes care of this deficiency by starting the quotient with decimal point followed by zeroes equivalent to the deficiency. A few examples of such partitions are shown.

For example:

Eg(i): 789 + 23451

$$\begin{array}{r|l}
3451 & :\;\; 7\;8\;9\\
2 & : \\ \hline
 & .\,0\;\;.\;\text{Quotient Digits}\\
 & \;\;Q_1
\end{array}$$

As there are four digits in the Dhwajanka and three digits in the Dividend, one zero is to be placed after the decimal point in the quotient is (after the partition of the divided).

Eg.(ii): 89 ÷ 23451

$$\begin{array}{r|l}
3451 & :\;\;\;\; 8\;9\\
2 & : \\ \hline
 & .\,0\;\;0\;\;\;\text{Quotient Digits}\\
 & \;\;Q_1\,Q_2
\end{array}$$

As there are four digits in the Dhwajanka and two digits in the Dividend, two zeroes are to be placed after the decimal point in the quotient. i e (after the partition of the divided).

Eg(iii): 9 + 23451

$$\begin{array}{r|l}
3451 & :\;\;\;\;\;\; 9\\
2 & : \\ \hline
 & .\,0\;\;0\;\;0\ldots\text{Quotient Digits}\\
 & \;\;\;Q_1Q_2\,Q_3
\end{array}$$

As there are four digits in the Dhwajanka and one digit in the Dividend, three zeroes are to be placed after the decimal point in the quotient. (after the partition of the divided).

If an intrinsic decimal point is present in the dividend or divisor or both, then the following rules for partition are to be considered

(3) <u>When dividend alone has intrinsic decimal</u>, the partition of the dividend should be counted from its decimal point to the left side so that the number of digits is same as that in the Dhwajanka The decimal in the quotient starts from the partition

For example.

Eg(i): 782.693 + 425

$$\begin{array}{rr|l}
 & & \;\;\;\text{Partition}\\
 & & \;\;\;\;\;\downarrow\\
\text{Dhwajanka}\rightarrow & 25 & 7\;:\;82,\,693\\
\text{Part Divisor}\rightarrow & 4 & \;\;\;: \\ \hline
 & & \;\;\;.\;\leftarrow\text{Decimal starting in the Quotient}
\end{array}$$

Eg(ii) 82. 693 ÷ 425

$$\begin{array}{r|l} 25 & : \quad 82.693 \\ \underline{4} & \underline{:\qquad\quad} \\ & \;. \end{array}$$

In case, there is a deficiency (i.e , the number of digits of the dividend on to the left side of its intrinsic decimal in comparison with the Dhwajanka) the above clause (2) is to be followed. The decimal in the quotient starts from the partition. Refer working details of 89 69 ÷ 243 in Example 14 case b(i) Page 65.

Eg(iii): 2. 693 + 425

$$\begin{array}{r|l} 25 & : 2.693 \\ \underline{4} & \underline{:\qquad\quad} \\ & .\,0 \,..\ \text{Quotient digits} \\ & \quad Q_1 \end{array}$$

Dividend has only one digit on the left of decimal, one zero has to be included after the decimal in the answer, as the Dhwajanka has two digits. (Refer example. page No·)

Eg(iv): 0. 2693 + 425

$$\begin{array}{r|l} 25 & : \quad 0.\ 2\ 6\ 9\ 3 \\ \underline{4} & \underline{:\qquad\qquad\qquad} \\ & .\,0\ \ 0\quad \text{Quotient Digits} \\ & \quad Q_1\ Q_2 \end{array}$$

Two zeros are to be placed on to the right of the decimal in the quotient digits, as the Dhwajanka has two digits

Eg(v): 0. 2693 ÷ 425321

$$\begin{array}{r|l} 25321 & : \qquad\qquad 0.\ 2\ 6\ 9\ 3 \\ \underline{4} & \underline{:\qquad\qquad\qquad\qquad\qquad} \\ & .\,0\ 0\ 0\ 0\ 0\ .\text{Quotient Digits} \\ & \ Q_1 Q_2 Q_3 Q_4 Q_5 \end{array}$$

Five zeros are to be placed after the decimal of the quotient digits, as the Dhwajanka has five digits.

(4) If in the problem, <u>the divisor only has intrinsic decimal</u>, the partition of the dividend is carried out in the usual way but by not considering the decimal point in the divisor, in the first instance i e , taking the divisor as a whole, then partition the divisor. Now the partition in the dividend is according to the general rule. At the end in the result, the decimal in the quotient is shifted towards the right side of the quotient by the number of digits after the decimal in the divisor. This is shown clearly in the worked out examples. (Refer working details of 15628 + 23.4 in example 16 page)

Eg.(i): 9856 + 12. 3

$$\begin{array}{r|l} 3 & 9\ 8\ 5\ \ :\quad\ 6 \\ \underline{12} & \underline{\qquad\qquad\qquad\quad} \\ & \qquad\quad\ : \end{array} \qquad \text{or} \qquad \begin{array}{r|l} 23 & 9\ 8\ :\ 5\ 6 \\ \underline{1} & \underline{\qquad :\qquad} \\ & \qquad : \end{array}$$

Eg. (ii) 1757 + 523.7

$$\begin{array}{r|l} 37 & 1\ 7\quad :\ 5\ \ 7 \\ \underline{52} & \underline{\qquad\ :\qquad\ } \\ & \end{array} \qquad\qquad \begin{array}{r|l} 237 & 1\ \ :\ 7\ 5\ 7 \\ \underline{5} & \underline{\quad\ :\qquad\ } \\ & \quad\ : \end{array}$$

Decimal is to be shifted to the right by one digit in the quotient to get the final result

Eg. (iii) 98476 ÷ 0.423

$$\begin{array}{r|cccccc} 3 & 9 & 8 & 4 & 7 & : & 6 \\ 42 & & & & & : & \\ \hline & & & & & : & \end{array}$$

or

$$\begin{array}{r|cccccc} 23 & 9 & 8 & 4 & : & 7 & 6 \\ 14 & & & & : & & \\ \hline & & & & : & & \end{array}$$

Decimal is to be shifted to the right by 3 digits to get the final result

Eg. (iv) 17574 ÷ 0 . 0012

$$\begin{array}{r|cccccc} 2 & 1 & 7 & 5 & 7 & : & 4 \\ 1 & & & & & : & \\ \hline & & & & & : & \end{array}$$

Decimal is to be shifted to the right by 4 digits in the quotient to get the final result

(5) When both the dividend and the divisor have intrinsic decimal, consideration of the divisor as a whole and followed by its partition helps in partitioning the dividend as per clause (3). In the final result one has to take cognisance of shifting of the decimal appropriately as given in clause (4) (Refer working details of 134.289 ÷ 2 76 and 2 1387 ÷ 0 312 in examples 17, 18 in page No)

Eg. (i) 374. 8 ÷ 98. 2

$$\begin{array}{r|ccccc} 82 & 3 & : & 7 & 4. & 8 \\ 9 & & : & & & \\ \hline & & : & & & \end{array}$$

In the final answer the decimal has to be shifted to the right by one digit.

Eg. (ii) 8972. 2 ÷ 12. 34

$$\begin{array}{r|cccccc} 34 & 8 & 9 & : & 7 & 2. & 2 \\ 12 & & & : & & & \\ \hline & & & : & & & \end{array}$$

In the final answer the decimal has to be shifted to the right by two digits

Eg. (iii) 0 . 8972 ÷ 1. 34

$$\begin{array}{r|cccccc} 34 & : & 0.8 & 9 & 7 & 2 & \\ 1 & : & & & & & \\ \hline & . & 0 & 0 & & & \\ & & Q_1 & Q_2 & & & \end{array}$$

Two zeros are to be placed on to the right of the decimal in the quotient digits as the Dhwajanka has two digits In the final answer the decimal has to be shifted to the right by two digits.

Eg. (iv) 0 . 0089 ÷ 1. 23

$$\begin{array}{r|ccccc} 3 & : & .0 & 0 & 8 & 9 \\ 12 & : & & & & \\ \hline & . & 0 & & & \\ & & Q_1 & & & \end{array}$$

One zero is to be placed after the decimal point of the quotient as the Dhwajanka has one digit. In the final answer the decimal has to be shifted by two digits.

Working Details:

The following general points need to be considered for division.

(1) The divisor partition, dividend partition and position of the decimal point are to be first determined

(2) The division is carried out digit by digit of the dividend by the part divisor While doing so, if the first digit of the dividend is not divisible by the part divisor, then one may consider the

minimum number of required digits for the divisibility to obtain the first quotient or one may report the division digit by digit This procedure is adaptable only in the quotient part of the dividend. However in the remainder part it should be digit by digit division

(3) In case the division starts with the decimal after the effective partition, the division should be digit by digit.

(4) When the divisor consists of decimal or the dividend consists of decimal or in certain cases both may consists of decimals, the rules are clearly given while describing the partition and placement of the decimal. The exact working of the division giving various quotients and remainders, intermediate dividends and new dividends at each stage of division can be demonstrated as follows. However, the above rules are to be strictly adhered to.

Step1: Divide the first digit of the dividend by the part divisor giving quotient Q_1 and remainder R_1 The quotient Q_1 is placed in the answer line The remainder R_1 is kept between the first and the second digits of the dividend and below the dividend, leading to the formation of first intermediate dividend If the first digit is not divisible by the part divisor (2nd clause in general points) , then one may consider the minimum number of digits for the divisibility and the remainder R_1 is to be kept accordingly leading to the formation of first intermediate dividend (ID) and so on.

Step 2: The first intermediate dividend is formed by the remainder R_1 and the digit of the dividend immediately following the first dividend / first group of digits taken as the first dividend

Step 3: Now the Urdhva multiplication of the allowed first digit of the Dhwajanka with the first quotient digit (Q_1) is carried out and the result so obtained is subtracted from the first intermediate dividend (ID) to get the first new dividend (ND) and the process is continued to obtain corresponding intermediate dividends and new dividends

Step 4: For getting the remaining new dividends, the following principles are to be adopted. In case the Dhwajanka consists of more than one digit, then the new dividends are to be formed by subtracting the results of multiplication of the quotients $Q_1, Q_2, Q_3, \ldots\ldots, Q_n$ as per the Tiryak or Urdhva and Tiryak taking into consideration in succession the number of quotients according as the number of digits in the Dhwajanka $D_1\ D_2 D_3$ .... The procedure is indicated by means of a diagram in case Dhwajanka having 1 or 2 or 3 digits The same is to be extended for any number of digits in Dhwajanka as follows. These are the steps required for subtraction in arriving the new dividends from the respective intermediate dividends. It is to be noticed that while the Dhwajanka remains constant, the quotient digits successively vary in the multiplication to get the new dividends.

Step 5: If the quotients after the decimal point are zeroes consequent on the deficiency, which is invariably due to the number of digits in Dhwajanka being greater than the dividend, then the formation of new dividends by subtractions are according to the following principles.

(a) Count zeroes also as quotients.

(b) All such zero quotients will be only passive and will not contribute anything for either intermediate dividend or for subtraction

(c) If a zero quotient results due to division, such zeroes will not contribute to the subtraction

The steps required for subtraction in arriving at the new dividends can be diagrammatically shown as given below.

i) Dhwajanka has one digit:

If Dhwajanka is D_1 and the quotient digits are Q_1, Q_2, ,Q_n, then the subtracting quantities are as follows

$$\begin{matrix} D_1 \\ \uparrow \\ Q_1 \end{matrix}$$

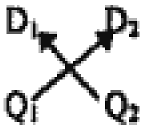
$$\begin{matrix} D_1 \\ \uparrow \\ Q_2 \end{matrix}$$

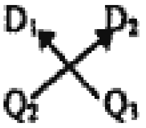
$$\begin{matrix} D_1 \\ \uparrow \\ Q_n \end{matrix}$$ etc

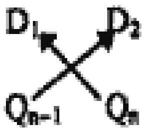
ii) Dhwajanka has two digits:

Dhwajanka is D_1D_2 and quotients digits are Q_1,Q_2, ,Q_n

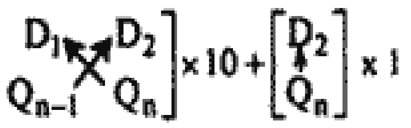
$$\begin{matrix} D_1 \\ \uparrow \\ Q_1 \end{matrix}$$







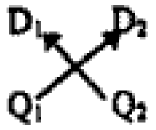
If one wants the absolute remainder, then we have to subtract the following from the total remainder region.

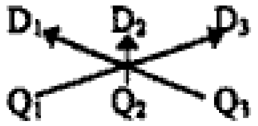


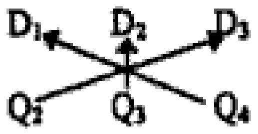
$$\left[\begin{matrix} D_1 & \times & D_2 \\ Q_{n-1} & & Q_n \end{matrix}\right] \times 10 + \left[\begin{matrix} D_2 \\ \uparrow \\ Q_n \end{matrix}\right] \times 1$$

(ii) Dhwajanka has three digits:

Dhwajanka is $D_1D_2D_3$ and quotients digits are $Q_1, Q_2, \quad , Q_n$

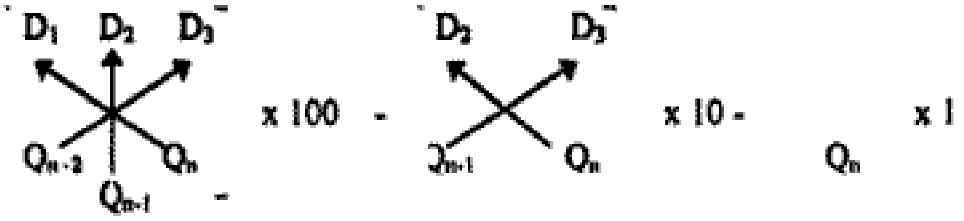






……

If one wants the absolute remainder, then we have to subtract the following multiplications from the total remainder region.



and so on for the multidigited Dhwajanka problem

Step 6: While working the new dividends, one may come across a negative value as a consequence of subtraction in which case one has to reduce the quotient by 1. To quote one example for the negative dividend, refer example 3. In the example 3, we can come up to the 4th quotient, i.e., 2, we get the intermediate dividend as 24 On computation to obtain new dividend, we have to subtract $\overset{4}{\underset{2}{\uparrow}} = 8$ from 24, thereby we are left with 16 Dividing this new dividend by 3 we get the quotient 5 and remainder 1 giving 15 as the intermediate dividend. Continuing the process of subtraction of multiplication of this quotient and Dhwajanka, $\overset{4}{\underset{5}{\uparrow}}$ one gets 20, which is greater than 15 (ID), resulting in negative new dividend, which is not accepted in this method Hence one has to reduce the quotient 5 by 1 resulting in the modified value as 4.

Proceeding similarly we will get the new dividend as 45 – 16 = 29. Divide this by 3, we can try 9, but with this also one can see again a negative dividend and hence the quotient 9 is to be further reduced by 1 giving the value 8. This gives a remainder 5 Hence intermediate dividend is 56 We have to subtract $\overset{4}{\underset{8}{\uparrow}} = 32$ from 56, giving a value of 24 as the new dividend.

Similar procedure is carried out in problems when a repeated occurrence of negative value results by this method In all such cases one has to go on reducing the quotient by 1 step by step until the negative dividend ceases.

Another method is suggested when negative new dividends are formed. That is the negative result is written in vinculum form and the entire procedure is adopted with the vinculum number At the end, one has to necessarily come out of vinculum to give the final result. Examples are given for this also.

In the examples the formation of intermediate dividends and new dividends are clearly shown The same is to be understood for the other examples.

Step 7: The new dividends are subjected to division by part divisor and the procedure of earlier steps are repeated until one enters into the remainder region.

Step 8: At this stage the remainder so obtained together with the remainder part of the dividend can be considered as intermediate remainder From this one has to subtract the value obtained by multiplying Dhwajanka and the last digit of the quotient to get the final remainder, as explained diagrammatically earlier in case of one digit two digits etc. in Dhwajanka (refer step 4 page)

The procedure is still extendable to the remainder part for obtaining the decimals If the number of decimals is specified in the beginning itself, then the problem can be worked out until the specification is reached.

A number of examples are worked out to cover as many varieties as possible in this type of straight division. Keeping in view that the number of digits of the Dhwajanka is the criterion for the partition of the dividend, a number of problems are worked out.

The division is also extendable to the case where the dividend alone or divisor alone or both have decimals.

The division is also carried out by converting the numbers into vinculum forms

The division is carried out for different cases such as the number of digits in the Dhwajanka is equal to or greater than or less than that in the dividend.

The division is worked out to obtain finally

i As quotient and remainder

ii As quotient having decimal

The division is also explained when the remainder is also subjected to division to include a specific number of decimals in the quotient.

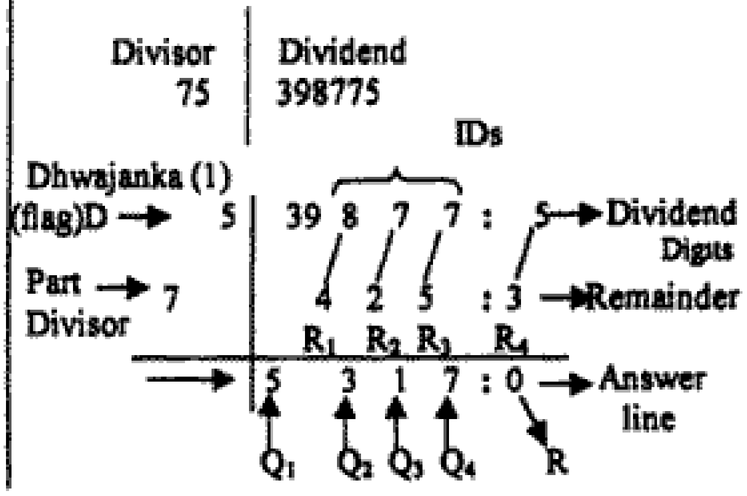
The proof for all the above details is given in terms of a polynomial in x where x value is taken as 10 to identify the given number

Example 1: 398775 ÷ 75

Current Method

```
75) 398775 (5317
    375
     237
     225
      127
       75
       525
       525
         0
```

Vedic Method



Quotient = 5317
Remainder, R =0(Exactly Divisible)

V.M.

In the above example, the part divisor 7 is active in division and 5, the Dhwajanka, is active in multiplication. The dividend is also to be partitioned from right end of the dividend such that the remainder part consists of as many digits as the Dhwajanka has.

The steps are as follows:

(1) The first division is to be carried out in the quotient region as 3 ÷ 7. But as it is not divisible one should consider 39 as first dividend. 39 divided by the part divisor 7 gives 5 as first quotient Q_1 and 4 as the remainder R_1.

```
7) 39 (5 (Q1)
   35
    4 (R1)
```

Quotient Q_1, i.e., 5, is kept in the answer. Remainder, 4, is placed between the first dividend 39 as a unit and the next digit 8 as shown in the example which can be read as 48,the intermediate dividend (ID). From this, the new dividend (ND) can be computed as given below.

2) The first quotient Q_1, 5, is multiplied by Dhwajanka, 5, and the result is subtracted from 48, intermediate dividend, i.e., 48 – 5×5 = 23. This is new dividend

$$\text{(ID) } 48 - \overset{D}{\underset{Q_1}{\begin{pmatrix} 5 \\ \uparrow \\ 5 \end{pmatrix}}} = 48 - 25 = 23 \text{ (ND)}$$

The new dividend 23 is divided by the part divisor 7 giving 3 as the next quotient digit Q_2 and 2 as the remainder R_2.

7) 23 (3 (Q_2)
$\underline{21}$
2 (R_2)

The placement of the remainder R_2 obtained in this step is similar as given above The next intermediate dividend is 27

3) Next new dividend is calculated by subtracting the multiplication result of Q_2 and Dhwajanka 5 from the intermediate dividend 27

$$\text{(ID) } 27 - \overset{D}{\underset{Q_2}{\begin{bmatrix} 5 \\ \uparrow \\ 3 \end{bmatrix}}} = 27 - 15 = 12 \text{ (ND)}$$

This new dividend is divided by 7 giving 1 as the next quotient digit Q_3 and intermediate dividend as 57

7) 12 (1 (Q_1)
$\underline{7}$
5 (R_3)

4) The new dividend is calculated as

$$\text{(ID) } 57 - \overset{D}{\underset{Q_3}{\begin{bmatrix} 5 \\ \uparrow \\ 1 \end{bmatrix}}} = 57 - 5 = 52 \text{ (ND)}$$

7) 52 (7 (Q_4)
$\underline{49}$
3 (R_4)

5) Here the problem has entered into the remainder region Remainder is calculated as given below.

$$\text{(Intermediate Remainder) } 35 - \underset{Q_4}{\overset{D}{\begin{bmatrix}5\\ \uparrow\\ 7\end{bmatrix}}} = 35 - 35 = 0$$

$\therefore$ Remainder = 0

Proof is given by converting the numbers into polynomials in x (x being 10) A little reorientation in placements of products of divisor and quotient and bringing down a part of the original dividend will explain the Vedic method of straight division

For example, in the proof given below the product of the first quotient $5x^3$ with the divisor, is subtracted from the original dividend. This is followed by bringing down a part of the original dividend so that it can be written as difference of two terms (refer A in the proof). The term with minus sign can be identified with the result obtained by Urdhva Multiplication of $5x^3$ with 5 step1 in the proof For the following steps also, the term with minus sign can be identified with the corresponding Urdhva multiplications as shown in the problem $7x + 5$ can be understood as the Dhwajanka 5 and the part divisor, 7. This subtraction can be seen throughout the working with D_1

Proof:

Divisor

$$\begin{array}{lll}
D_1 & \text{Dividend} & \text{Quotient} \\
7x+5) & 39x^4 + 8x^3 + 7x^2 + 7x + 5 & (5x^3 + 3x^2 + x + 7 \\
 & \underline{35x^4 + 25x^3} & \quad Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \\
 & \left.\begin{array}{l} 4x^4 + \ 8x^3 \ \boxed{-25x^3} \\ = x^3(4x+8) - 25x^3 \\ = 48x^3 - 25x^3 \end{array}\right\} \xrightarrow{\quad} (1) \ Ⓐ & \\
 & \underline{= 23x^3} & \\
 & = 23x^3 + 7x^2 & \\
 & \underline{21x^3 + 15x^2} & \\
 & 2x^3 + 7x^2 \ \boxed{-15x^2} \xrightarrow{\quad} (2) & \\
 & = x^2(2x+7) - 15x^2 & \\
 & = 27x^2 - 15x^2 & \\
 & \underline{= 12x^2} & \\
 & 12x^2 + 7x & \\
 & \underline{7x^2 + 5x} & \\
 & 5x^2 + 7x \ \boxed{-5x} \xrightarrow{\quad} (3) & \\
 & = x(5x+7) - 5x & \\
 & = \quad 57x - 5x & \\
 & \underline{= 52x} & \\
 & 52x + 5 & \\
 & \underline{49x + 35} & \\
 & 3x + 5 \ \boxed{-35} \xrightarrow{\quad} (4) & \\
 & \underline{= 35 - 35} & \\
 & = 0 &
\end{array}$$

Subtracting

$$\underset{Q_1}{\overset{D_1}{\begin{array}{c}5\\ \uparrow\\ 5x^3\end{array}}} = 25x^3 \quad (1)$$

$$\underset{Q_2}{\overset{D_1}{\begin{array}{c}5\\ \uparrow\\ 3x^2\end{array}}} = 15x^2 \quad (2)$$

$$\underset{Q_3}{\overset{D_1}{\begin{array}{c}5\\ \uparrow\\ x\end{array}}} = 5x \quad (3)$$

$$\overset{D_1}{\begin{array}{c}5\\ \uparrow\\ 7\end{array}} = 35 \quad (4)$$

Some examples are given below:

Example 2: 79335 ÷ 123

Current Method

$$\begin{array}{l} 123)\,79335\,(645 \\ \quad\;\underline{738} \\ \quad\;\;553 \\ \quad\;\;\underline{492} \\ \quad\;\;\;615 \\ \quad\;\;\;\underline{615} \\ \quad\;\;\;\;\underline{0} \end{array}$$

Vedic Method

$$\begin{array}{r|l} \text{Divisor} & \text{Dividend} \\ 123 & 79335 \end{array}$$

$$\begin{array}{r|lllll} 3 & 79 & 3 & 3 & : & 5 \\ & & / & / & & / \\ 12 & & 7 & 7 & : & 1 \\ & & R_1 & R_2 & & R_3 \\ \hline & 6 & 4 & 5 & : & 0 \\ & Q_1 & Q_2 & Q_3 & & \end{array}$$

Quotient = 645
Remainder = 0 (exactly divisible)

Vedic Method Steps:

Step 1:

$$\begin{array}{l} 12)\,79\,(6\,(Q_1) \\ \quad\;\underline{72} \\ \quad\;\;7\,(R_1) \end{array}$$

$$\boxed{Q_1 = 6}$$

Step 2:

$$(\text{ID})\ 73 - \underset{Q_1}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 6 \end{pmatrix}}} = 73 - 18 = 55\ (\text{ND})$$

$$\begin{array}{l} 12)\,55\,(4\,(Q_2) \\ \quad\;\underline{48} \\ \quad\;\;7\,(R_2) \end{array}$$

$$\boxed{Q_2 = 4}$$

Step 3:

$$(\text{ID})\ 73 - \underset{Q_2}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 4 \end{pmatrix}}} = 73 - 12 = 61\ (\text{ND})$$

$$\begin{array}{l} 12)\,61\,(5\,(Q_3) \\ \quad\;\underline{60} \\ \quad\;\;1\,(R_3) \end{array}$$

$$\boxed{Q_3 = 5}$$

Quotient = 645

Remainder part

Step 4:

$$15 - \underset{Q_3}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 5 \end{pmatrix}}} = 15 - 15 = 0$$

∴ Remainder = 0

Example 3: 7896456 ÷ 34 (The answer is represented as quotient and remainder)

Current Method

34) 7896456 (232248
$\underline{68}$
109
$\underline{102}$
76
$\underline{68}$
84
$\underline{68}$
165
$\underline{136}$
296
$\underline{272}$
$\underline{24}$

Vedic Method

D - Dhwajanka

$$\begin{array}{c|l} \text{Divisor} & \text{Dividend} \\ 34 & 7896456 \end{array}$$

$$\begin{array}{ll|cccccccc} D & 4 & 7 & 8 & 9 & 6 & 4 & 5 & : & 6 \\ & & / & / & / & / & / & & & / \\ (Pd) & 3 & 1 & 1 & 1 & 2 & 4 & & : & 5 \\ & & R_1 & R_2 & R_3 & R_4 & R_5 & & & R_6 \\ & & & & & & (m) & & & (m) \\ \hline & & 2 & 3 & 2 & 2 & 4 & 8 & : & 24 \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & & \\ & & & & & & (m) & (m) & & \end{array}$$

Quotient = 232248

$$\text{Remainder} = 56 - \overset{D_1}{\underset{Q_6}{\begin{pmatrix} 4 \\ \uparrow \\ 8 \end{pmatrix}}} = 56 - 32 = 24$$

Q = 232248
R = 24

Vedic Method Steps:

Step 1:

3) 7 (2 (Q_1)
$\underline{6}$
1 (R_1)

$\boxed{Q_1 = 2}$

Step 2:

$$(ID)\ 18 \quad \overset{D_1}{\underset{Q_1}{\begin{pmatrix} 4 \\ \uparrow \\ 2 \end{pmatrix}}} - 18 - 8 = 10\ (ND)$$

3) 10 (3 (Q_2)
$\underline{9}$
1 (R_2)

$\boxed{Q_2 = 3}$

Step 3:

$$(\text{ID})\ 19 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix}4\\ \uparrow\\ 3\end{pmatrix}}} = 19 - 12 = 7\ (\text{ND})$$

$3)\ 7\ (2\ (Q_3)$
$\underline{6}$
$1\ (R_3)$

$$\boxed{Q_3 = 2}$$

Step 4:

$$(\text{ID})\ 16 - \overset{D_1}{\underset{Q_3}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 16 - 8 = 8\ (\text{ND})$$

$3)\ 8\ (2\ (Q_4)$
$\underline{6}$
$2\ (R_4)$

$$\boxed{Q_4 = 2}$$

Step 5:

$$(\text{ID})\ 24 - \overset{D_1}{\underset{Q_4}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 24 - 8 = 16\ (\text{ND})$$

$3)\ 16\ (5\ (Q_5)$
$\underline{15}$
$1\ (R_5)$

Step 6:

$$(\text{ID})\ 15 - \overset{D_1}{\underset{Q_5}{\begin{pmatrix}4\\ \uparrow\\ 5\end{pmatrix}}} = 15 - 20 = -5\ (\text{negative value})$$

proceed for the reduction of Q_5 by 1 to get modified $Q_5 \rightarrow Q_5(m)$

Step6:

$3)\ \bar{5}\ \ (\bar{2}(Q_6)$
$\underline{\bar{6}}$
$\underline{1}\ (R_6)$

conversion to vinculum at the stage of step 6

$\therefore$ We reduce the quotient 5 by 1 giving the modified quotient $Q_5(m)$ value as 4.

$3)\ 16\ (4\ [Q_5(m)]$ (m = modified)
$\underline{12}$
$4\ [R_5(m)]$

$$\boxed{Q_5\,(m) = 4}$$

$$(ID)\ 45 - \underset{Q_5(m)}{\overset{D_1}{\begin{pmatrix}4\\ \uparrow\\ 4\end{pmatrix}}} = 45 - 16 = 29\ (ND)$$

3) 29 (9 (Q_6)
$\underline{27}$
2 (R_6)

Remainder part entry

Step 7:

$$(IR)\ 26 - \underset{Q_6}{\overset{D_1}{\begin{pmatrix}4\\ \uparrow\\ 9\end{pmatrix}}} = 26 - 36 = -10 \text{ (negative value)}$$

∴ We reduce the quotient by 1

3) 29 (8 $[Q_6(m)]$
$\underline{24}$
5 $R_6(m)$ $\boxed{Q_6(m) = 8}$

$$\text{Remainder} = (IR)\ 56 - \underset{Q_6(m)}{\overset{D_1}{\begin{pmatrix}4\\ \uparrow\\ 8\end{pmatrix}}} = 56 - 32 = 24$$

∴ Quotient = 232248, Remainder = 24

Continuation of Vinculum

Step7:

$$(IR)16 - \underset{Q_6}{\overset{D_1}{\begin{pmatrix}4\\ \uparrow\\ \bar{2}\end{pmatrix}}} = 16+8 = 24$$

Vinculum.

$$\begin{array}{c|cccccccc}
4 & 7 & 8 & 9 & 6 & 4 & 5 & : & 6 \\
 & & / & / & / & / & / & & / \\
3 & & 1 & 1 & 1 & 2 & 1 & : & 1 \\
\hline
 & 2 & 3 & 2 & 2 & 5 & \bar{2} & : & 24 \\
 & 2 & 3 & 2 & 2 & 4 & 8 & : & 24
\end{array}$$

One can avoid the process of reduction of Quotient stepwise until one gets a positive ID, if the negative Quotient or the remainder is used directly in the calculations.

Example 4: 6974 + 7 (Single digit divisor) this divisor needs to be converted to Vinculum to facilitate straight Division

Current Method

$$\begin{array}{l}
7)\,6974\,(996 \\
\quad \underline{63} \\
\quad\; 67 \\
\quad \underline{63} \\
\quad\;\; 44 \\
\quad\;\; \underline{42} \\
\quad\;\;\; 2
\end{array}$$

Quotient = 996, Remainder = 2

Vedic Method

$$\begin{array}{ll|lllll}
\text{Divisor} & & \text{Dividend} & & & & \\
7 = 1\bar{3} & & 6974 & & & & \\
D & \bar{3} & 6 & 9 & 7 & : & 4 \\
 & & & / & / & & / \\
Pd & 1 & & 0 & 0 & : & 0 \\
 & & & R_1 & R_2 & & R_3 \\ \hline
 & & 6 & 27 & 88 & : & 268 \\
 & & Q_1 & Q_2 & Q_3 & &
\end{array}$$

Q = Quotient = 958
R = Remainder

$$4 - \begin{pmatrix} \bar{3} \\ \uparrow \\ 88 \end{pmatrix} = 4 + 264 = 268$$

R = 268 > 7 (Divisor)

Hence further division by $1\bar{3}$ is continued treating, the final reminder R as the dividend.

__Various steps for the division of R__

$$\begin{array}{ll|llll}
(1) & \bar{3} & 2 & 6 & : & 8 \\
 & & & / & & / \\
0 & 1 & & 0 & : & 0 \\
 & & & R_1' & & R_2' \\ \hline
 & & 2 & 12 & : & \longrightarrow Q' \\
 & & Q_1' & Q_2' & &
\end{array}$$

. Quotient, $Q' = 32$

$$\text{Remainder, } R' = 8 - \begin{pmatrix} \bar{3} \\ \uparrow \\ 12 \end{pmatrix} = 44$$

again the reminder $R' = 44 > 7$ (Divisor)

$$\begin{array}{ll|lll}
(2) & \bar{3} & 4 & : & 4 \\
 & & & & / \\
 & & & : & 0 \\
 & 1 & & & R_1'' \\ \hline
 & & 4 & : & \longrightarrow Q'' \\
 & & Q_1'' & & Q_1'' = \text{Quotient} = 4
\end{array}$$

R'' = Remainder

$$= 4 - \begin{pmatrix} \bar{3} \\ \uparrow \\ 4 \end{pmatrix} = 4 + 12 = 16$$

The reminder $R'' = 16 > 7$ (Divisor)

$$(3) \quad \begin{array}{c|ll} \bar{3} & 1 : & 6 \\ 1 & : & 0 \\ & & R_1''' \\ \hline & 1 : & \end{array} \longrightarrow Q'''$$

Q''' = Quotient = 1

R''' = Remainder = 9

$$= 6 - \begin{pmatrix} \bar{3} \\ \uparrow \\ 1 \end{pmatrix} = 6 + 3 = 9$$

The reminder R''', $9 > 7$ (Divisor)

The dividend 9 is converted into vinculum to facilitate partition

(4) $9 = 1\bar{1}$

$$\begin{array}{c|ll} \bar{3} & 1 : & \bar{1} \\ 1 & : & 0 \\ & & R_1'''' \\ \hline & 1 : & \end{array} \longrightarrow Q''''$$

Q'''' = Quotient = 1

R'''' = Remainder = 2

$$= \bar{1} = \begin{pmatrix} \bar{3} \\ \uparrow \\ 1 \end{pmatrix} = -1 + 3 = 2$$

By adding Q''' to the quotients obtained in the above steps we get the final quotient

$\therefore$ Quotient = 958 + 32 + 4 + 1 + 1 = 996

Q Q' Q''

Q''' Q''''

Quotient = $Q + Q' + Q'' + Q''' + Q''''$
$(Q = Q_1 + Q_2 + Q_3) + (Q' = Q_1' + Q_2')$
$+ (Q'' = Q_1'') + (Q''' = Q''')$
$+ (Q'''' = Q_1'''') = 996$
Remainder = 2

xample 5: 7652 ÷ 23 The answer is represented as quotient and remainder and continued for ecimals in the quotient)

urrent Method

```
3)7652(332
  69
   75
   69
    62
    46
    16
```

uotient = 332
emainder = 16

Vedic Method

```
Divisor | Dividend
  23    | 7652
```

```
a)  3 | 7  6  5  :  2
      |     /  /     /
    2 |   1  1   :  2
      |   R1 R2     R3 (m)
   ---+--------------------
      | 3  3  2
        Q1 Q2 Q3
                (m)
```

Quotient = 332

If Q_3 = 3 R_3 = 0 then ND = 2-9 = -7(-ve) hence one can consider a reduction of Q_3 by 1 i.e. 2

$$\text{Remainder} = 22 - \overset{D_1}{\underset{Q_3}{\begin{pmatrix}3\\ \uparrow\\ 2\end{pmatrix}}} = 16$$

If one wants to work the problem of division to a specific number say 4 decimal places, in the uotient then one has to include as many zeros at the end of the dividend as to accommodate the umber of decimal places and continue for decimals.

Current Method

```
23)7652(332.695652
   69
    75
    69
     62
     46
     160
     138
      220
      207
        130
        115
          150
          138
           120
           115
              50
              46
               4
```

Vedic Method

```
Divisor | Dividend
  23    | 7652

   3 | 7   6  5 :  2   0   0   0   0
     |    /  /    /   /   /   /   /
   2 |  1   1   : 2   4   4   3   3
     |  R1  R2    R3  R4  R5  R6  R7
     |           (m) (m) (m) (m) (m)
  ---+--------------------------------
     | 3   3    2.  6   9   5   6
       Q1  Q2   Q3  Q4  Q5  Q6  Q7
               (m) (m) (m) (m) (m)
```

Quotient = 332.6956

Vedic Method Steps:

Step 1:

$$\begin{array}{l} 2)\ 7\ (3\ (Q_1) \\ \ \ \ \underline{6} \\ \ \ \ \underline{1}(R_1) \end{array} \qquad \boxed{Q_1 = 3}$$

Step 2:

$$(ID)\ 16 - \overset{D_1}{\underset{Q_1}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}}} = 16 - 9 = 7\ (ND)$$

$$\begin{array}{l} 2)\ 7\ (3\ (Q_2) \\ \ \ \ \underline{6} \\ \ \ \ \underline{1}(R_2) \end{array} \qquad \boxed{Q_2 = 3}$$

Step 3:

$$(ID)\ 15 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}}} = 15 - 9 = 6\ (ND)$$

$$\begin{array}{l} 2)\ 6\ (3\ (Q_3) \\ \ \ \ \underline{6} \\ \ \ \ 0\ (R_3) \end{array}$$

<u>Remainder Part:</u>

Step 4:

$$2\ \ - \overset{D_1}{\underset{Q_3}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}}} = 2 - 9 = -7\ \text{(negative value)}$$

∴ We reduce the quotient Q_3 by 1.

$$\begin{array}{l} 2)\ 6\ (2\ [Q_3\ (m)] \\ \ \ \ \underline{4} \\ \ \ \ \underline{2}[R_3\ (m)] \end{array} \qquad \boxed{Q_3\ (m) = 2}$$

Step4: Replacement by vinculum

ID = 02 ND = 02 − 9 = -7

$$\begin{array}{l} 2\ )\ \ \bar{7}\ (\ \bar{3}\ (Q_4) \\ \ \ \ \ \ \ \underline{\bar{6}} \\ \ \ \ \ \ \ \underline{\bar{1}}\ \ (R_4) \end{array}$$

$$\text{(Intermediate Remainder) } 22 - \overset{D_1}{\underset{Q_3(m)}{\begin{pmatrix}3\\ \uparrow\\ 2\end{pmatrix}}} = 22 - 6 = 16\ \text{(Remainder)}$$

Quotient = 332, Remainder = 16

If decimal points in the quotient are needed then continue the process by treating the remainder 16 as new dividend.

2) 16 (8 (Q_4)
$\underline{16}$
$\underline{0}$ (R_4)

Step 5:

$$\text{(ID) } 0 - \underset{Q_4}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 8 \end{pmatrix}}} = 0 - 24 = -24 \text{ (negative value)}$$

∴ We reduce the quotient Q_4 by 1

2)16 (7 $[Q_4(m)]$
$\underline{14}$
$\underline{2}$ R_4 (m)

Step 5:

$$\text{(ID) } \bar{1}0 - \underset{Q_4}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ \bar{3} \end{pmatrix}}} = \bar{1}0 - \bar{9} = \bar{1} \text{ (ND)}$$

2) $\bar{1}$ ($\bar{1}$ (Q_5)
$\underline{\bar{2}}$
$\underline{1}$ (R_5)

ND = $\bar{1}$
2) $\bar{1}$ (0 (Q_5)
$\underline{0}$
$\underline{\bar{1}}$ (R_5)

with this set of Q_5, R_5 the remaining problem when worked out gives the same final result

$$\text{(ID) } 20 - \underset{Q_4(m)}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 7 \end{pmatrix}}} = 20 - 21 = -1 \text{ (negative value)}$$

. We reduce the modified quotient Q_4 further by 1

2) 16 (6 $[Q_4(m)]$
$\underline{12}$
$\underline{4}$ $[R_4(m)]$

$\boxed{Q_4(m) = 6}$

$$\text{(ID) } 40 - \underset{Q_4(m)}{\begin{pmatrix} \uparrow \\ 6 \end{pmatrix}} = 40 - 18 = 22 \text{ (ND)}$$

2) 22 (11 (Q_5)
$\underline{22}$
$\underline{0}$ (R_5)

Step 6:

$$\text{(ID) } 0 - \underset{Q_5}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ 11\end{pmatrix}}} = 0 - 33 = -33 \text{ (negative value)}$$

∵ We reduce the quotient Q_5 by 1

2) 22 (10 [Q_5(m)]
20
2 [R_5 (m)]

$$\text{(ID) } 20 - \underset{Q_5(m)}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ 10\end{pmatrix}}} = 20 - 30 = -10 \text{ (negative value)}$$

∴ We reduce the quotient Q_5 further by 1

2) 22 (9 [Q_5(m)]
18
4 [R_5(m)]

$$\text{(ID) } 40 - \underset{Q_5(m)}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ 9\end{pmatrix}}} = 40 - 27 = 13 \text{ (ND)}$$

2) 13 (6 (Q_6)
12
1 (R_6)

Step6:

$$\text{(ID) } \bar{1}0 - \underset{Q_5}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ 0\end{pmatrix}}} = \bar{1}0 - 0 = \bar{1}0$$

2) $\bar{1}0$ ($\bar{5}$ (Q_6)
$\bar{1}0$
0 (R_6)

Step 7:

$$\text{(ID) } 10 - \underset{Q_6}{\overset{D_1}{\begin{pmatrix} \\ \uparrow\\ 6\end{pmatrix}}} = 10 - 18 = -8 \text{ (–ve value)}$$

∴ We reduce quotient Q_6 by 1.
2) 13 (5 [Q_6(m)]
10
3 [R_6(m)]

Step7:

$$\text{(ID) } 0 - \underset{Q_6}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ \bar{5}\end{pmatrix}}} \quad 15$$

2) 15 (7 (Q_7)
14
1 (R_7)

$$(\text{ID})\ 30 - \overset{D_1}{\underset{Q_6(m)}{\begin{pmatrix}3\\ \uparrow\\ 5\end{pmatrix}}} = 30 - 15 = 15\ (\text{ND})$$

2) 15 (7 (Q_7)
$\underline{14}$
$\underline{1}$ (R_7)

Step 8:

$$(\text{ID})\ 10 - \left|\underset{Q_7}{\underset{7}{\uparrow}}\right| = 10 - 21 = -9\ (-\text{ve value})$$

∴ We reduce quotient Q_7 by 1

2) 15 (6 [Q_7(m)]
$\underline{12}$
$\underline{3}$ [R_7(m)]

Step8:

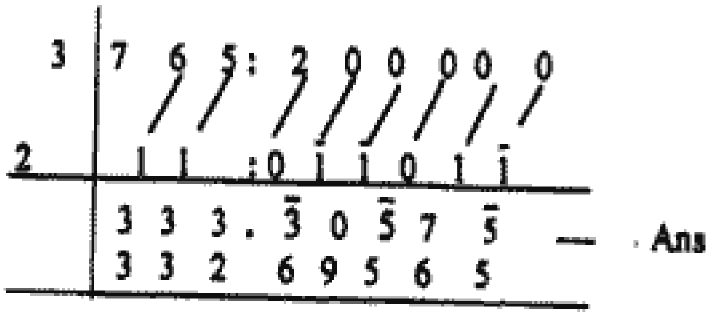
$$(\text{ID})\ 10 - \left|\underset{-}{\uparrow}\right| = 10 - 21 = \overline{11}$$

2) $\overline{11}$ ($\bar{5}$ (Q_8)
$\underline{\bar{1}0}$
$\bar{1}$ __ (R_8)

$$(\text{ID})\ 30 - \overset{D_1}{\underset{Q_7(m)}{\begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix}}} = 30 - 18 = 12\ (\text{ND})$$

∴ Quotient = 332.6956

Vinculum:



we can see the ease
with which the
problem is worked
out using Vinculum

Proof:

$D_1 \uparrow$ $\quad Q_1 \uparrow \quad Q_2 \uparrow \quad Q_3 \uparrow$

$2x+3)7x^3 + 6x^2 + 5x + 2\ (3x^2 + 3x + 2$

$\underline{6x^3 + 9x^2}$

$x^3 + 6x^2\ \boxed{-9x^2} \longrightarrow (1)$

D_1
3
$\uparrow = 9x^2 \quad (1)$
$3x^2$
Q_1

$= x^2(x + 6) - 9x^2$

$\cdot\, 16x^2 - 9x^2$ Here $x = 10$

$\underline{= 7x^2}$

$7x^2 + 5x$

$\underline{6x^2 + 9x}$

$x^2 + 5x\ \boxed{-9x} \longrightarrow (2)$

D_1
3
$\uparrow = 9x \quad (2)$
$3x$
Q_2

$= x(x + 5) - 9x$

$= 15x - 9x$

$\underline{= 6x}$

$6x + 2$

$\underline{4x + 6}$

$2x + 2\ \boxed{-6} \longrightarrow (3)$

$D_1 \quad (3)$
3
$\uparrow = 6$
2
Q_3

$= 22 - 6$

$= 16$

Remainder = 16

If decimal points are needed then one has to continue the procedure as given below

The remainder is again converted into polynomial and one has to consider it only after multiplying with 10 in order to proceed into the decimal working Hence the remainder 16 becomes 16x and is divided by 2x + 3 (Here x = 10)

Division of the remainder

$D_1 \downarrow$ $\quad Q_4 \downarrow \quad Q_5 \downarrow \quad Q_6 \downarrow \quad Q_7 \downarrow \quad Q_8 \downarrow$

$2x+3)16x \qquad (0.6 \quad 9 \quad 5 \quad 6 \quad 5$

$\underline{12x + 18}$

$4x\ \boxed{-18} \longrightarrow (1)$

D_1
3
$\uparrow = 18$
$6 \quad (1)$
Q_4

$= 40 - 18$

$\underline{= 22}$

$22x$

$\underline{18x + 27}$

$4x\ \boxed{-27} \longrightarrow (2)$

D_1
3
$\uparrow = 27\ (2)$
9
Q_5

$= 40 - 27$

$\underline{= 13}$

$13x$

$\underline{10x + 15}$

$3x\ \boxed{-15} \longrightarrow (3)$

D_1
3
$\uparrow = 15\,(3)$
5
Q_6

$= 30 - 15$

$= 15$

$= 15x$

$\underline{12x + 18}$

$3x\ \boxed{-18} \longrightarrow (4)$

D_1
3
$\uparrow = 18\,(4)$
6
Q_7

$= 30 - 18$

$= 12$

Example 6: 8954 + 89 (Division resulting in remainder and the answer is represented as quotient and remainder The work is continued for decimals in the quotient) upto 5 places of decimals.

Current Method

89) 8954 (100 60674
8900
540
534
600
534
660
623
370
356
14

Vedic Method

Divisor | Dividend
89 | 8954

9	8	9	5 :	4	0	0	0	0
8	0	0	:5	6	6	12	10	5
	R_1	R_2	R_3	R_4	R_5	R_6 (m)	R_7 (m)	R_8
	1	0	0.	6	0	6	7	4
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6 (m)	Q_7 (m)	Q_8

Quotient = 100

$$\text{Remainder} = 54 - \underset{Q_3}{\overset{D_1}{\begin{pmatrix} 9 \\ \uparrow \\ 0 \end{pmatrix}}} = 54$$

Quotient in decimals = 100.60674

One can keep more than one digit in the Dhwajanka, if the divisor has more than two digits. In such a case the procedure is as follows.

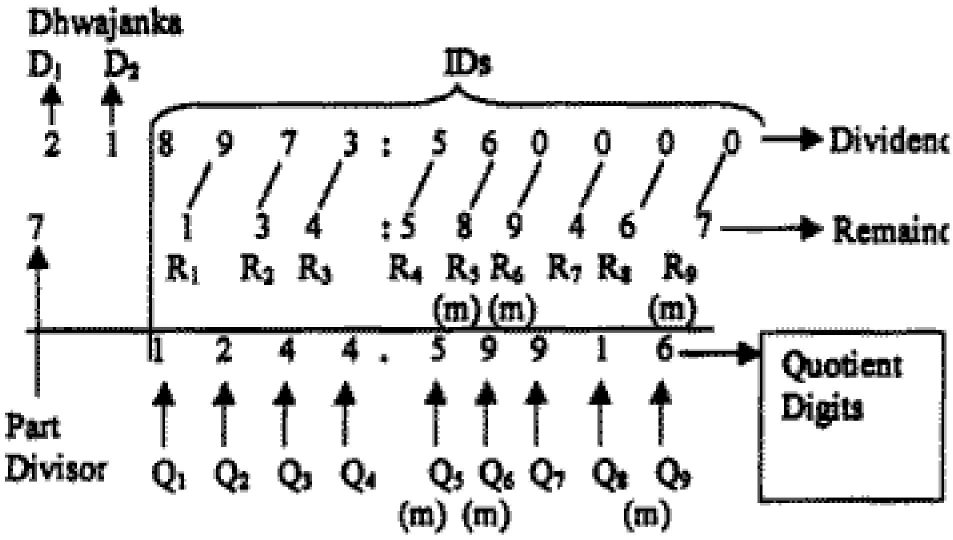
Example 7: 897356 ÷ 721 (Division where the Dhwajanka has two digits and the answer is represented as quotient and remainder and continued up to 5 decimals in the quotient)

Current Method

```
721)897356(1244.599
    721
     1763
     1442
      3215
      2884
       3316
       2884
        4320
        3605
         7150
         6489
          6610
          6489
           121
```

Vedic Method

Divisor | Dividend
721 | 897356



Quotient = 1244, Remainder = 432
Quotient in decimals = 1244.59916

Vedic Method Steps:
The first step is same as in previous cases.

Step 1:

```
7) 8 (1 (Q1)
   7
   1 (R1)
```

This gives rise to the intermediate dividend 19

Step 2:

After the first step is over, while the multiplication part is taken up, as there is only one quotient digit, one has to multiply the quotient digit (Q_1) with the first digit of the Dhwajanka. The product is subtracted from the intermediate dividend, 19, to give rise to the new dividend.

$$(ID)\ 19 - \left[\overset{D_1}{\begin{matrix}2\\ \uparrow\\ 1\end{matrix}}\right] = 17\ (ND)$$

Divide this new dividend 17 by 7, then 2 is the quotient and the remainder is 3, giving an intermediate dividend 37

7) 17 (2 (Q_2)
$\underline{14}$
3 (R_2)

Step 3:

Consider the multiplication of the Q_1, Q_2 so far obtained with the Dhwajanka digits D_1, D_2 by Tiryak multiplication. The new dividend is obtained by subtracting the result of this Tiryak-multiplication from the intermediate dividend, 37.

$$\text{(ID) } 37 - \begin{array}{c} D_1D_2 \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 1 & & 2 \end{pmatrix} \\ Q_1\ Q_2 \end{array} = 32 \text{ (ND)}$$

Divide the new dividend, 32 by 7 to get the next intermediate dividend, 43.

7) 32 (4 (Q_3)
$\underline{28}$
4 (R_3)

Step 4:

As the Dhwajanka contains two digits, we now consider two quotient digits Q_2 and Q_3 for ryak -multiplication with the Dhwajanka digits D_1D_2. The Tiryak-multiplication result is subtracted m the corresponding intermediate dividend, 43 to get the corresponding new dividend, 33 as lows:

$$\text{(ID) } 43 - \begin{array}{c} D_1\ \ D_2 \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 2 & & 4 \end{pmatrix} \\ Q_2\ Q_3 \end{array} = 33 \text{ (ND)}$$

7) 33 (4 (Q_4)
$\underline{28}$
5 (R_4)

The working has now entered into the remainder part.

In order to get the remainder, one has to stop at the stage of entering into the remainder part after part of the dividend is considered as intermediate remainder (556).

One has to compute from this value the actual remainder by subtracting the result of Tiryak and Urdhva multiplication as follows.

$$\text{Remainder is} 556 - \begin{array}{c} D_1 \qquad D_2 \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 4 & & 4 \end{pmatrix} \\ Q_3 \qquad Q_4 \end{array} \text{x } 10 - \begin{array}{c} D_2 \\ \begin{pmatrix} 1 \\ \uparrow \\ 4 \end{pmatrix} \\ Q_1 \end{array} \text{x } 1 \quad (D_1 \text{ and } Q_3 \text{ are in tens place})$$

= 556 – 120 – 4 = 432

Quotient = 1244, Remainder = 432.

Similar procedure is continued to get the other dividends in the decimal region.

Step 5:

$$(ID)\ 55 - \overset{D_1\ D_2}{\underset{Q_3\ Q_4}{\left(\begin{array}{cc} 2 & 1 \\ \multicolumn{2}{c}{\times} \\ 4 & 4 \end{array}\right)}} = 43\ (ND)$$

7) 43 (6 (Q_5)
$\underline{42}$
1 (R_5)

Step 6:

$$(ID)\ 16 - \overset{D_1\ D_2}{\underset{Q_4\ Q_5}{\left(\begin{array}{cc} 2 & 1 \\ \multicolumn{2}{c}{\times} \\ 4 & 6 \end{array}\right)}} = 0\ (ND)$$

7) 0 (0 (Q_6)
$\underline{0}$
0 (R_6)

Step 7:

$$(ID)\ 0 - \overset{D_1\ D_2}{\underset{Q_5}{\left(\begin{array}{cc} 2 & \\ \multicolumn{2}{c}{\times} \\ 6 & 0 \end{array}\right|}} = 0 - 6 = -6\ (\text{negative value})$$

Step 7:

$$ID\ 0 - \left|\begin{array}{cc} \multicolumn{2}{c}{\times} \\ & 0 \end{array}\right) = 0 - 6 = 6$$

But the quotient is zero, so if we reduce this value it becomes negative. Therefore, we reduce previous quotient 6 (obtained in step 5) as 5.

7) 43 (5 [Q_5(m)]
$\underline{35}$
8 [R_5 (m)]

$$(ID)\ 86 - \overset{D_1\ D_2}{\underset{Q_4\ Q_5(m)}{\left(\begin{array}{cc} 2 & 1 \\ \multicolumn{2}{c}{\times} \\ 4 & 5 \end{array}\right)}} = 86 - 14 = 72\ (ND)$$

7) 72 (10 [Q_6(m)]
$\underline{70}$
2 [R_6 (m)]

7) $\bar{6}$ ($\bar{1}$ [Q_7]
$\underline{\bar{7}}$
1 (R_7)

$$)\,20 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 5 & & 10 \end{pmatrix} \\ \begin{array}{cc} Q_5 & Q_6 \\ (m) & (m) \end{array} \end{array} = 20 - 25 = -5 \text{ (–ve value)}$$

Ve reduce quotient 10 by 1.

72 (9 [Q_6(m)]
<u>53</u>
9 [R_6 (m)]

Q_6 = (m) = 9

$$)\,90 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 5 & & 9 \end{pmatrix} \\ \begin{array}{cc} Q_5(m) & Q_6(m) \end{array} \end{array} = 90 - 23 = 67 \text{ (ND)}$$

57 (9 (Q_7)
<u>53</u>
4 (R_7)

Q_7 = 9

p 8:

$$)\,40 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 9 & & 9 \end{pmatrix} \\ \begin{array}{cc} Q_6(m) & Q_7 \end{array} \end{array} = 40 - 27 = 13 \text{ (ND)}$$

13 (1 (Q_8)
<u>7</u>
6 (R_8)

Q_8 = 1

Step 8:

$$\text{(ID) } 10 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 0 & & \bar{1} \end{pmatrix} \\ \begin{array}{cc} Q_6 & Q_7 \end{array} \end{array} = 12 \text{ (ND)}$$

7) 12 (1 Q_8
<u>7</u>
5 R_8

p 9:

$$)\,60 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ 9 & & 1 \end{pmatrix} \\ \begin{array}{cc} Q_7 & Q_8 \end{array} \end{array} = 60 - 11 = 49 \text{ (ND)}$$

49 (7 (Q_9)
<u>49</u>
<u>0</u> (R_9)

Step 9:

$$\text{(ID) } 50 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 2 & & 1 \\ & \times & \\ \bar{1} & & 1 \end{pmatrix} \\ \begin{array}{cc} Q_7 & Q_8 \end{array} \end{array} = 49 \text{ (ND)}$$

7) 49 (7 (Q_9)
<u>49</u>
<u>0</u> (R_9)

Step 10:

$$\begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ (ID)\ 0 - \begin{bmatrix} 2 & & 1 \\ & \times & \\ 1 & & 7 \end{bmatrix} = 0 - 15 = -15 \text{ (negative value)} \\ \begin{array}{cc} Q_8 & Q_9 \end{array} \end{array}$$

Step 10:

$$\begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ 00 - \begin{bmatrix} 2 & & 1 \\ & \times & \\ 1 & & 7 \end{bmatrix} = -15 \\ \begin{array}{cc} Q_8 & Q_9 \end{array} \end{array}$$

$$\begin{array}{l} 7\ )\ \overline{15}\ (\bar{2}\ \ Q_{10} \\ \quad\ \ \underline{\overline{14}} \\ \quad\ \ \ 1\ R_{10} \end{array}$$

∴ We reduce the quotient 7 by 1.

$$\begin{array}{l} 7)\ 49\ (6\ [Q_9(m)] \\ \quad\ \underline{42} \\ \quad\ 7\ [R_9\,(m)] \end{array} \qquad \boxed{Q_9\,(m) = 6}$$

$$\begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ (ID)\ 70 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ 1 & & 6 \end{pmatrix} = 70 - 13 = 57 \text{ (ND)} \\ \begin{array}{cc} Q_8 & Q_9(m) \end{array} \end{array}$$

Vinculum: (Direct)

$$\begin{array}{r|cccccccccc} 21 & 8 & 9 & 7 & 3: & 5 & 6 & 0 & 0 & 0 & 0 \\ & & / & / & / & / & / & / & / & / & / \\ 7 & 1 & 3 & 4 & :5 & 1 & 0 & 1 & 5 & 0 & \bar{1} \\ & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} \\ \hline & 1 & 2 & 4 & 4. & 6 & 0 & \bar{1} & 1 & 7 & \bar{2} \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} \end{array}$$

$= 1244.599168$

∴ Quotient $= 1244.59916$

In case of Dhwajanka having two digits the procedure is as described in the diagrams and the difference term ('–' term) can be identified with the Urdhva or cross multiplication as the case may be. It is exemplified in the following steps. The proof is given below:

1897356 + 721

Proof:

$$7x^2 + \overset{\overset{D_1}{\uparrow}}{2x} + \overset{\overset{D_2}{\uparrow}}{1}\,)\,8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6\,(\overset{\overset{Q_1}{\uparrow}}{x^3} + \overset{\overset{Q_2}{\uparrow}}{2x^2} + \overset{\overset{Q_3}{\uparrow}}{4x} + \overset{\overset{Q_4}{\uparrow}}{4}\,.\,\overset{\overset{Q_5}{\uparrow}}{5}\;\;\overset{\overset{Q_6}{\uparrow}}{9}\;\;\overset{\overset{Q_7}{\uparrow}}{9}\;\;\overset{\overset{Q_8}{\uparrow}}{1}\;\;\overset{\overset{Q_9}{\uparrow}}{6}$$

$$\begin{array}{l} \underline{7x^5 + 2x^4} \\ x^5 + 9x^4 \boxed{-2x^4} \longrightarrow (1) \\ = x^4(x+9) - 2x^4 \\ = 19x^4 - 2x^4 \\ \underline{= 17x^4} \end{array}$$

$$\begin{array}{c} D_1 \\ 2x \\ \uparrow \\ x^3 \\ Q_1 \end{array} = 2x^4 \longrightarrow \quad (1)$$

$$\begin{array}{l} 17x^4 + 7x^3 \\ \underline{14x^4 + 4x^3 + x^3} \\ 3x^4 + 7x^3 \boxed{-4x^3 - x^3} \longrightarrow (2) \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ x^3 & & 2x^2 \\ Q_1 & & Q_2 \end{array} = 4x^3 + x^3 = 5x^3 \longrightarrow (2)$$

$$\begin{array}{l} = x^3(3x+7) - 5x^3 \\ = 37x^3 - 5x^3 \\ \underline{= 32x^3} \\ 32x^3 + 3x^2 \\ \underline{28x^3 + 8x^2 + 2x^2} \\ 4x^3 + 3x^2 \boxed{-8x^2 - 2x^2} \longrightarrow (3) \\ = x^2(4x+3) - 10x^2 \\ = 43x^2 - 10x^2 \\ \underline{= 33x^2} \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ 2x^2 & & 4x \\ Q_2 & & Q_3 \end{array} = 8x^2 + 2x^2 = 10x^2 \longrightarrow \quad (3)$$

$$\begin{array}{l} 33x^2 + 5x \\ \underline{28x^2 + 8x + 4x} \\ 5x^2 + 5x \boxed{-8x - 4x} \longrightarrow (4) \\ = x(5x+5) - 12x \\ = 55x - 12x \\ \underline{= 43x + 6} \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ 4x & & 4 \\ Q_3 & & Q_4 \end{array} = 8x + 4x = 12x \longrightarrow (4)$$

$$* \longleftarrow \boxed{\begin{array}{l} 43x + 6 \boxed{-4} \\ \underline{= 43x + 2} \end{array}} \longrightarrow (5)$$

$$\begin{array}{c} D_2 \\ 1 \\ \uparrow \\ 4 \\ Q_4 \end{array} = 4 \longrightarrow (5)$$

*x (= 10) is introduced through conversion of dividend under decimal working and is denoted with ($^{\prime}$) notation and is shown as $Q_4{}' \ldots Q_8{}'$.

$$\begin{array}{l} ➢ \longleftarrow 43x^2 + 6x \\ \underline{35x^2 + 10x + 4x^*} \\ 8x^2 + 6x \boxed{-10x - 4x} \longrightarrow (6) \\ = x(8x+6) - 14x \\ = 86x - 14x = 72x \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ {}^*4x & & 5 \\ Q_4{}' & & Q_5 \end{array} = 10x + 4x = 14x \longrightarrow (6)$$

$$\begin{array}{l} 72x^2 \\ \underline{63x^2 + 18x + 5x^*} \\ 9x^2 \boxed{-18x - 5x} \longrightarrow (7) \\ = x(9x) - 23x \\ = 90x - 23x = 67x \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ {}^*5x & & 9 \\ Q_5{}' & & Q_6 \end{array} = 18x + 5x = 23x \longrightarrow (7)$$

$$\begin{array}{l} 67x^2 \\ \underline{63x^2 + 18x + 9x^*} \\ 4x^2 \boxed{-18x - 9x} \longrightarrow (8) \\ x(4x) - 27x \\ = 40x - 27x = 13x \end{array}$$

$$\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ {}^*9x & & 9 \\ Q_6{}' & & Q_7 \end{array} = 18x + 9x = 27x \longrightarrow \quad (8)$$

*Intermediate Step to get there remainder or to extend to decimals

➢Multiplying by 10 ie, x for the decimal evaluaton

$$\begin{array}{l} 13x^2 \\ \underline{7x^2 + 2x + 9x^*} \\ 6x^2 \,\boxed{-2x - 9x} \end{array}$$

$$= x(6x) - 11x$$
$$= 60x - 11x = 49x$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ {}^*9x & 1 \\ Q_7{}' & Q_8 \end{array} = 2x + 9x \qquad (9)$$

$$\begin{array}{l} 49x^2 \\ \underline{42x^2 + 12x + x^*} \\ 7x^2 \,\boxed{-12x -} \end{array}$$

$$= x(7x) - 13x$$
$$= 70x - 13x = 57x$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ {}^*x & 6 \\ Q_8{}' & Q_9 \end{array} = 12x + x\cdot \qquad (10)$$

In the proof at the stage when we get 43x + 6, one should decide whether one has to stop at this stage to work out the remainder or one has to go still further to get the decimals in the quotients. In the first case, the completion of the step is arrived by subtracting $\begin{pmatrix}1\\ \uparrow \\ 4\end{pmatrix}$ from 43x + 6, i.e., $43x + 6 - 4 = 43x + 2$ Now the quotient $= x^3 + 2x^2 + 4x + 4 = 1244$.
Remainder $= 43x + 2 = 432$
But if one wants to go in for the decimals in the quotient, then one has to continue from the step 43x + 6.
We can continue like this to as many digits as per specifications.
One can work out for the remainder by getting down all the remaining dividend parts and then to work out the corresponding quotient and proceeding further. This is clearly shown in the working given below.

$$7x^2+\overset{\overset{D_1}{\uparrow}}{2x}+\overset{\overset{D_2}{\uparrow}}{1}\,)\; 8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6\; (\overset{\overset{Q_1}{\uparrow}}{x^3} + \overset{\overset{Q_2}{\uparrow}}{2x^2} + \overset{\overset{Q_3}{\uparrow}}{4x} + \overset{\overset{Q_4}{\uparrow}}{4}$$

$$\begin{array}{l} \underline{7x^5 + 2x^4} \\ x^5 + 9x^4 - 2x^4 \\ = x^4(x + 9) - 2x^4 \\ = 19x^4 - 2x^4 = 17x^4 \end{array}$$

$$\begin{array}{l} 17x^4 + 7x^3 \\ \underline{14x^4 + 4x^3 + x^3} \\ 3x^4 + 7x^3 - 4x^3 - x^3 \\ = x^3(3x + 7) - 5x^3 \\ = 37x^3 - 5x^3 = 32x^1 \end{array}$$

$$\begin{array}{l} 32x^3 + 3x^2 \\ \underline{28x^3 + 8x^2 + 2x^2} \\ 4x^3 + 3x^2 - 8x^2 - 2x^2 \\ = x^2(4x + 3) - 10x^2 \\ = 43x^2 - 10x^2 = 33x^2 \\ 33x^2 \,\boxed{+ 5x + 6} \text{ ——— } \rightarrow \text{ Remaining Dividend Part} \end{array}$$

$$\begin{array}{l} 33x^2 + 5x + 6 \\ \underline{28x^2 + 8x + 4x + 4} \end{array}$$

: Remainder region of the dividend

Remainder $= 5x^2 + 5x + 6 - 8x - 4x - 4$

$$* \leftarrow = \boxed{5x^2 + 5x + 6}\ \boxed{-(8x + 4x) - 4} -$$

$= 556 - 120 - 4$

$= 432.$

$$\begin{array}{ccccc} D_1 & & D_2 & & D_2 \\ \end{array}$$

$$-\begin{bmatrix} 2x & & 1 & & 1 \\ & \nwarrow\nearrow & & -\Big| & \uparrow \\ 4x & \times & 4 & & 4 \end{bmatrix} = -(8x+4x)+4$$

$$Q_3 \quad Q_4$$

If one wants decimal points in the quotient, then start the division by taking the remainder as the dividend and the same procedure explained above is followed.

The remainder is converted into polynomial and one has to consider it only after multiplying with 10 in order to proceed into the decimal working and hence it becomes:

$43x^2 + 2x$ (remainder)

The actual division repeats from this stage onwards.(Refer page for the remainder as $43x + 2$)

$$\begin{array}{l} 7x^2 + 2x + 1)\ 43x^2 + 2x\ (6 \\ \qquad\qquad\quad \underline{42x^2 + 12x} \\ \qquad\qquad\quad x^2 + 2x\ \boxed{-12x}\ - \quad \cdot (1) \\ \qquad\qquad\qquad x(x + 2) - 12x \\ \qquad\qquad\quad = 2x - 12x \\ \qquad\qquad\quad = x(12) - 12x \\ \qquad\qquad\quad = 0 \end{array}$$

$$\begin{array}{l} D_1 \\ 2x \\ \uparrow \quad = 12x \\ 6 \\ Q_5 \end{array} \qquad (1)$$

$$\begin{array}{l} 7x^2 + 2x + 1)\ 0 \qquad (0 \\ \qquad\qquad\quad \underline{0 + 6} \\ \qquad\qquad\quad \boxed{-6} \longrightarrow \quad (2) \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\nwarrow\!\!\nearrow\ -6} \\ 6 & 0 \\ Q_5 & Q_6 \end{array} \qquad (2)$$

As negative dividend is not acceptable, one has to consider previous quotient as 5 but not as 6

$$\begin{array}{l} 7x^2 + 2x + 1)\ 43x^2 + 2x\ (5 \\ \qquad\qquad\quad \underline{35x^2 + 1(} \\ \qquad\qquad\quad 8x^2 + 2x\ \underline{|-10x} \qquad \cdot (3) \\ \qquad\qquad\quad x\ (8x + 2) \\ \qquad\qquad\quad = x(82) - 10x \\ \qquad\qquad\quad = 72x \end{array}$$

$$\begin{array}{l} D_1 \\ 2x \\ \uparrow \quad = 10x \\ 5 \\ Q_5(m) \end{array} \qquad (3)$$

Now multiplying new dividend also by 10, we get $72x^2$

Again dividing by the divisor;

---

*Intermediate remainder

$$\begin{array}{l} 7x^2 + 2x + 1)\ 72x^2(10 \\ \quad \underline{70x^2 + 20x + 5x} \\ \quad 2x^2 - 20x - 5x \\ \quad = x(2x)\boxed{-20x - 5x} \longrightarrow (4) \\ \quad = 20x - 25x \\ \quad = -5x \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ {}^*5x & 10 \\ Q_5'(m) & Q_6(m) \end{array} = 20x + 5x \quad (4)$$

This remainder is also discarded. Hence the quotient should be 9. Proceeding further with the quotient:

$$\begin{array}{l} 7x^2 + 2x + 1)\ 72x^2(9 \\ \quad \underline{63x^2 + 18x + 5x} \\ \quad 9x^2 \boxed{-18x - 5x} \longrightarrow (5) \\ \quad = x(9x) - 23x \\ \quad = 90x - 23x \\ \quad = 67x\ (R_3) \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ {}^*5x & 9 \\ Q_5'(m) & Q_6(m) \end{array} = 18x + 5x \quad (5)$$

Further proceeding we get 9 as next quotient:

$$\begin{array}{l} 7x^2 + 2x + 1)\ 67x^2(9 \\ \quad \underline{63x^2 + 18x + 9x} \\ \quad 4x^2 \boxed{-18x - 9x} \longrightarrow (6) \\ \quad = x(4x) - 18x - 9x \\ \quad = 40x - 27x \\ \quad = 13x\ (R_4) \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ {}^*9x & 9 \\ Q_6'(m) & Q_7 \end{array} = 18x + 9x \quad (6)$$

$$\begin{array}{l} 7x^2 + 2x + 1)\ 13x^2(1 \\ \quad \underline{7x^2 + 2x + 9x} \\ \quad 6x^2 - \boxed{2x - 9x} \longrightarrow (7) \\ \quad = x(6x) - 11x \\ \quad = 60x - 11x = 49x\ (R_5) \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ 9x & 1 \\ Q_7' & Q_8 \end{array} = 2x + 9x \quad (7)$$

$$\begin{array}{l} 7x^2 + 2x + 1)\ 49x^2(6 \\ \quad \underline{42x^2 + 12x + x} \\ \quad 7x^2 \boxed{-12x - x} \longrightarrow (8) \\ \quad = x(7x) - 13x \\ \quad = 70x - 13x = 57x \end{array}$$

$$\begin{array}{cc} D_1 & D_2 \\ 2x & 1 \\ \multicolumn{2}{c}{\times} \\ x & 6 \\ Q_8' & Q_9 \end{array} = 12x + x \quad (8)$$

In this process also one can work out the decimal points of the division as per one's choice.

One can extend the procedure to any number of digits in Dhwajanka for multiplication or as a matter of fact even to any number of digits, which are considered to be active in the division part. In this method it appears that the entire problem is divided into different working units applying simple division, simple multiplication and also the Urdhva Tiryak multiplication. Each time getting the value of the quotient and the corresponding remainder, an intermediate dividend, new dividend, followed by the corrected dividend, if necessary. With the help of these, the process is continued.

Example 8: 549876 + 1246 (Division where the Dhwajanka has two digits and Part divisor has two digits and the answer is represented as quotient and remainder and continued up to 6 decimals in the quotient)
(Up to 6 decimal places)

Current Method

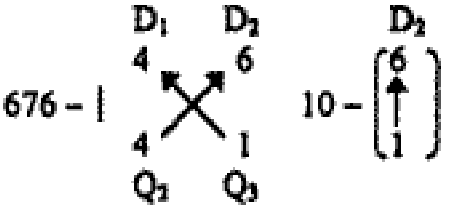
1246) 549876 (441.313001
4984
5147
4984
1636
1246
3900
3738
1620
1246
3740
3738
2000
1246
754

Vedic Method

```
     46  54   9  8 :  7  6  0  0  0  0
12            6  5 :6  3  6  2  2  2  8
              R1 R2  R3 R4 R5 R6 R7 R8 R9
         4   4  1 .  3  1  3  0  0  1
         Q1  Q2 Q3   Q4 Q5 Q6 Q7 Q8 Q9
```

Quotient = 441

Remainder =



= 676 – (280 + 6)
= 390

Quotient in decimals = 441.313001

Q = 441
R = 390

We can consider even more than two digits in the Dhwajanka, in which case some of the steps deal with three-digited-multiplications with the quotients. (or even more depending on the digits in the Dhwajanka)

The following example shows the details. In the example given below the fourth step is the remainder step. Here we come across three-digited-multiplication together with two-digited and one-digited, giving the final remainder as 154 and quotient 288.

Example 9: 985342 + 4321 (Division where Dhwajanka has three digits and the answer is represented as quotient and remainder.)

Current Method

4321) 985342 (228
$\underline{8642}$
12114
$\underline{8642}$
34722
$\underline{34568}$
154

Vedic Method

$$\begin{array}{cccc|ccccccc}
 & D_1 & D_2 & D_3 & & & & & & & \\
 & \uparrow & \uparrow & \uparrow & & & & & & & \\
 & 3 & 2 & 1 & 9 & 8 & 5 & : & 3 & 4 & 2 \\
 & & & & & / & / & & / & & \\
4 & & & & & 1 & 4 & :3 & & & \\
 & & & & & R_1 & R_2(m) & & R_3 & & \\
\hline
 & & & & 2 & 2 & 8 & : & & & \\
 & & & & Q_1 & Q_2 & Q_3 & & & & \\
 & & & & & (m) & & & & &
\end{array}$$

Quotient = 228
Remainder = 154

Vedic Method Steps:

Step 1:

4) 9 (2 (Q_1)
$\underline{8}$
1 (R_1)

$\boxed{Q_1 = 2}$

Step 2:

$$(ID)\ 18 - \overset{D_1}{\underset{Q_1}{\left(\begin{array}{c} 3 \\ \uparrow \\ 2 \end{array}\right)}} = 18 - 6 = 12\ (ND)$$

4) 12 (3 (Q_2)
$\underline{12}$
0(R_2)

Step 3:

$$(ID)\ 5 - \left(\begin{array}{ccc} D_1 & & D_2 \\ 3 & & 2 \\ & \times & \\ 2 & & 3 \\ Q_1 & & Q_2 \end{array}\right) = 5 - 4 - 9 = -8 \text{ (negative value)}$$

Step 3:

4) $\bar{8}$ ($\bar{2}$
$\underline{\bar{8}}$
0

$\therefore$ We reduce quotient Q_2 by 1.

4) 12 (2 [Q_2(m)]
$\underline{8}$
4 [R_2(m)]

$\boxed{Q_2(m) = 2}$

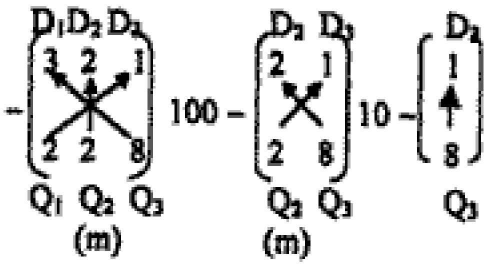
ID) 45 – 

= 45 – 6 – 4 = 35 (ND)

4) 35 (8 (Q_3)
$\underline{32}$
3 (R_3)

$\boxed{Q_3 = 8}$

ep 4: Remainder step

* 3342 – 

= 3342 – 3188 = 154
Quotient = 228, Remainder = 154

Vinculum: (Direct)

$$\begin{array}{r|cccccccccccc} 321 & 9 & 8 & 5 & : & 3 & 4 & 2 & 0 & 0 & 0 & 0 & 0 \\ & / & / & & / & / & / & / & / & / & / & / & \\ 4 & 1 & 0 & & 0 & 1 & 3 & 1 & \bar{2} & \bar{2} & \bar{2} & \bar{1} & \\ \hline & 2 & 3 & \bar{2} & : & 0 & 3 & 6 & \bar{3} & \bar{6} & 0 & \bar{1} & \end{array}$$

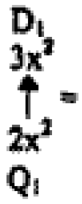
Q = 228 0356399

oof is given below:

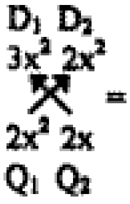
$$\begin{array}{l} \quad D_1 \quad D_2 \quad D_3 \qquad\qquad\qquad\qquad\qquad\qquad\qquad Q_1 \quad Q_2 \quad Q_3 \\ ^3 + 3x^2 + 2x + 1)\ 9x^5 + 8x^4 + 5x^3 + 3x^2 + 4x + 2\ (2x^2 + 2x + 8 \end{array}$$

$$\underline{8x^4 + 6x^4}$$
$$x^5 + 8x^4\ \boxed{-6x^4} \longrightarrow (1)$$
$$= x^4(x + 8) - 6x^4$$
$$\underline{= 18x^4 - 6x^4}$$
$$12x^4 + 5x^3$$
$$\underline{8x^4 + 6x^3 + 4x^3}$$
$$4x^4 + 5x^3 - 10x^3$$
$$= x^3(4x + 5)\ \boxed{-10x^3} \longrightarrow (2)$$
$$\underline{= 45x^3 - 10x^3}$$
$$35x^3 + 3x^2 + 4x + 2$$
$$\underline{32x^3 + 24x^2 + 4x^2 + 2x^2 + 16x + 2x + 8}$$

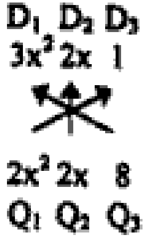
emainder
$$= 3x^3 + 3x^2 + 4x + 2\ -\ 24x^2 - 4x^2 - 2x^2\ -\ 16x - 2x\ -\ 8$$
$$= (3x^3 + 3x^2 + 4x + 2) - \underset{\downarrow\,(3)}{\boxed{24x^2 + 4x^2 + 2x^2}} - \underset{\downarrow\,(4)}{\boxed{16x + 2x}}\ \boxed{-\ 8} \longrightarrow (5)$$
$$= 3342 - 3000 - 180 - 8$$
$$= 3342 - 3188 = 154$$



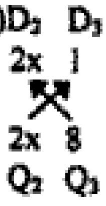
$= 6x^4$ (1)



$= 6x^3 + 4x^3$ (2)

(3) 

$= 24x^2 + 4x^2 + 2x^2$

(4) 

$= 16x + 2x$

(5) 

= 8

An example for decimal working where Dhwajanka contains three digits is given below The proof r this is also given.

* Intermediate Remainder

Example10: 89124 + 5378 (Division where Dhwajanka has three digits and the answer is represented as quotient and remainder and is continued to decimals in the quotient.)

Current Method

$$\begin{array}{l} 5378)\,89124\,(16.5719 \\ \qquad \underline{5378} \\ \qquad 35344 \\ \qquad \underline{32268} \\ \qquad\ \ 30760 \\ \qquad\ \ \underline{26890} \\ \qquad\quad 38700 \\ \qquad\quad \underline{37646} \\ \qquad\qquad 10540 \\ \qquad\qquad\ \underline{5378} \\ \qquad\qquad\ 51620 \\ \qquad\qquad\ \underline{48402} \\ \qquad\qquad\ \ \underline{3218} \end{array}$$

Vedic Method

$$\begin{array}{r|llllll} 378 & 8\,9: & 1 & 2 & 4 & 0 & 0 \\ & / & / & / & / & / & / \\ 5 & 3 & :6 & 11 & 12 & 15 & 13 \\ & R_1 & R_2 & R_3 & R_4 & R_5 & \\ & & (m) & (m) & (m) & (m) & \\ \hline & 1\,6. & 5 & 7 & 1 & 9 & \\ & Q_1Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & \\ & & (m) & (m) & (m) & (m) & \end{array}$$

Quotient = 16

Remainder =

$$6124 - \underset{\substack{Q_1\quad Q_2 \\ (m)}}{\overset{D_1\ D_2\ D_3}{\begin{pmatrix} 3 & 7 & 8 \\ \nwarrow & \uparrow & \nearrow \\ 0 & 1 & 6 \end{pmatrix}}}100 - \underset{\substack{Q_1\ Q_2 \\ (m)}}{\overset{D_2\ D_3}{\begin{pmatrix} 7 & 8 \\ \multicolumn{2}{c}{\times} \\ 1 & 6 \end{pmatrix}}}10 - \underset{\substack{Q_3 \\ (m)}}{\overset{D_3}{\begin{pmatrix} 8 \\ \uparrow \\ 6 \end{pmatrix}}}$$

= 6124 – 2500 – 500 – 48

= 6124 – 3048 = 3076

Quotient in decimals = 16 5719

Vedic Method Steps:

Step 1:

$$\begin{array}{l} 5)\,8\,(1\ (Q_1) \\ \quad \underline{5} \\ \quad 3\ (R_1) \end{array} \qquad \boxed{Q_1 = 1}$$

Step 2:

$$(ID)\ 39 - \underset{Q_1}{\overset{D_1}{\begin{bmatrix} 3 \\ \uparrow \\ 1 \end{bmatrix}}} = 39 - 3 = 36\ (ND)$$

$$\begin{array}{l} 5)\,36\,(7\ (Q_2) \\ \quad \underline{35} \\ \quad\ 1\ (R_2) \end{array}$$

Step 3:

$$(ID)\ 11 - \underset{Q_1\quad Q_2}{\overset{D_1\quad D_2}{\begin{pmatrix} 3 & 7 \\ \multicolumn{2}{c}{\times} \\ 1 & 2 \end{pmatrix}}} = 11 - 21 - 7 = -17\ (\text{negative value})$$

∴ We reduce quotient by 1.

Step3:

$$\begin{array}{l} 5\,)\,\overline{17}\,(\,\bar{3} \qquad (Q_3) \\ \quad\ \underline{\overline{15}} \\ \qquad \bar{2}\ \ (R_3) \end{array}$$

5) 36 (6 [Q_2(m)]
$\underline{30}$
6 [R_3 (m)]

$\boxed{Q_2\text{ (m)} = 6}$

$$\text{ID) } 61 - \left(\begin{array}{cc} D_1 & D_2 \\ 3 & 7 \\ 1 & 6 \\ Q_1 & Q_2(m) \end{array}\right) = 61 - 18 - 7 = 36 \text{ (ND)}$$

5) 36 (7 (Q_3)
$\underline{35}$
1 (R_3)

Step 4:

$$\text{ID) } 12 - \left(\begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 1 & 6 & 7 \\ Q_1 & Q_2 & Q_3 \\ & (m) & \end{array}\right) = 12 - 21 - 8 - 42 = -59$$

(negative value)

∴ We reduce quotient by 1.

5) 36 (6 [Q_3(m)]
$\underline{30}$
6 [R_3(m)]

Step4:

$$\bar{2}2 - \left(\begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 1 & 7 & \bar{3} \\ Q_1 & Q_2 & Q_3 \end{array}\right) = \bar{2}2 - 48 = \bar{2}\,2 + \bar{4}\,\bar{8} = \overline{66}$$

5) $\overline{66}$ ($\overline{13}$
$\underline{\overline{65}}$
$\bar{1}$

$$\text{(ID) } 62 - \left(\begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 1 & 6 & 6 \\ Q_1 & Q_2 & Q_3 \\ & (m) & (m) \end{array}\right) = 62 - 18 - 8 - 42 = -6 \text{ (negative value)}$$

∴ We reduce the quotient further.

5) 36 (5 [Q_3(m)]
$\underline{25}$
11 [R_3 (m)]

$\boxed{Q_3\text{ (m)} = 5}$

$$\text{(ID) } 112 - \left(\begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 1 & 6 & 5 \\ Q_1 & Q_2 & Q_3 \\ & (m) & (m) \end{array}\right) = 112 - 15 - 8 - 42 = 47 \text{ (ND)}$$

5) 47 (9 (Q_4)
$\underline{45}$
2 (R_4)

Step 5:

$$\text{(ID) } 24 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ 6 & 5 & 9 \end{pmatrix} \\ \begin{array}{ccc} Q_2 & Q_3 & Q_4 \\ (m) & & (m) \end{array} \end{array} = 24 - 27 - 48 - 35 = -86 \quad \text{(negative value)}$$

∴ We reduce quotient by 1.

5) 47 (8 [Q_4(m)]
<u>40</u>
7 R_4 (m)

Step5:

$$\bar{1}4 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ 7 & \bar{3} & \overline{\overline{13}} \end{pmatrix} \\ \begin{array}{ccc} Q_2 & Q_3 & Q_4 \end{array} \end{array} = \bar{2}$$

5) $\bar{2}$ (0 Q_5
<u>0</u>
$\bar{2}$

$$\text{(ID) } 74 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ 6 & 5 & 8 \end{pmatrix} \\ \begin{array}{ccc} Q_2 & Q_3 & Q_4 \\ (m) & (m) & (m) \end{array} \end{array} = 74 - 24 - 48 - 35 = -33 \quad \text{(negative value)}$$

∴ We reduce the quotient further.

5) 47 (7 [Q_4(m)]
<u>35</u>
12 [R_4 (m)]

$$\boxed{Q_4\,(m) = 7}$$

$$\text{(ID) } 124 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ 6 & 5 & 7 \end{pmatrix} \\ \begin{array}{ccc} Q_2 & Q_3 & Q_4 \\ (m) & (m) & (m) \end{array} \end{array} = 124 - 21 - 48 - 35 = 20 \text{ (ND)}$$

5) 20 (4 (Q_5)
<u>20</u>
0 (R_5)

Step 6:

$$\text{(ID) } 0 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ 5 & 7 & 4 \end{pmatrix} \\ \begin{array}{ccc} Q_3 & Q_4 & Q_5 \\ (m) & (m) & \end{array} \end{array} = 0 - 12 - 40 - 49 = -101$$

Step6:

$$\bar{2}0 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 3 & 7 & 8 \\ \bar{3} & \overline{13} & 0 \end{pmatrix} \\ \begin{array}{ccc} Q_3 & Q_4 & Q_5 \end{array} \end{array} \begin{aligned} &= \bar{2}0 - (\bar{1}\,\bar{1}\,\bar{5}) \\ &= \bar{2}0 + 115 = 95 \end{aligned}$$

'e reduce the quotient by 1.

5) 20 (3 [Q_5 (m)]
<u>15</u>
5 [R_5 (m)]

5) 95 (19 (Q_6)
<u>95</u>
0 (R_6)

$$\text{D) } 50 - \begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 5 & 7 & 3 \\ Q_3 & Q_4 & Q_5 \\ (m) & (m) & (m) \end{array} = 50 - 9 - 40 - 49 = -48 \text{ (negative value)}$$

5) 20 (2 [Q_5(m)]
<u>10</u>
10 [R_5 (m)]

$$\text{D) } 100 - \begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 5 & 7 & 2 \\ Q_3 & Q_4 & Q_5 \\ (m) & (m) & (m) \end{array} = 100 - 6 - 40 - 49 = 5 \text{ (ND)}$$

5) 5 (1 (Q_6)
<u>5</u>
0 (R_6)

p 7:

$$\text{ID) } 0 - \begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 7 & 2 & 1 \\ Q_4 & Q_5 & Q_6 \\ (m) & (m) & \end{array} = 0 - 3 - 14 - 56 = -73 \text{ (negative value)}$$

'e reduce quotient by 1.

5) 5 (0 [Q_6(m)]
<u>0</u>
5 [R_6 (m)]

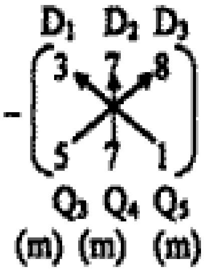
$$\text{ID) } 50 - \begin{array}{ccc} D_1 & D_2 & D_3 \\ 3 & 7 & 8 \\ 7 & 2 & 0 \\ Q_4 & Q_5 & Q_6 \\ (m) & (m) & (m) \end{array} = 50 - 0 - 14 - 56 = -20 \text{ (negative value)}$$

$Q_6(m)$ in this procedure of reduction, cannot be further reduced because as the reduced quotient leads to '-' ve value. Hence we have to go back to immediate quotient Q_5 and reduce it by 1. ie Q_5 (m) = 1

∴ We reduce the Q_5 value 2 by 1.

5) 20 (1 [Q_5(m)]
 5
 15 [R_5 (m)]

$\boxed{Q_5 \text{ (m)} = 1}$

(ID) 150 –


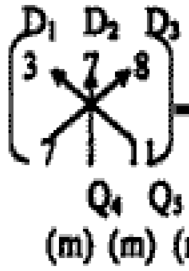
= 150 – 3 – 49 – 40 = 58 (ND)

Vinculum

$$\begin{array}{c|cccccccc}
378 & 8 & 9 & : & 1 & 2 & 4 & 0 & \\
 & & / & & / & / & / & / & \\
5 & & 3 & : & 1 & \bar{2} & \bar{1} & \bar{2} & 0 \\
\hline
 & 1 & 7 & : & \bar{3} & \overline{\overline{13}} & 0 & 19 & \\
 & 1 & 7 & : & & \bar{4} & \bar{3} & 19 & \\
\hline
 & 1 & 6: & & & 5 & 1 & 19 &
\end{array}$$

We continue the calculatings to get the value of Q_6 also

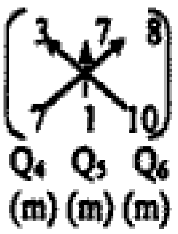
5)58(11 [Q_6(m)]
 55
 3[R_6(m)]

30 -


= 30 – 33 – 56 – 78 = - 66

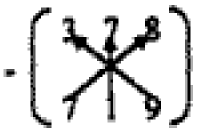
Q_6(m) is further reduced by 1.
5)58(10
 50
 8

Step 8:

80


$= 80 - 30 - 56 - 7 = -13$ ∴ We reduce the value of Q_6 by 1 i.e. $Q_6 = 9$

$5)58(9 \quad Q_6(m)$
$\underline{45}$
13

130 -


$= 130 - 27 - 56 - 7$
$130 - 90 = 40$

∴ Quotient = 16. 5719

oof:

$D_1 \quad D_2\, D_3 \qquad\qquad Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5$

$5x^3+3x^2+7x+8)\ 8x^4 + 9x^3 + x^2 + 2x + 4\ (x + \ 6. \ \ 5 \ \ 7 \ \ 1$

$\underline{5x^4 + 3x^3}$

$3x^4 + 9x^3 - \boxed{3x^3} \longrightarrow (1)$

$= x^3(3x + 9) - 3x^3$

$= 39x^3 - 3x^3$

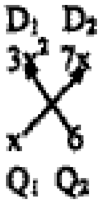
$\underline{= 36x^3}$

D_1
$3x^2$
$\uparrow = 3x^3 \qquad (1)$
x
Q_1

$36x^3 + x^2$

$\underline{30x^3 + 18x^2 + 7x^2}$

$6x^3 + x^2\ \boxed{-18x^2 - 7x^2} \longrightarrow (2)$



$= 18x^2 + 7x^2 \quad (2)$

$= x^2(6x + 1) - 25x^2$

$= 61x^2 - 25x^2$

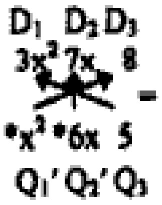
$= 36x^2$

$\leftarrow \boxed{= 36x^2 + 2x} \longrightarrow$ Remainder region

$36x^3 + 2x^2$

$\underline{25x^3 + 15x^2 + 42x^2 + 8x^2}$

$11x^3 + 2x^2\ \boxed{-15x^2 - 42x^2 - 8x^2} \longrightarrow (3)$



$= 15x^2 + 42x^2 + 8x^2 \qquad (3)$

$= x^2(11x + 2) - 65x^2$

$= 112x^2 - 65x^2$

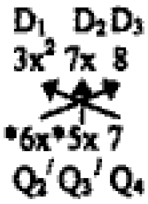
$\underline{= 47x^2}$

$\underline{47x^2 + 4x}$

$47x^3 + 4x^2$

$\underline{35x^3 + 21x^2 + 35x^2 + 48x^2}$

$12x^3 + 4x^2\ \boxed{-21x^2 - 35x^2 - 48x^2} \longrightarrow (4)$

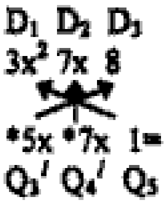


$= 21\,x^2 + 35\,x^2 + 48x^2 \qquad (4)$

$= x^2(12 + 4) - 104x^2$

$= 124x^2 - 104x^2$

$\underline{= 20x^2}$



$= 3x^2+49x^2+40x2$

$20x^3$

$\underline{5x^3 + 3x^2 + 49x^2 + 40x^2}$

$15x^3\ \boxed{-3x^2 - 49x^2 - 40x^2} \longrightarrow (5) \qquad (5)$

$= x^2(15x) - 92x^2$

$= 150x^2 - 92x^2 = 58x^2$

$*\ \ 36x^2 + 2x + 4$
$\underline{\qquad\qquad + 42x + 8x + 48}$
$36x^2 + 2x + 4 - 42x - 8x - 48 = 36x^2 - 48x - 44$

For remainder:

$D_1 \quad D_2D_3 \qquad\qquad\qquad\qquad Q_1 \; Q_2$

$5x^3+3x^2+7x+8)\ 8x^4 + 9x^3 + x^2 + 2x + 4\ (x + 6$

$\underline{5x^4 + 3x^3}$

$3x^4 + 9x^3 - 3x^3$

$= x^3(3x + 9) - 3x^3$

$\underline{= 39x^3 - 3x^3}$

$36x^3 + x^2 + 2x + 4$

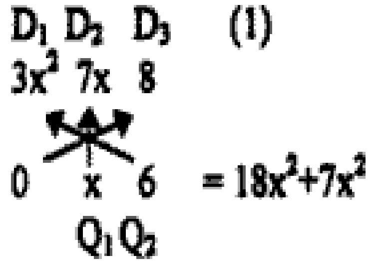
$\underline{30x^3 + 18x^2 + 7x^2 + 42x + 8x + 48}$

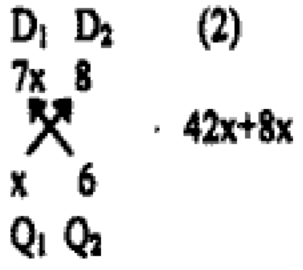
Remainder $= 6x^3 + x^2 + 2x + 4 - 18x^2 - 7x^2 - 42x - 8x - 48$

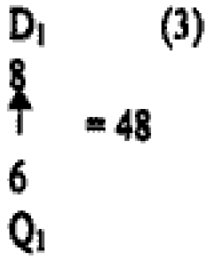
$= (6x^3 + x^2 + 2x + 4)\ \boxed{-18x^2 + 7x^2}\ \boxed{-42x + 8x}\ \boxed{-48}$

$= 6124 - 2500 - 500 - 48 \qquad \uparrow(1) \quad \uparrow(2) \quad \uparrow(3)$

$= 6124 - 3048 = 3076$







(1)

Decimal points from the remainder: $(30x^2 + 7x + 6)$

$D_1 \quad D_2D_3 \qquad\qquad\qquad\qquad Q_3 \quad Q_4 \quad Q_5$

$5x^3+3x^2+7x+8)\ 30x^3 + 7x^2 + 6x\ (0.\ 5 \quad 7 \quad 1$

$\underline{25x^3 + 15x^2}$

$5x^3 + 7x^2\ \boxed{-15x^2} \longrightarrow (1)$

$= x^2(5x + 7) - 15x^2$

$= 57x^2 - 15x^2$

$\underline{= 42x^2}$

$\underline{42x^2 + 6x}$

$42x^3 + 6x^2$

$\underline{35x^3 + 21x^2 + 35x^2}$

$7x^3 + 6x^2\ \underline{-21x^2 - 35x^2} \qquad (2)$

$= x^2(7x + 6) - 56x^2$

$= 76x^2 - 56x^2$

$\underline{= 20x^2}$

$20x^3$

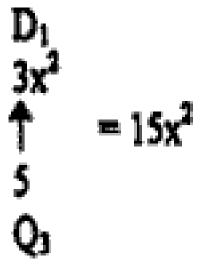
$\underline{5x^3 + 3x^2 + 49x^2 + 40x^2}$

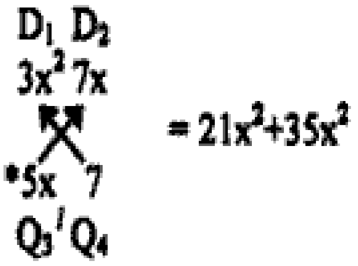
$15x^3\ \underline{-3x^2 - 49x^2 - 40x^2} \quad \blacktriangleright(3)$

$= x^2(15x) - 92x^2$

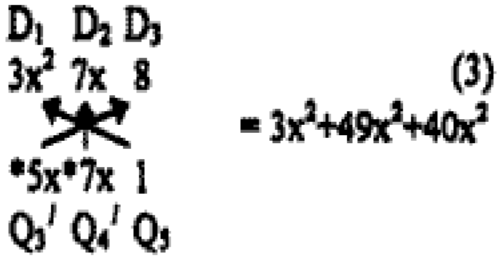
$= 150x^2 - 92x^2$

$= 58x^2$





(2)



(3)

mple 11: 789421 + 10321 (Division where the Dhwajanka has three digits and the part divisor has two digits and the answer is represented as quotient and remainder.)

Current Method

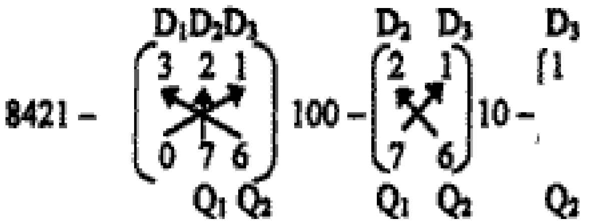
10321) 789421 (76
72247
66951
61926
5025

Vedic Method

321	78	9 :	4	2	1
10		8 :	8		
		R_1	R_2		
	7	6 :			
	Q_1	Q_2			

Quotient = 76

Remainder =



R = 5025

mple 12: 6543 + 89798 (Division where the Dhwajanka has four digits.)

Current Method

89798) 654300 (0.0728
628586
257140
179596
775440
718384
570556

Vedic Method

9798	:	6	5	4	3	0		→ Dividend
8	:		6	9	15	22	3	→ Remainders
			R_1	R_2	R_3	R_4	R_5	
				(m)	(m)	(m)		
	.	0	7	2	8	6		→ Quotient Digits
		Q_1	Q_2	Q_3	Q_4	Q_5		
			(m)	(m)	(m)			

Quotient = 0.07286

Refer 4 (2) – Straight Division Page: 12

Vedic Method Steps:

Step 1:

8) 6 (0 (Q_1)

6 (R_1)

Step 2:

$$\text{(ID) } 65 - \begin{array}{c} D_1 \\ \begin{bmatrix} 9 \\ \\ 0 \end{bmatrix} \\ Q_1 \end{array} = 65 \text{ (ND)}$$

8) 65 (8 (Q_2)
   $\underline{64}$
   1 (R_2)

Step 3:

$$\text{(ID) } 14 - \begin{array}{c} \begin{array}{ccc} D_1 & & D_2 \end{array} \\ \begin{pmatrix} 9 & & 7 \\ & \times & \\ 0 & & 8 \end{pmatrix} \\ \begin{array}{ccc} Q_1 & & Q_2 \end{array} \end{array} \quad 14 - 72 = -58 \text{ (negative value)}$$

∴ We reduce the quotient by 1

8) 65 (7 $[Q_2(m)]$
   $\underline{56}$
   9 $[R_2 (m)]$

$$\text{(ID) } 94 - \begin{array}{c} \begin{array}{ccc} D_1 & & D_2 \end{array} \\ \begin{pmatrix} 9 & & 7 \\ & \times & \\ 0 & & 7 \end{pmatrix} \\ \begin{array}{ccc} Q_1 & & Q_2 \\ & & (m) \end{array} \end{array} = 94 - 63 = 31 \text{ (ND)}$$

8) 31 (3 (Q_3)
   $\underline{24}$
   7 (R_3)

Step3:

$$8\ )\ \overline{58}\ (\ \overline{7} \qquad (Q_3)$$
$$\underline{\overline{56}}$$
$$\overline{2}\ (R_3)$$

Step 4:

$$\text{(ID) } 73 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 9 & 7 & 9 \\ & \times & \\ 0 & 7 & 3 \end{pmatrix} \\ \begin{array}{ccc} Q_1 & Q_2 & Q_3 \\ & (m) & \end{array} \end{array} = 73 - 76 = -3 \quad \text{(negative value)}$$

∴ We reduce the quotient.

8) 31 (2 $[Q_3(m)]$
   $\underline{16}$
   15 R_3 (m)

$\boxed{Q_3 (m) = 2}$

$$\text{(ID) } 153 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 9 & 7 & 9 \\ & \times & \\ 0 & 7 & 2 \end{pmatrix} \\ \begin{array}{ccc} Q_1 & Q_2 & Q_3 \\ & (m) & (m) \end{array} \end{array} = 153 - 67 = 86 \text{ (ND)}$$

Step4:

$$\overline{2}3 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 9 & 7 & 9 \\ & \times & \\ 0 & 8 & \overline{7} \end{pmatrix} \\ \begin{array}{ccc} Q_1 & Q_2 & Q_3 \end{array} \end{array} = \overline{10}$$

$$8\ )\ \overline{10}\ (\ \overline{1}\ (Q_4)$$
$$\underline{\overline{8}}$$
$$\overline{2}\ (R_4)$$

8) 86 (10 (Q_4)
80
6 (R_4)

Step 5:

$$(\text{ID})\ 60-\left(\begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 9 & 7 & 9 & 8 \\ 0 & 7 & 2 & 10 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ & (m) & (m) & \end{array}\right)=60-167=-107 \text{ (negative value)}$$

∴ We reduce the quotient by 1.

8) 86 (9 [Q_4(m)]
72
14 R_4 (m)

$$(\text{ID})\ 140-\left(\begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 9 & 7 & 9 & \\ 0 & 7 & 2 & 9 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ & (m) & (m) & (m) \end{array}\right) \cdot 140-158=-18 \text{ (negative value)}$$

∴ We reduce the quotient further.

8) 86 (8 [Q_4(m)]
64
22 R_4 (m)

$$(\text{ID})\ 220-\left(\begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 9 & 7 & 9 & 8 \\ 0 & 7 & 2 & 8 \\ Q_1 & Q_2 & Q_3 & Q_4 \\ & (m) & (m) & (m) \end{array}\right)=220-149=71 \text{ (ND)}$$

8) 71 (8 (Q_5)
64
7 (R_5)

$\boxed{Q_5 = 8}$

∴ Quotient = 0.0728

Step5:

$$\bar{2}0-\left(\begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ 9 & 7 & 9 & 8 \\ 0 & 8 & \bar{7} & \bar{1} \\ Q_1 & Q_2 & Q_3 & Q_4 \end{array}\right) \cdot \overline{34}$$

8) $\overline{34}$ ($\bar{4}$ (Q_5)
$\overline{32}$
$\bar{2}$ (R_5)

Vinculum:

$$\begin{array}{c|cccccc} 9798 & .6 & 5 & 4 & 3 & 0 & 0 \\ & & / & / & / & / & / \\ 8 & . & 6 & 1 & 2 & 2 & 2 \\ \hline & .0 & 8 & \bar{7} & \bar{1} & \bar{4} & \end{array}$$

$Q = .08\bar{7}\bar{1}\bar{4}$
$= .07286$

We can do the above problem by using Vinculum in the divisor also

Current Method

6543 ÷ 89798

$$\begin{array}{l} 89798\,)\,657300\,(\,.07286 \\ \quad\;\;\underline{628586} \\ \quad\;\;\;257140 \\ \quad\;\;\;\underline{179596} \\ \quad\;\;\;\;775440 \\ \quad\;\;\;\;\underline{718384} \\ \quad\;\;\;\;570560 \\ \quad\;\;\;\;\;\underline{538788} \\ \quad\;\;\;\;\;\;31772 \end{array}$$

Vedic Method using Vinculum in the Divisor

$89798 = 9\,0\,\bar{2}\,0\,\bar{2}$

$$\begin{array}{c|ccccc} 0\,\bar{2}\,0\,\bar{2} & 6 & 5 & 4 & 3 & 0\;\;0 \\ & / & / & / & / & / \\ & 6 & 2 & 6 & 5 & 0 \\ & R_1 & R_2 & R_3 & R_4 & R_5 \\ \hline & .0 & 7 & 2 & 8 & 6 \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 \end{array}$$

Quotient = 0.07286

Vedic Method Steps:

Step 1:

9) 6 (0 (Q_1)
$\underline{0}$
6 (R_1)

$\boxed{Q_1 = 0}$

Step 2:

(ID) 65 – ↑| = 65 (ND)

9) 65 (7 (Q_2)
$\underline{63}$
2 (R_2)

$\boxed{Q_2 = 7}$

Step 3:

$$(\text{ID})\ 24 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{pmatrix} 0 & \bar{2} \\ 0 & 7 \end{pmatrix} \\ \begin{array}{cc} Q_1 & Q_2 \end{array} \end{array} = 24\ (\text{ND})$$

9) 24 (2 (Q_3)
$\underline{18}$
6 (R_3)

$\boxed{Q_3 = 2}$

Step 4:

$$(\text{ID})\ 63 - \begin{array}{c} \begin{array}{ccc} D_1 & D_2 & D_3 \end{array} \\ \begin{pmatrix} 0 & \bar{2} & 0 \\ 0 & 7 & 2 \end{pmatrix} \\ \begin{array}{ccc} Q_1 & Q_2 & Q_3 \end{array} \end{array} = 63 - (-14) = 77\ (\text{ND})$$

9) 77 (8 (Q_4)
$\underline{72}$
5 (R_4)

Step 5:

$$\text{(ID) } 50 - \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \\ \left(\begin{array}{cccc} 0 & \bar{2} & 0 & \bar{2} \\ 0 & 7 & 2 & 8 \end{array} \right) \\ Q_1 & Q_2 & Q_3 & Q_4 \end{array} = 50 - (-4) = 54 \text{ (ND)}$$

9) 54 (6 (Q_5)
$\underline{54}$
0 (R_5)

Case (a): if the dividend has less number of digits than the Dhwajanka

<u>Example 13</u>: 78 ÷ 21345 (Up to 7 decimal places)
(Refer. 4 (2) – Straight Division for partition rules. Page:)

Current Method

21345) 78000 (0.0036542
$\underline{64035}$
139650
$\underline{128070}$
115800
$\underline{106725}$
90750
$\underline{85380}$
53700
$\underline{42690}$
$\underline{11010}$

Vedic Method

$$\begin{array}{lccccccc} 1345 & & & 7 & 8 & 0 & 0 & 0 \quad 0 \\ & & & & / & / & / & / \quad / \\ & & & \underline{\quad} & \underline{1} & \underline{3} & \underline{5} & \underline{7 \quad 8} \\ & .0 & 0 & 3 & 6 & 5 & 4 & 2 \\ & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\ & & & & (m) & (m) & (m) & (m) \end{array}$$

Vedic Method Steps:

Step 1:

2) 7 (3 (Q_3)
$\underline{6}$
1 (R_3)

Step 2:

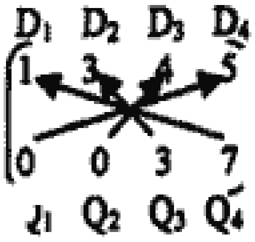
$$(ID)18-\begin{pmatrix}D_1\\ 1\\ \uparrow\\ 3\end{pmatrix}=18-3=15(ND)$$

2) 15 (7 (Q_4)
$\underline{14}$
1 (R_4)

Step 3:

$$ID10-\left[\begin{array}{cc}D_1 & D_2\\ 1 & 3\\ \multicolumn{2}{c}{\times}\\ 3 & 7\\ Q_3 & Q_4\end{array}\right]=10-16=-6\text{ (negative value)}$$

(or) 10 -

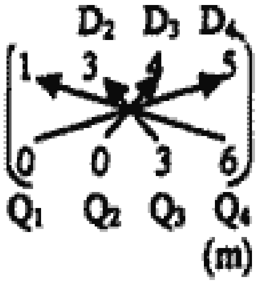


∴ We reduce the quotient Q_4 by 1.

2) 15 (6 [Q_4(m)]
$\underline{12}$
3 (R_4)

$$(ID)\ 30-\left[\begin{array}{cc}D_1 & D_2\\ 1 & 3\\ \multicolumn{2}{c}{\times}\\ 3 & 6\\ Q_3 & Q_4(m)\end{array}\right]=30-15=15\ (ND)$$

(or) 30 -



2) 15 (7 (Q_5)
$\underline{14}$
1 (R_5)

Step 4:

$$(\text{ID})10-\begin{array}{c}D_1\;D_2\;D_3\\ \left(\begin{array}{ccc}1&3&4\\3&6&7\end{array}\right)\\ Q_3\;Q_4\;Q_5\\ (m)\end{array}=10-37=-27\ \text{(negative value)(or)}10-\begin{array}{c}D_1\;D_2\;D_3\;D_4\\ \left(\begin{array}{cccc}1&3&4&5\\0&3&6&7\end{array}\right)\\ Q_2\;Q_3\;Q_4\;Q_5\\ (m)\end{array}$$

∴ We reduce the quotient Q_5 by 1.

2) 15 (6 [Q_5(m)]
 12
 3 (R_5)

$$(\text{ID})\ 30-\begin{array}{c}D_1D_2D_3\\ \left(\begin{array}{ccc}1&3&4\\3&6&6\end{array}\right)\\ Q_3\;Q_4\;Q_5\\ (m)(m)\end{array}=30-36=-6\ \text{(negative value) (or) }30-\begin{array}{c}D_1\;D_2\;D_3\;D_4\\ \left(\begin{array}{cccc}1&3&4&5\\0&3&6&6\end{array}\right)\\ Q_2\;Q_3\;Q_4\;Q_5\\ (m)\end{array}$$

∴ We reduce the quotient Q_5 further by 1.

2) 15 (5 [Q_5(m)]
 10
 5 [R_5 (m)]

$$(\text{ID})\ 50-\begin{array}{c}D_1D_2\;D_3\\ \left(\begin{array}{ccc}1&3&4\\3&6&5\end{array}\right)\\ Q_3\;Q_4Q_5\\ (m)(m)\end{array}=50-35=15\ \text{(ND)}$$

2) 15 (7 (Q_6)
 14
 1 (R_6)

Step 5:

$$(\text{ID})\ 10-\begin{array}{c}D_1\;D_2\;D_3\;D_4\\ \left(\begin{array}{cccc}1&3&4&5\\3&6&5&7\end{array}\right)\\ Q_3\;Q_4\;Q_5\;Q_6\\ (m)\;(m)\end{array}=10-61=-51\ \text{(negative value)}$$

∴ We reduce the quotient Q_6 by 1.

2) 15 (6 [Q_6(m)]
$\underline{12}$
3 [R_6 (m)]

$$\text{(ID) } 30 - \begin{array}{c} D_1 D_2\ D_3 D_4 \\ \left(\begin{array}{cccc}1&3&4&5\\3&6&5&6\end{array}\right) \\ Q_3 Q_4 Q_5\ Q_6 \\ (m)(m)(m) \end{array} = 30 - 60 = -30 \text{ (negative value)}$$

∴ We reduce the quotient Q_6 further by 1.

2) 15 (5 [Q_6(m)]
$\underline{10}$
5 [R_6 (m)]

$$\text{(ID) } 50 - \begin{array}{c} D_1 D_2\ D_3 D_4 \\ \left(\begin{array}{cccc}1&3&4&5\\3&6&5&5\end{array}\right) \\ Q_3 Q_4 Q_5\ Q_6 \\ (m)\ (m)(m) \end{array} = 50 - 59 = -9 \text{ (negative value)}$$

∴ We reduce the quotient Q_6 still further by 1

2) 15 (4 [Q_6(m)]
$\underline{8}$
7 [R_6 (m)]

$$\text{(ID) } 70 - \begin{array}{c} D_1\ D_2\ D_3 D_4 \\ \left(\begin{array}{cccc}1&3&4&5\\3&6&5&4\end{array}\right) \\ Q_3 Q_4\ Q_5\ Q_6 \\ (m)\ (m)(m) \end{array} \begin{array}{l} = 70 - [15 + 24 + 15 + 4] \\ = 70 - 58 = 12 \text{ (ND)} \end{array}$$

2) 12 (6 (Q_7)
$\underline{12}$
0 (R_7)

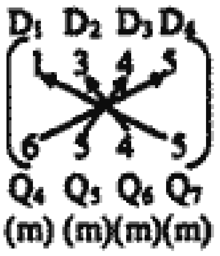
Step 6:

$$\text{(ID) } 0 - \begin{array}{c} D_1\ D_2\ D_3\ D_4 \\ \left(\begin{array}{cccc}1&3&4&5\\6&5&4&6\end{array}\right) \\ Q_4\ Q_5\ \ Q_6\ Q_7 \\ (m)\ (m)\ (m) \end{array} \begin{array}{l} = 0 - [30 + 20 + 12 + 6] \\ = 0 - 68 = -68 \text{ (negative value)} \end{array}$$

∴ We reduce the quotient Q_7 by 1

2) 12 (5 [Q_7 (m)]
<u>10</u>
2 [R_7 (m)]

(ID) 20 –

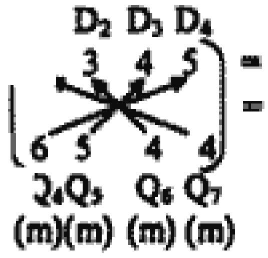


= 20 – [30 + 20 + 12 + 5]
= 20 – 67 = –47 (negative value)

∴ We reduce the quotient Q_7 further by 1.

2) 12 (4 [Q_7(m)]
<u>8</u>
4 [R_7 (m)]

(ID) 40 –

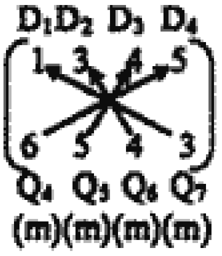


= 40 – [30 + 20 + 12 + 4]
= 40 – 66 = – 26 (ND)

∴ We reduce the quotient Q_7 further still by 1

2) 12 (3 [Q_7 (m)]
<u>6</u>
<u>6</u> [R_7 (m)]

(ID) 60 –



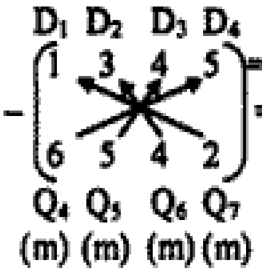
= 60 – [30 + 20 + 12 + 3]
= 60 – 65 = –5 (negative value)

∴ We reduce the quotient Q_7 further still more by 1.

2) 12 (2 [Q_7 (m)[
<u>4</u>
8 [R_7 (m)]

Vinculum

(ID) 80 –



= 80 – [30+20+12+2]

= 80 – 64 = 16 (ND) 1 3 4 5

7 8 0 0 0 0 0 0

/ / / / / / /

1 1 0 0 $\bar{1}$ 1 0

: 0 0 3 7 $\bar{3}$ 15 $\bar{9}$ 10 51

$Q_1 Q_2 Q_3\ Q_4\ Q_5\ Q_6\ Q_7\ Q_8\ Q_9$

0 0 3 7 $\bar{4}$ 5 $\bar{8}$ 5 1

. 0 0 3 6 5 4 2 5 1

Refer Example: 14 Page No·
for Vinculum working details

Case (b)(i): Where the dividend only has decimals

Refer 4 (3) – Straight Division for partition rules Page No.

Example 14: 89.69 + 243
(Up to 4 decimal places)

Current Method

24300) 89690 (0.3690946
72900
167900
145800
221000
218700
230000
218700
113000
97200
158000
145800
12200

Vedic Method

43 : 8 9. 6 9 0 0

2 : 2 5 5 5 5

$R_1\ R_2\ R_3\ R_4\ R_5$

(m)(m)(m) (m)(m)

.3 6 9 0 9

$Q_1\ Q_2\ Q_1\ Q_4\ Q_5$

Quotient = 0.36909

Vedic Method Steps:

Step 1:

2) 8 (4 (Q_1)
8
0 (R_1)

Step 2:

$$\text{(ID) } 9 - \left|\overset{D_1}{\overset{4}{\uparrow}}\right| = 9 - 16 = -7 \text{ (negative value)}$$

∴ We reduce the quotient by 1.

2) 8 (3 $[Q_1(m)]$
$\underline{6}$
2 R_1 (m)

$$\text{(ID) } 29 - \left(\overset{D_1}{\underset{\underset{Q_1(m)}{3}}{\overset{4}{\uparrow}}}\right) = 29 - 12 = 17 \text{ (ND)}$$

2) 17 (8 (Q_2)
$\underline{16}$
1 (R_2)

Step 3 :

$$\text{(ID) } 16 - \begin{array}{c} D_1\ \ D_2 \\ \begin{pmatrix} 4 & 3 \\ 3 & 8 \end{pmatrix} \\ Q_1\ \ Q_2 \\ (m) \end{array} = 16 - 41 = -25 \text{(negative value)}$$

D_1 D_2 = 16 – [9 + 32]

∴ We reduce the quotient by 1.

2) 17 (7 $[Q_2(m)]$
$\underline{14}$
3 $R_2(m)$

ND

Step2:

2) $\underline{\bar{7}}$ ($\bar{3}$ (Q_2)

$\underline{\bar{1}}$ (R_2)

Step 3 :

$$\bar{1}6 - \begin{array}{c} D_1\ \ D_2 \\ \begin{pmatrix} 4 & 3 \\ 4 & \bar{3} \end{pmatrix} \\ Q_1\ \ Q_2 \end{array} = \bar{1}6-[12+\bar{1}2] = \bar{4}$$

2) $\bar{4}$ ($\bar{2}$ (Q_3)
$\underline{\bar{4}}$
0 (R_3)

$$\text{(ID) } 36 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{ccc} 4 & & 3 \\ & \times & \\ 3 & & 7 \end{array}\right) \\ \begin{array}{cc} Q_1 & Q_2 \\ (m) & (m) \end{array} \end{array} \begin{array}{l} = 36 - [9 + 28] \\ = 36 - 37 = -1 \text{ (negative value)} \end{array}$$

∴ We reduce the quotient further.

2) 17 (6 [Q_2(m)]
12
5 R_2 (m)

$$\text{(ID) } 56 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left|\begin{array}{ccc} 4 & & 3 \\ & \times & \\ 3 & & 6 \end{array}\right| \\ \begin{array}{cc} Q_1 & Q_2 \\ (m) & (m) \end{array} \end{array} \begin{array}{l} = 56 - [9 + 24] \\ = 56 - 33 = 23 \text{ (ND)} \end{array}$$

2) 23 (11 (Q_3)
22
1 (R_3)

Step 4·

$$\text{(ID) } 19 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{ccc} 4 & & 3 \\ & \times & \\ 6 & & 11 \end{array}\right) \\ \begin{array}{cc} Q_2 & Q_1 \\ (m) & \end{array} \end{array} \begin{array}{l} = 19 - [18 + 44] \\ = 19 - 62 = 43 \text{ (negative value)} \end{array}$$

∴ We reduce quotient by 1.

2) 23 (10 [Q_3(m)]
20
3 R_3 (m)

$$\text{(ID) } 39 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{ccc} 4 & & 3 \\ & \times & \\ 6 & & 10 \end{array}\right) \\ \begin{array}{cc} Q_2 & Q_3 \\ (m) & (m) \end{array} \end{array} \begin{array}{l} = 39 - [18 + 40] \\ = 39 - 58 = 19 \text{ (negative value)} \end{array}$$

∴ We reduce the quotient further

Step4

$$9 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{ccc} 4 & & 3 \\ & \times & \\ \bar{3} & & \bar{2} \end{array}\right) \\ \begin{array}{cc} Q_2 & Q_1 \end{array} \end{array} = 26$$

2) 26 (13 (Q_4)
26
0 (R_4)

2) 23 (9 [Q_3(m)]
18
5 R_3 (m)

$$\boxed{Q_3\ (m) = 9}$$

$$(\text{ID})\ 59 - \begin{array}{c} \begin{pmatrix} 4 & & 3 \\ & \times & \\ 6 & & 9 \end{pmatrix} \\ Q_2 \quad Q_3 \\ (m) \quad (m) \end{array} \begin{array}{l} = 59 - [18 + 36] \\ = 59 - 54 = 5\ (\text{ND}) \end{array}$$

2) 5 (2 (Q_4)
4
1 (R_4)

$$\boxed{Q_4 = 2}$$

Step 5·

$$(\text{ID})\ 10 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 4 & & 3 \\ & \times & \\ 9 & & 2 \end{pmatrix} \\ Q_3 \quad Q_4 \\ (m) \qquad \end{array} \begin{array}{l} = 10 - [27 + 8] \\ = 10 - 35 = -25\ (\text{negative value}) \end{array}$$

∴ We reduce the quotient by 1

2) 5 (1 [Q_4(m)]
2
3 R_4 (m)

$$(\text{ID})\ 30 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 4 & & 3 \\ & \times & \\ 9 & & 1 \end{pmatrix} \\ Q_3 \quad Q_4 \\ (m) \quad (m) \end{array} \begin{array}{l} = 30 - [27 + 4] \\ \ \ 30 - 31 = -1\ (\text{negative value}) \end{array}$$

∵ We reduce quotient further.

2) 5 (0 [Q_4(m)]
0
5 R_4 (m)

$$(\text{ID})\ 50 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 4 & & 3 \\ & \times & \\ 9 & & 0 \end{pmatrix} \\ Q_3 \quad Q_4 \\ (m) \quad (m) \end{array} \begin{array}{l} = 50 - [27 + 0] \\ = 50 - 27 = 23\ (\text{ND}) \end{array}$$

Step5:

$$0 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 4 & & 3 \\ & \times & \\ \overline{2} & & 13 \end{pmatrix} \\ Q_3\, Q_4 \end{array} \begin{array}{l} = 0 - [\overline{6} + 52] \\ = 0 - 46 = \overline{46} = \overline{46} \end{array}$$

2) $\overline{46}$ ($\overline{23}$ (Q_5)
$\overline{46}$
0 (R_5)

2) 23 (11 (Q_5)
22
1 (R_5)

Step 6:

$$(\text{ID})\ 10 - \begin{array}{cc} D_1 & D_2 \\ \left(\begin{array}{cc} 4 & 3 \\ 0 & 11 \end{array}\right) \\ Q_4 & Q_5 \\ (m) & \end{array} = 10 - [0 + 44] = 10 - 44 = -33 \text{ (negative value)}$$

∴ We reduce the quotient by 1.

2) 23 (10 [Q_5(m)]
20
3 R_5 (m)

$$(\text{ID})\ 30- \begin{array}{cc} D_1 & D_2 \\ \left(\begin{array}{cc} 4 & 3 \\ 0 & 10 \end{array}\right) \\ Q_4 & Q_5 \\ (m) & (m) \end{array} = 30 - 40 = -10 \text{ (negative value)}$$

∴ We reduce the quotient further.

2) 23 (9 [Q_5(m)] $\boxed{Q_5\ (m) = 9}$
18
5 R_5 (m)

$$(\text{ID})\ 50- \begin{array}{cc} D_1 & D_2 \\ \left(\begin{array}{cc} 4 & 3 \\ 0 & 9 \end{array}\right) \\ Q_4 & Q_5 \\ (m) & (m) \end{array} = 50 - [0 + 36] = 50 - 36 = 14$$

∴ Quotient = 0.36909

Step6.

$$0 - \begin{array}{cc} D_1 & D_2 \\ \left(\begin{array}{cc} 4 & 3 \\ 13 & \overline{23} \end{array}\right) \\ Q_4 & Q_5 \end{array} = 0 - [39 + \overline{92}] = -\overline{53}$$

$$= 53$$

2) 53 (26 (Q_6)
52
1 (R_6)

Refer Page for further details of Vinculum Method.

Direct Vinculum

$$\begin{array}{ccccccccccc} 43 & 8 & 9 & . & 6 & ..9 & 0 & 0 & 0 & 0 & 0 \\ & & / & & / & / & / & / & / & / & / \\ & & 0 & & \bar{1} & 0 & 0 & 0 & 1 & \bar{1} & 0 \end{array}$$

$$\begin{array}{cccccccc} 4 & \bar{3} & \bar{2} & 13 & \overline{23} & 26 & 12 & \overline{20} \\ Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 \end{array}$$

$= 4\ \bar{3}\ \bar{2}\ 13\ \overline{23}\ 26\ 1\underline{2}\ \ 20$
$= 4\ \bar{3}\ \bar{1}\ 1\ \bar{1}\ 5\ \bar{4}\ 0$
$= 3\ 6\ 90946$

Example 15: 0.8927124 ÷ 96218734 (Dividen ting w decimal and Dhwajanka has seven digits)

Current Method

$$\frac{0\,8927124}{96218734}=\frac{8927124}{962187340000000}$$

$$\begin{array}{l}
962187340000000\ )\ 892712400000000\ (0.0000000927794 \\
\underline{865968606000000} \\
267437940000000 \\
\underline{192437468000000} \\
750004720000000 \\
\underline{673531138000000} \\
764735820000000 \\
\underline{673531138000000} \\
.912046820000000 \\
\underline{865968606000000} \\
460782140000000 \\
\underline{384874936000000} \\
759072040000000
\end{array}$$

Vedic Method

$$\begin{array}{r|ccccccccccccccccc}
6218734 & : & & & & & & & . & 8 & 9 & 2 & 7 & 1 & 2 & 4 & 0 \\
 & & & & & & & & & & / & / & / & / & / & / & / \\
9 & : & & & & & & & & & 8 & 8 & 10 & 14 & 23 & 21 & 24 \\
\hline
 & & .0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 2 & 7 & 7 & 9 & 4 & \\
 & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} & Q_{12} & Q_{13} & Q_{14} &
\end{array}$$

Quotient = 0.0000000927794

Vinculum:

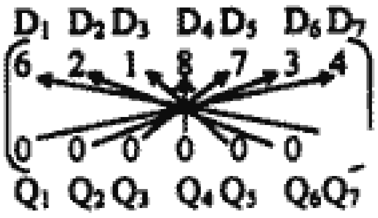
$$\begin{array}{r|ccccccccccccccc}
6218734 & & & & & & & 8 & 9 & 2 & 7 & 1 & 2 & 4 & 0 & 0 \\
 & & & & & & & & / & / & / & / & / & / & / & / \\
9 & & & & & & & & 8 & 8 & 1 & \bar{1} & \bar{3} & \bar{3} & \bar{4} & 0 \\
\hline
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 3 & \bar{2} & \bar{1} & \overline{10} & \bar{5} & \bar{2} & \bar{2} \\
 & & & & & & & & 9 & 3 & \bar{2} & \bar{1} & \bar{1}0\bar{5} & \bar{3} & &
\end{array}$$

Quotient = .00000000 9277 947 8

Vedic Method Steps:

Step 1: $Q_1 \rightarrow Q_7$ (Seven zero)

(ID) 8 –



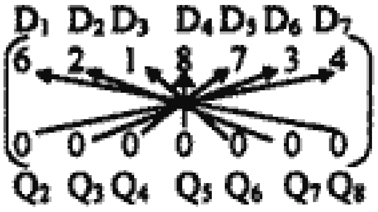
= 8 (ND)

Step 2:

9) 8 (0 (Q_8)
0
8 (R_8)

Step 3:

(ID) 89 –

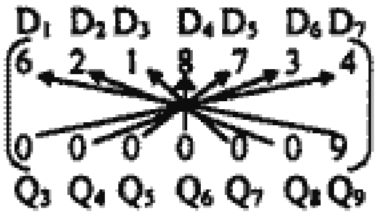


= 89 (ND)

9) 89 (9 (Q_9) | $Q_9 = 9$
81
8 (R_9)

Step 4:

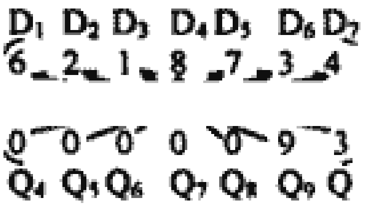
(ID) 82 –



= 82 – 54 = 28 (ND)

9) 28 (3 (Q_{10}) $\boxed{Q_{10} = 3}$
27
1 (R_{10})

Step 5:

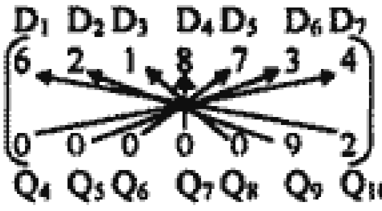
(ID) 17 −


− 17 · 36 19 (negative value)

9) $\bar{1}\ \bar{9}$ ($\bar{2}$ (Q$_{11}$)
$\underline{\bar{1}\ \bar{8}}$
$\bar{1}$ (R$_{11}$)

∴ We reduce the quotient by 1

9) 28 (2 [Q$_{10}$(m)]
$\underline{18}$
10 [R$_{10}$ (m)]

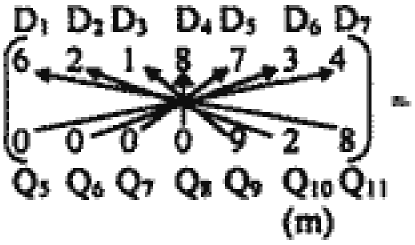
(ID)107 −


= 107 − 30 = 77 (ND)

9) 77 (8 (Q$_{11}$)
$\underline{72}$
5 [R$_{11}$]

$$\boxed{Q_{11} = 8}$$

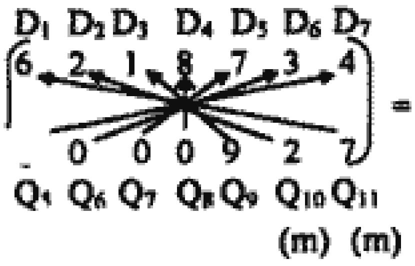
Step 6.

(ID) 51 −


= 51 − 61 = −10 (negative value)

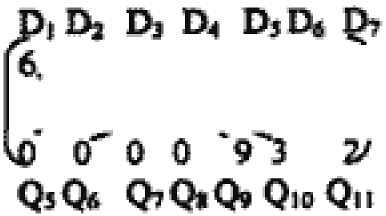
∴ We reduce the quotient by 1.

9) 77 (7 [Q$_{11}$(m)]
$\underline{63}$
14 [R$_{11}$ (m)]

ID 141 −


= 141 − 55 = 86 (ND) $\bar{1}$1−

Vinculum Continued. .



$= \bar{1}1 - [\overline{12} + 6 + 9]$
$= \bar{1}1 - [3] = \bar{1}\,\bar{2}$

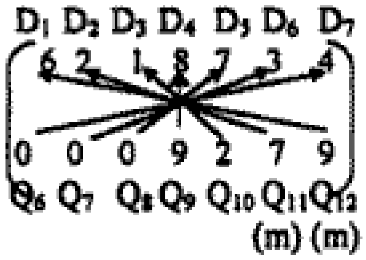
9) 86 (9 (Q_{12})
$\underline{81}$
5 (R_{12})

$\boxed{Q_{12} = 9}$

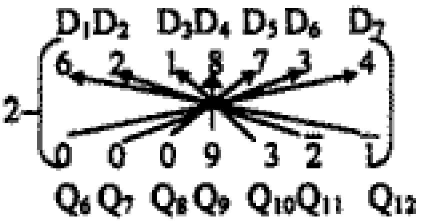
9) $\overline{12}$ ($\overline{1}$ (Q_{12})
$\underline{\overline{9}}$
$\overline{3}$ (R_{12})

Step 7:

(ID) 52–



=52–142= –90(or) $\overline{3}$2–
(negative value)



$= \overline{3}2 - [\overline{6} + \overline{4} + 3 + 72]$
$= \overline{3}2 - 65 = \overline{93}$

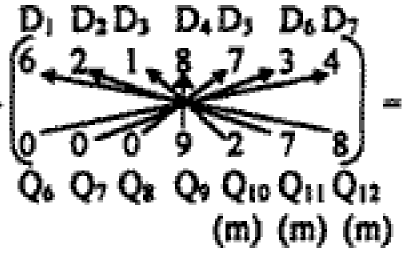
∴ We reduce the quotient by 1

9) 86 (8 [Q_{12}(m)]
$\underline{72}$
14 [R_{12} (m)]

$\boxed{Q_{12}\text{ (m)} = 8}$

9) $\overline{93}$ ($\overline{10}$ (Q_{13})
$\underline{90}$
$\underline{\overline{3}}$ (R_{13})

(ID) 142–



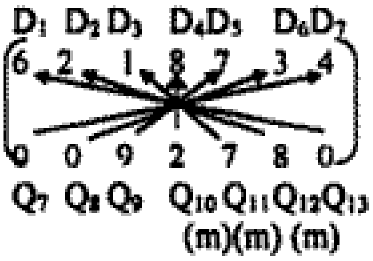
= 142 – 136 = 6 (ND)

9) 6 (0 (Q_{13})
$\underline{0}$
$\underline{6}$ (R_{13})

$\boxed{Q_{13} = 0}$

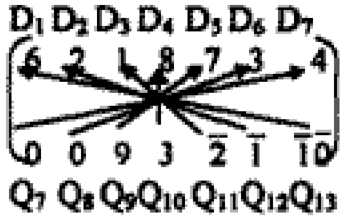
Step 8:

(ID) 64 –



= 64 – 102 = –38
(negative value)

$\overline{3}$4 –



$= \overline{3}4 - [\overline{60} + \overline{2} + \overline{2} + 24 + 63]$

$= \overline{3}4 - 23 = \overline{5}1 = \overline{49}$

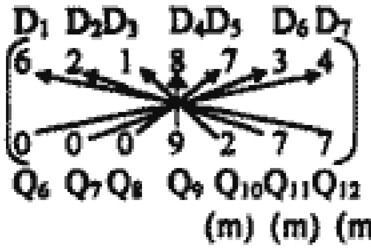
$9)\ \overline{4}\overline{9}\ (\overline{5}$ (Q_{14})
$\underline{\overline{4}\overline{5}}$
$\underline{+\overline{4}}$ (R_{14})

If we reduce the quotient, 0, by 1, then it becomes negative
Therefore, we reduce previous quotient, 8, by 1

9) 86 (7 [Q_{12}(m)]
$\underline{63}$
23 [R_{12} (m)]

$\boxed{Q_{12}\ (m) = 7}$

ID 232 –

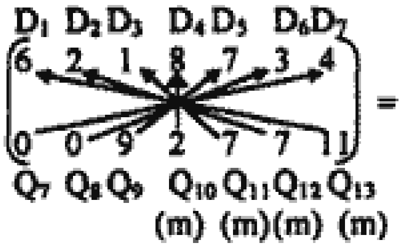


= 232 – 130 = 102 (ND)

9) 102 (11 [Q_{13}(m)]
$\underline{99}$
3 [R_{13} (m)]

$\boxed{Q_{13} - 11}$

(ID) 34 –



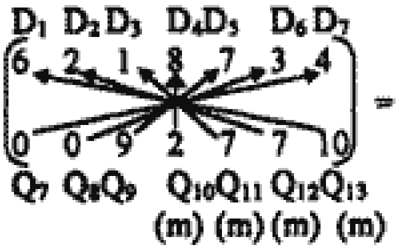
= 34 – 166 = –132 (–ve value)

∴ We reduce the quotient 11 by 1

9) 102 (10 [Q_{13}(m)]
$\underline{90}$
12 [R_{13} (m)]

$\boxed{Q_{13}\ (m) = 10}$

ID 124 –



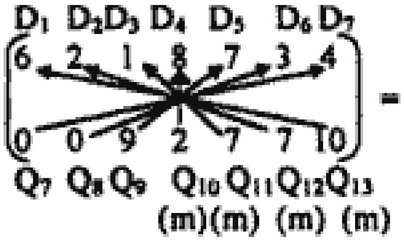
= 124 – 160 = –36 (–ve value)

∴ We reduce the quotient further

9) 102 (9 [Q_{13} (m)]
$\underline{81}$
21 [R_{13} (m)]

$\boxed{Q_{13}\ (m) = 9}$

ID 214 –



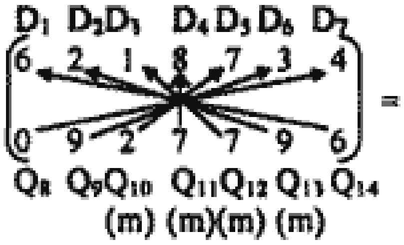
= 214 – 154 = 60 (ND)

9) 60 (6 (Q_{14})
<u>54</u>
6 [R_{14}]

$\boxed{Q_{14} = 6}$

Step 9:

ID 60 –



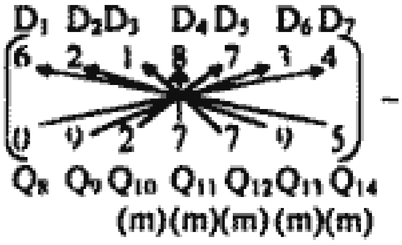
= 60 – 158 = –98
(negative value)

∴ We reduce the quotient by 1.

9) 60 (5 [Q_{14}(m)]
<u>45</u>
15 [R_{14} (m)]

$\boxed{Q_{14}(m) = 5}$

ID 150 –



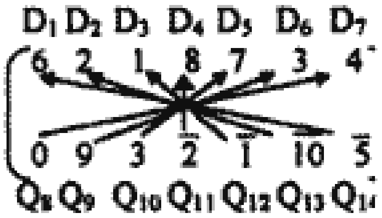
– 150 152 - 2 (negative value)

∴ We reduce the quotient further

9) 60 (4 [Q_{14}(m)]
<u>36</u>
24 [R_{14} (m)]

$\boxed{Q_{14} (m) = 4}$

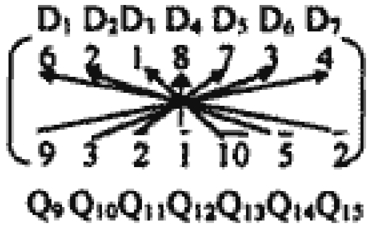
$\bar{4}0$ -



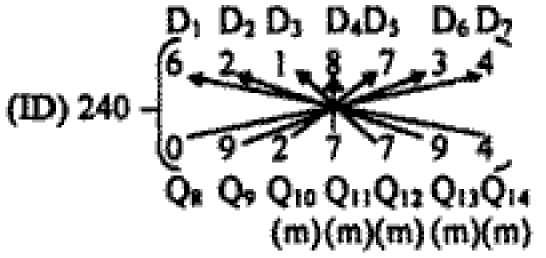
$= \bar{4}0 - [\bar{1}9] = \bar{3}9$
$= \bar{3} \quad 1\bar{1} = \bar{2}\bar{1}$

9) $\bar{2}\bar{1}$ ($\bar{2}$ (Q_{15})
$\underline{\bar{1}\bar{8}}$
$\underline{\bar{3}}$

$\bar{3}0$ -



9) $\bar{2}0$ ($\bar{2}$ (Q_{16})
$\underline{\bar{1}\bar{8}}$
$\underline{\bar{1}\bar{8}}$ = $\bar{2}$ (R_{16})



240 − 146 = 94 (ND)

∴ Quotient = 0.000000009277948

Case (b)(ii): Where the divisor only has decimals.

The division is carried out in the usual way not taking cognisence of the decimal while dividing. After the division is over, the decimal point is shifted towards right side in the quotient through the same number of digits that are existing in the divisor.

Example 16: 15628 ÷ 23.4

Current Method

$\frac{15628}{23.4}$ $\frac{156280}{234}$

23.4) 156280 (667 86324
$\underline{1404}$
1588
$\underline{1404}$
1840
$\underline{1638}$
2020
$\underline{1872}$
1480
$\underline{1404}$
760
$\underline{702}$
580
$\underline{468}$
1120

Vedic Method

34 | 15 6: 2 0 0 0 0
/ / / / / / / /
3 : 6 6 7 6 4 3 4
R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8
(m) (m) (m) (m) (m) (m) (m)

6 6 : 7 8 6 3 2 4
Q_1 Q_2 Q_3 Q_4 Q_5 Q_6 Q_7 Q_8
(m) (m) (m) (m) (m) (m)(m) (m)

Quotient = 667.86324

Q = 667.86324

Vedic Method Steps:

Step 1:

2) 15 (7 (Q_1)
$\underline{14}$
1 (R_1)

Step 2:

$$\text{(ID) } 16-\begin{array}{c}\left.\begin{matrix}\uparrow\\ 7\end{matrix}\right|\\ Q_1\end{array} = 16-21 = -5 \text{(negative value)}$$

∴ We reduce the quotient by 1

Step2:

$$\begin{array}{l} 2\ )\ \bar{5}\ (\ \bar{2}(Q_2) \\ \quad\ \ \underline{\bar{4}} \\ \quad\ \ \bar{1}\ \ (R_2) \end{array}$$

$$\begin{array}{l} 2)\ 15\ (6\ [Q_1(m)] \\ \quad \underline{12} \\ \quad\ 3\ R_1(m) \end{array}$$

$$\boxed{Q_1\ (m) = 6}$$

$$\text{(ID) } 36-\begin{array}{c} D_1 \\ \begin{bmatrix} 3\\ \uparrow\\ 6\end{bmatrix} \\ Q_1(m)\end{array} = 36 - 18 = 18 \text{ (ND)}$$

$$\begin{array}{l} 2)\ 18\ (9\ (Q_2) \\ \quad \underline{18} \\ \quad\ 0\ (R_2) \end{array}$$

Step 3:

$$\text{(ID) } 2-\begin{array}{c} D_1\ \ D_2 \\ \begin{bmatrix} 3 & & 4\\ & \times & \\ 6 & & 9\end{bmatrix} \\ Q_1(m)\ \ Q_2\end{array} \begin{array}{l} -2 \quad (27+24) \\ = 2 - 51 - \cdot 49 \text{ (negative value)} \end{array}$$

· We reduce the quotient

Step3:

$$\bar{1}2-\begin{array}{c} D_1\ \ D_2 \\ \left(\begin{matrix} 3 & & 4\\ & \times & \\ 7 & & \bar{2}\end{matrix}\right. \\ Q_1\ \ Q_2\end{array}$$

$$\begin{aligned} &= \bar{1}2 - (\bar{6}+28) \\ &= \bar{1}2 - (22) = \bar{1}2 + \bar{2}\bar{2} \\ &= \bar{3}0 \end{aligned}$$

$$\begin{array}{l} 2)\ 18\ (8\ [Q_2(m)] \\ \quad \underline{16} \\ \quad\ 2\ R_2\ (m) \end{array}$$

$$\begin{array}{l} 2\ )\ \bar{3}0\ (\ \overline{15}\ \ (Q_3) \\ \quad\ \ \underline{\bar{3}0} \\ \quad\ \ \ 0\ \ \ (R_3) \end{array}$$

$$(\text{ID})\ 22 - \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 6 & 8 \end{array}\right) \\ Q_1 \quad Q_2 \\ (m) \quad (m) \end{array} = 22 - 48 = -26 \text{ (negative value)}$$

∴ We reduce the quotient further

$$\begin{array}{l} 2)\ 18\ (7\ [Q_2(m)] \\ \quad \underline{14} \\ \quad\ 4\ R_2(m) \end{array}$$

$$(\text{ID})\ 42 - \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 6 & 7 \end{array}\right) \\ Q_1 \quad Q_2 \\ (m) \quad (m) \end{array} = 42 - 45 = -3 \text{ (negative value)}$$

∴ We reduce the quotient further

$$\begin{array}{l} 2)\ 18\ (6\ [Q_2\,(m)] \\ \quad \underline{12} \\ \quad\ 6\ R_2\,(m) \end{array}$$

$$(\text{ID})\ 62 - \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 6 & 6 \end{array}\right) \\ Q_1 \quad Q_2 \\ (m) \quad (m) \end{array} = 62 - 42 = 20$$

$$\begin{array}{l} 2)\ 20\ (10\ (Q_3) \\ \quad \underline{20} \\ \quad\ 0\ (R_3) \end{array}$$

Step 4:

$$(\text{ID})\ 8 - \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 6 & 10 \end{array}\right) \\ Q_2 \quad Q_3 \\ (m) \quad \end{array} = 8 - 54 = -46 \text{ (negative value)}$$

We reduce the quotient by 1

Step4:

$$\begin{array}{ccl} D_1 & D_2 & \\ 3 & 4 & \\ \multicolumn{2}{c}{\times} & = 8 - [\overline{45} + \bar{8}] \\ \bar{2} & \overline{15} & = 8 - [\bar{5}\bar{3}] = 8 + 53 \\ Q_2 & Q_3 & = 61 \end{array}$$

2) 20 (9 [Q_3(m)]
<u>18</u>
2 R_3(m)

$$(\text{ID})\ 28 - \begin{array}{c}\begin{array}{cc} D_1 & D_2 \end{array}\\ \begin{pmatrix} 3 & & 4\\ & \times & \\ 6 & & 9 \end{pmatrix}\\ \begin{array}{cc} Q_2 & Q_3\\ (m) & (m)\end{array}\end{array} = 28 - 51 = -23 \text{ (negative value)}$$

∴ We reduce the quotient further

2) 20 (8 [Q_3(m)]
<u>16</u>
4 R_3 (m)

$$(\text{ID})\ 48 - \begin{array}{c}\begin{array}{cc} D_1 & D_2 \end{array}\\ \begin{pmatrix} 3 & & 4\\ & \times & \\ 6 & & 8 \end{pmatrix}\\ \begin{array}{cc} Q_2 & Q_3\\ (m) & (m)\end{array}\end{array} = 48 - 48 = 0 \text{ (ND)}$$

2) 0 (0 (Q_4
<u>0</u>
0 R_4

Step 5:

$$(\text{ID})\ 0 - \begin{array}{c}\begin{array}{cc} D_1 & D_2 \end{array}\\ \left|\begin{matrix} 3 & & 4\\ & \times & \\ 8 & & \end{matrix}\right|\\ \begin{array}{cc} Q_3 & Q\\ (m) & \end{array}\end{array} = 0 - 32 = -\ 32 \text{ (negative value)}$$

∴ If we reduce the quotient, 0, it becomes negative Therefore, we reduce the previous quotient, 8, by 1.

2) 20 (7 [Q_3(m)]
<u>14</u>
6 [R_3 (m)]

$\boxed{Q_3(m) = 7}$

2) 61 (30 (Q_4)
<u>60</u>
1 (R_4)

Step5:

$$10 - \begin{array}{c}\begin{array}{cc} D_1 & D_2 \end{array}\\ \begin{pmatrix} 3 & & 4\\ & \times & \\ \overline{15} & & 30 \end{pmatrix}\\ \begin{array}{cc} Q_3 & Q_4 \end{array}\end{array} \begin{array}{l} = 10 - [90 + 60]\\ = 10 - [30] = \overline{20}\end{array}$$

$Q_3\ Q_4 = \overline{20}$

2) $\overline{20}$ ($\overline{10}$ (Q_5)
$\overline{20}$
0 (R_5)

$$\text{(ID) } 68 - \underset{\substack{Q_2\ \ Q_3\\(m)\ (m)}}{\overset{D_1\ \ D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 6 & & 7 \end{pmatrix}}} = 68 - 45 = 23 \text{ (ND)}$$

2) 23 (11 [Q_4 (m)]
$\underline{22}$
1 [R_4 (m)]

$$\text{(ID) } 10 - \underset{\substack{Q_3\ \ Q_4\\(m)\ (m)}}{\overset{D_1\ \ D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 7 & & 11 \end{pmatrix}}} = 10 - 61 = -51 \text{ (negative value)}$$

∴ We reduce the quotient by 1.

2) 23 (10 [Q_4(m)] $\boxed{Q_4(m) = 10}$
$\underline{20}$
3 [R_4 (m)]

$$\text{(ID) } 30 - \underset{\substack{Q_3\ \ Q_4\\(m)\ (m)}}{\overset{D_1\ \ D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 7 & & 10 \end{pmatrix}}} = 30 - 58 = -28 \text{ (negative value)}$$

∴ We reduce quotient further

2) 23 (9 [Q_4(m)]
$\underline{18}$
5 [R_4 (m)]

$$\text{(ID) } 50 - \underset{\substack{Q_3\ \ Q_4\\(m)\ (m)}}{\overset{D_1\ \ D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 7 & & 9 \end{pmatrix}}} = 50 - 55 = -5 \text{ (negative value)}$$

∴ We reduce quotient further

2) 23 (8 [Q_4(m)]
<u>16</u>
7 [R_4 (m)]

$\boxed{Q_4\ (m) = 8}$

$$(\text{ID})\ 70 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 7 & 8 \end{array}\right) \\ \begin{array}{cc} Q_3 & Q_4 \\ (m) & (m) \end{array} \end{array} = 70 - 52 = 18\ (\text{ND})$$

2) 18 (9 (Q_5)
<u>18</u>
0 (R_5)

Step 6:

$$(\text{ID})\ 0 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 8 & 9 \end{array}\right) \\ \begin{array}{cc} Q_4 & Q_5 \\ (m) & \end{array} \end{array} = -59\ (\text{negative value})$$

∴ We reduce the quotient by 1.

2) 18(8 [Q_5(m)]
<u>16</u>
2 [R_5 (m)]

$$(\text{ID})\ 20 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 8 & 8 \end{array}\right) \\ \begin{array}{cc} Q_4 & Q_5 \\ (m) & (m) \end{array} \end{array} = 20 - 56 = -36\ (\text{negative value})$$

∴ We reduce the quotient further.

2) 18 (7 [Q_5(m)]
<u>14</u>
4 [R_5 (m)]

$$(\text{ID})\ 40 - \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 8 & 7 \end{array}\right) \\ \begin{array}{cc} Q_4 & Q_5 \\ (m) & (m) \end{array} \end{array} = 40 - 53 = -13\ (\text{negative value})$$

Step6:

$$0 - \begin{array}{c} D_1D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 30 & \overline{10} \end{array}\right) \\ Q_4\ Q_5 \end{array} = 0 - [\,\overline{30} + 120] = \overline{90}$$

2) $\overline{90}$ ($\overline{45}$ (Q_6)
<u>$\overline{90}$</u>
0 (R_6)

2) 18 (6 [Q_5(m)]
<u>12</u>
6 [R_5 (m)]

$\boxed{Q_5\ (m) = 6}$

$$\text{(ID) } 60 - \begin{array}{cc} D_1 & D_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 8 & & 6 \end{pmatrix}} \\ Q_4 & Q_5 \\ (m) & (m) \end{array} = 60 - 50 = 10 \text{ (ND)}$$

2) 10 (5 (Q_6)
<u>10</u>
0 (R_6)

Step 7:

$$\text{(ID) } 0 - \begin{array}{cc} D_1 & D_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 6 & & 5 \end{pmatrix}} \\ Q_5 & Q_6 \\ (m) & \end{array} = -39 \text{ (negative value)}$$

∴ We reduce the quotient

2) 10 (4 [Q_6(m)]
<u>8</u>
2 [R_6 (m)]

$$\text{(ID) } 20 - \begin{array}{cc} D_1 & D_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 6 & & 4 \end{pmatrix}} \\ Q_5 & Q_6 \\ (m) & (m) \end{array} = 20 - 36 = -16 \text{ (negative value)}$$

2) 10 (3 [Q_6 (m)]
<u>6</u>
4 [R_6 (m)]

$$\text{(ID) } 40 - \begin{array}{cc} D_1 & D_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 6 & & 3 \end{pmatrix}} \\ Q_5 & Q_6 \\ (m) & (m) \end{array} = 40 - 33 = 7 \text{ (ND)}$$

Step7:

$$0 - \begin{array}{cc} D_1 & D_2 \\ \multicolumn{2}{c}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ \overline{10} & & \overline{45} \end{pmatrix}} \\ Q_5 & Q_6 \end{array} = 0 - [\,\overline{40} + \overline{135}\,] = 175$$

2) 175 (87 (Q_7)
<u>174</u>
1 (R_7)

2) 7 (3 (Q_7)
   $\underline{6}$
   1 [R_7]

Step 8:

$$\text{(ID) } 10 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ 3 & & 3 \end{pmatrix} \\ Q_6 \quad Q_7 \\ (m) \quad \end{array} = 10 - 21 = -11 \text{ (negative value)}$$

∴ We reduce the quotient.

2) 7 (2 [Q_7(m)]
   $\underline{4}$
   3 [R_7 (m)]

$\boxed{Q_7\text{ (m)} = 2}$

$$\text{(ID) } 30 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ 3 & & 2 \end{pmatrix} \\ Q_6 \quad Q_7 \\ (m)(m) \end{array} = 30 - 18 = 12 \text{ (ND)}$$

∴ Quotient = 667.8632

Step 8:

$$10 - \begin{pmatrix} \underline{3} & & 4 \\ & \times & \\ 45 & & 87 \end{pmatrix} = 10 - (261 + \bar{1}\bar{8}0)$$

$$= 10 - 1\bar{2}1$$

$$= 10 + \bar{1}2\bar{1}$$

$$= \bar{1}3\bar{1} = \bar{7}\bar{1}$$

$$\begin{array}{l} 2\,)\ \bar{7}\,\bar{1}\ (\ \bar{3}\,\bar{5} \\ \quad\ \underline{\bar{6}\quad} \\ \quad\ \bar{1}\bar{1} \\ \quad\ \underline{\bar{1}0} \\ \quad\ \ \underline{\ \bar{1}\ } \end{array}$$

Vinculum:

$$\begin{array}{r|l} 34 & 15\ \ 6:\ \ 2\ \ 8\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 \\ & \quad\ /\ \ /\ \ /\ \ /\ \ /\ \ /\ \ /\ \ / \\ 2 & 1\ \ \ :\bar{1}\ \ 0\ \ 1\ \ 0\ \ 0\ \ 1\ \ \bar{1} \\ \hline & 7\ \ \bar{2}:\ \ \overline{15}.30\ \ \overline{10}\ \ \overline{45}\ \ 87\ \ \overline{35} \\ & Q_1\ Q_2\ \ Q_3\ Q_4\ \ Q_5 Q_6 Q_7\ Q_8 \end{array}$$

As the divisor has one digit after decimal the decimal point is shifted by one digit towards right.

Quotient = $7\overline{32}.\overline{14}34\bar{5}$
           = 667 86335

Case (b)(iii): In case the decimal point exists both in the divisor as well as in the dividend, then both Case (b)(i) and Case (b)(ii) are applied in order.

Example 17: 134 289 ÷ 2 76

Current Method

$$\frac{134.289}{2\ 76} = \frac{134289}{2760}$$

```
2760)134289(48.6554
     11040
      23889
      22080
       18090
       16560
        15300
        13800
         15000
         13800
          12000
          11040
            960
```

Vedic Method [as per b(i)]

```
76 | 1 : 3  4. 2  8  9  0  0
   |      /  /  /  /  /  /  /
2  |   : 1  5 10 10  8  8  7
----------------------------
   | 0 .   4  8  6  5  5  4
```

Quotient = 48 6554 (as per b[ii])

Example 18: 2.1387 ÷ 0. 312

Current Method

$$\frac{2.1387}{0.312} = \frac{21387}{3120}$$

```
3120) 21387(6.85480769
      18720
       26670
       24960
        17100
        15600
         15000
         12480
          25200
          24960
           24000
           21840
            21600
            18720
             28800
             28080
               720
```

Vedic Method (as per b[ii])

```
12 | :   2 .  1  3  8  7  0  0  0  0
   |         /  /  /  /  /  /  /  /
3  | :       2  3  3  3  4  2  4  3
------------------------------------
   | .  0  0    6  8  5  4  8  0  7
```

Quotient = 6.85480769 (as per b[ii])

Example 19: 0.461397 ÷ 123 4

Current Method

```
0 461397     461397
  123 4      123400000

123400000)  561397000  (0.00373903
            370200000
            911970000
            863800000
             481700000
             370200000
             1115000000
             1110600000
                 440000000
                 370200000
                  69800000
```

Vedic Method (as per b [i])

```
  34              .4  6   1 3   9  7  0  0
                     /   / /   /  /  /  /
  12                 4  10 8  14  4  8  8
            .  0  0  0  3   7 3   9  0  3
```

Quotient = 0.00373903 (as per b [ii])

Working details of Vinculum Method and reduction method are shown in page No

(b) Reduction Method (Simplified)for Straight Division:

While working out a division problem using straight division, some of intermediate dividends give rise to negative values, on subtraction of the Urdhva – Tiryak multiplication value of the Dhwajanka (D_1D_2.....etc)and quotients (Q_1Q_2... ...etc). Reduction of this negative value, by modifying the previous quotients or by considering the negative value itself as vinculum, one can carry out the problem These methods are already explained earlier

There is one more method for reduction and the procedure is as follows

(1) The partition rules of dividend and divisor are same as explained earlier

(2) The reduction starts when one arrives at a negative value, in the process of subtraction of Urdhva – Tiryak multiplication result from the intermediate dividend

At this Stage·

(a) One has to reduce the previous quotient by '1'

(b) Add the part divisor to the current intermediate remainder

(c) Carry out the process of subtraction , with the new quotient and remainder

(d) If one again gets a negative value, repeat steps (a),(b),(c) until the negative value vanishes This procedure is elaborately explained in the following examples.

In the straight division, we come across the modifications of dividend and reduction of quotients

Example 1: 7896456 + 34 (Example 3 Page No)

D							
4	7	8	9	6	4	5	6
		/	/	/	/	/	/
		1	1	1	2	1	2
(P.D)3						④	⑤
	2	3	2	2	5	9	
					④	⑧	
	Q_1	Q_2	Q_1	Q_4	Q_5	Q_6	

The first five steps do not involve any modification, so they are the same

Step1:

$3\ )\ 7\ (\ 2\ (Q_1)$

$\underline{6}$

$\underline{1}\ (R_1)$

Step 2:

$$18 - \underset{Q_2}{\overset{D}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 10$$

$3\ )\ 10\ (\ 3\ (Q_2)$

$\underline{9}$

$\underline{1}\ (R_2)$

Step3:

$$19 - \underset{Q_2}{\overset{D}{\begin{pmatrix}4\\ \uparrow\\ 3\end{pmatrix}}} = 7$$

$3\ )\ 7\ (\ 2\ (Q_3)$

$\underline{6}$

$\underline{1}\ (R_3)$

Step 4:

$$16 - \underset{Q_3}{\overset{D}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 8$$

$3\ )\ 8\ (\ 2\ (Q_4)$

$\underline{6}$

$\underline{2}\ (R_4)$

Step 5:

$$24 \quad \underset{Q_4}{\overset{D}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} \quad 16$$

$3\ )\ 16\ (\ 5\ (Q_5)$

$\underline{15}$

$\underline{1}\ (R_5)$

Step 6: Here the Urdhva multiplication and its subtraction from the intermediate dividend gives a negative value

$$\text{i.e.}\quad 15 - \overset{D}{\underset{Q_5}{\begin{pmatrix} 4 \\ \uparrow \\ 5 \end{pmatrix}}} = 15 - 20 = -5.$$

So a reduction of '1' is made in the quotient Q_5 and the part divisor (PD) is added to the remainder R_5 So $Q_5(m) = 4$, $R_5(m) = 1 + 3 = 4$

Now the new intermediate dividend (ID) is 45

D
(4)
45 - 29
$Q_5(m)$

Thus the negative value is eliminated. Now following the usual procedure, the problem is carried out

$$\begin{array}{l} 3\ )\ 29\ (\ 9\ (Q_6) \\ \quad\ \underline{27} \\ \quad\ \ \ 2\ \ (R_6) \end{array}$$

Step7: We have entered into the remainder region

D

Again, 26 - 26 – 3 6 = - 10, a negative value is obtained

R_6

So a reduction of '1' in $Q_6 = 9$ and addition of (PD) to $R_6 = 2$ are made to obtain modified quotient $Q_6(m)$ as 8 and modified remainder $R_6(m)$ as 5.

Now the Urdhva multiplication and subtraction gives a positive value, 24 D

i.e., 56 - = 56 – 32 = 24

$Q_6(m)$

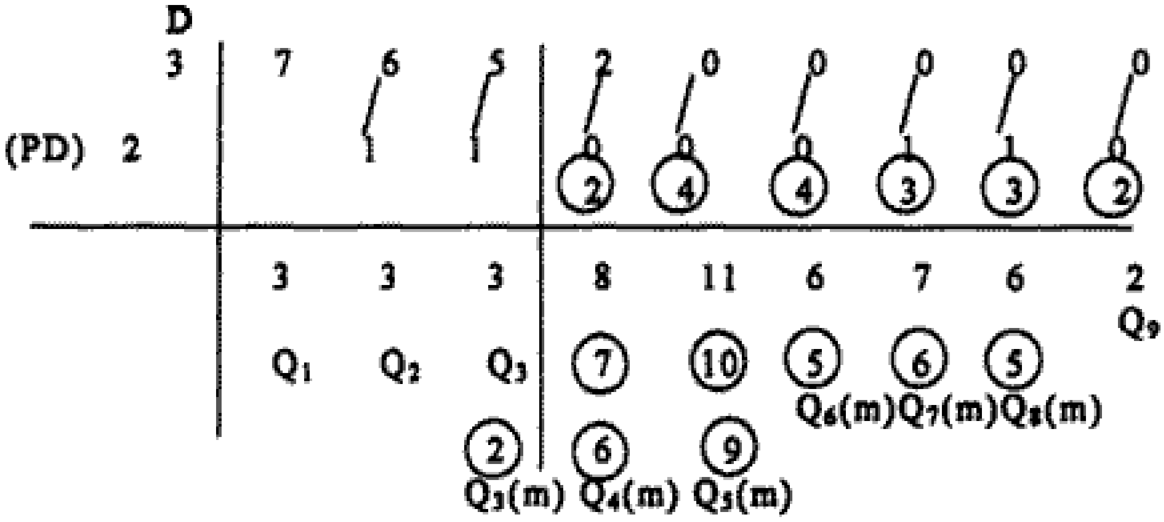
This is considered as the final remainder.

∴ Final Quotient = 232248
Final Remainder = 24

If one wishes to get decimals, the procedure can be continued as explained before. Some more examples are illustrated below and their working details are also included

Example 2 : 7652 + 23 (Example 5, Page No:)

Worked out for four decimals



Step1:

2) 7 (3 (Q_1)
<u>6</u>
<u>1</u> (R_1)

Step 3: $15 - \overset{D}{\underset{Q_2}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}}} = 6$

2) 6 (3 (Q_3)
<u>6</u>
<u>0</u> (R_3)

Step 2: $16 - \overset{D}{\underset{Q_1}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}}} = 7$

2) 7 (3 (Q_2)
∴ Modified $Q_3 = 2$
<u>6</u>
<u>1</u> (R_2)

Step 4: $02 - \overset{D}{\begin{pmatrix}3\\ \uparrow\\ 3\end{pmatrix}} = -7$, a negative value

∴ Modified $R_3 = 0 + 2 = 2$
Now the new ID is 22

ie, $22 - \overset{D}{\underset{Q_3(m)}{\begin{pmatrix}3\\ \\ 2\end{pmatrix}}} = 16$

2) 16 (8 (Q_4)
<u>16</u>
<u>0</u> (R_4)

Step 5: $$0 - \overset{D}{\underset{Q_4}{\begin{pmatrix} 3 \\ \uparrow \\ 8 \end{pmatrix}}} = -24, \text{ a negative value.}$$

Hence Q_4 value is reduced by 1

$Q_4(m) = 7$, $R_4(m) = 2$

$$20 - \overset{D}{\underset{Q_4(m)}{\begin{pmatrix} 3 \\ \uparrow \\ 7 \end{pmatrix}}} \; -1 \text{ again a negative value}$$

So reduce $Q_4(m)$ further to 6 and raise $R_4(m)$ to $2 + 2 = 4$

Now the ID is 40

$$40 - \overset{D}{\underset{Q_4(m)}{\begin{pmatrix} 3 \\ \uparrow \\ 6 \end{pmatrix}}} = 22$$

$$\begin{array}{l} 2\ )\ 22\ (\ 11\ (Q_5) \\ \quad\ \ \underline{22} \\ \quad\ \ \underline{\ 0\ }\ (R_5) \end{array}$$

Step 6: $$0 - \overset{D}{\underset{Q_5}{\begin{pmatrix} 3 \\ \uparrow \\ 11 \end{pmatrix}}} = -33$$

So reduce Q_5 to 10 and increase R_5 to 2.

$$20 - \overset{D}{\underset{Q_5(m)}{\begin{pmatrix} 3 \\ \uparrow \\ 10 \end{pmatrix}}} = -10$$

Again a negative a value.

So reduce Q_5 further to 9 and raise R_4 to 4.

$$\therefore\ 40 - \underset{Q_5(m)}{\overset{D}{\begin{pmatrix}3\\ \uparrow\\ 9\end{pmatrix}}} = 13$$

$2\ )\ 13\ (\ 6\ (Q_6)$
$\underline{12}$
$\underline{1}\ (R_6)$

Step 7: $$10 - \underset{Q_6}{\overset{D}{\begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix}}} = -8$$

∴ Q_6 is reduced to 5 and the remainder is raised to 3

$$30 - \underset{Q_6(m)}{\overset{D}{\begin{pmatrix}3\\ \uparrow\\ 5\end{pmatrix}}} = 15$$

$2)\ 15\ (\ 7\ (Q_7)$
$\underline{14}$
$\underline{1}\ (R_7)$

Step 8: $$10 - \underset{Q_7}{\overset{D}{\begin{pmatrix}3\\ \uparrow\\ 7\end{pmatrix}}} = -11$$

∴ Q_7 is reduced to 6 and R_7 is raised to 3.

$$30 - \underset{Q_7(m)}{\overset{D}{\begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix}}} = 12$$

$2)\ 12\ (\ 6\ (Q_8)$
$\underline{12}$
$\underline{0}\ (R_8)$

Quotient in Decimal form is 332.69562

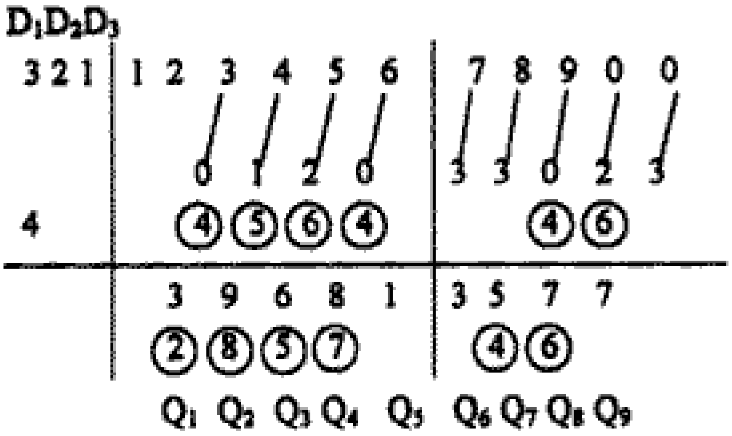
Note One has to stop the calculation after seeing to the correctness of the last quotient i.e., if one wants 4 decimal places, then calculate for the fifth decimal and see that there is no negative value. If the negativity continues reduce the previous quotient until the negativity vanishes and so on.

Example 3: 123456789 + 4321 (Example:9 , Page No. decimal calculation)

Current Method

4321) 123456789 (28571.346
8642
37036
34568
24687
21605
30828
30247
5819
4321
R_1 14980
12963
R_2 20170
17284
R_3 28860
25926
R_4 2934.

Vedic Method



Quotient = 28571.346

○ Indicates modification

If the part divisor cannot divide the first digit then one can consider first two digits or three digits as the case may be with reference to the number of digits in (PD) For example , in this problem PD (4) cannot divide the first digit, hence the first two digits 1.2 are considered for division by 4 giving a quotient 3 and remainder zero.

1. 12 + 4 = 3, (0) $Q_1 = 3, R_1 = 0$

2 03 – = –6(ve)

So reduce Q_1 = 3 by 1 and so $R_1(m) = 4$

$$43 - \begin{pmatrix} 3 \\ \uparrow \\ 2 \end{pmatrix} = 43 - 6 = 37 + 4 = 9\ (1)$$

$$Q_2 (R_2)$$

3. $$14 - \begin{pmatrix} 3 & & 2 \\ & \times & \\ 2 & & 9 \end{pmatrix} = 14 - 31 = -17(-ve)$$

Q_2 is reduced to 8 and R_2 is increased to 5.

$$54 - \begin{pmatrix} 3 & & 2 \\ & \times & \\ 2 & & 8 \end{pmatrix} = 54 - 28 = 26 + 4 = 6\ (2)$$

$$Q_3 (R_3)$$

4 $$25 - \begin{pmatrix} 3 & 2 & 1 \\ & \times & \\ 2 & 8 & 6 \end{pmatrix} = 25 - 36 = -11\ (-ve)$$

Q_3 is reduced to 5 and R_3 raised to 6.

$$65 - \begin{pmatrix} 3 & 2 & 1 \\ & \times & \\ 2 & 8 & 5 \end{pmatrix} = 65 - 33 = 32 + 4 = 8\ (0)$$

$$Q_4 (R_4)$$

5 $$06 - \begin{pmatrix} 3 & 2 & 1 \\ & \times & \\ 8 & 5 & 8 \end{pmatrix} = 6 - 42 = -36\ (-ve)$$

Q_4 is Reduced to 7 and R_4 is raised to 4

$$46 - \begin{pmatrix} 3 & 2 & 1 \\ & \times & \\ 8 & 5 & 7 \end{pmatrix} = 46 - 39 = 7 + 4 = 1\ ,\ (3)$$

$$Q_5 (R_5)$$

1st decimal calculation

6. $$37 - \begin{pmatrix} 3 & 2 & 1 \\ & \times & \\ 5 & 7 & 1 \end{pmatrix} = 37 - 22 = 15 + 4 = 3\ ,\ (3)$$

2nd decimal:

$$7 \qquad 38 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \\ 7&1&3\end{array} = 38 - 18 = 20 \div 4 = 5(0)$$

3rd decimal

$$8. \qquad 09 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \end{array} = 09 - 22 = -13 \;(-ve)$$

Hence 5 is reduced to 4 and remainder is increased to 4

$$49 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \\ 1&3&4\end{array} = 49 - 19 = 30 \div 4 = 7 \;(2)$$

$$9 \qquad 20 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \\ 3&4&7\end{array} \quad 20 - 32 = -12 \;(-ve)$$

Hence 7 is reduced to 6 and 2 is increased to 6.

$$60 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \\ 3&4&6\end{array} = 60 - 29 = 31 \div 4 = 7 \;(3)$$

Remainders:

At step 5 (before entering into decimals) one can calculate the remainder (absolute).

$$3789 - \begin{array}{ccc}3&2&1\\ \nwarrow&\uparrow&\nearrow\\ &\times& \\ 5&7&1\end{array} \times 100 - \begin{pmatrix}2&&1\\&\times&\\7&&1\end{pmatrix} \times 10 - \begin{pmatrix}\uparrow\\1\end{pmatrix} \times 1$$

$$3789 - 2200 - 90 - 1 = 1498 \;(R_1)$$

At step 6 (After first decimal)

$$3890 - \begin{array}{ccc}3&2&1\\ &\times& \\ 7&1&3\end{array} \times 100 - \begin{pmatrix}2&&1\\ &\times& \\ 1&&3\end{pmatrix} \times 10 - \begin{pmatrix}1\\ \uparrow\\ 3\end{pmatrix} \times 1$$

= 3890 – 1800 – 70 – 3 = 2017 (R_2)

At step 7 (After second decimal)

$$4900 - \begin{pmatrix}3&2&1\\ &\times& \\ 1&3&4\end{pmatrix} \times 100 - \qquad \times 10 \quad \begin{pmatrix}1\\ \uparrow\\ 3\end{pmatrix} \times 1$$

= 4900 – 1900 – 110 – 4 = 2886 (R_3)

At step 8 (After third decimal)

$$6000 - \begin{array}{ccc}3&2&1\\ &\times& \\ 3&4&6\end{array} \quad 100 - \begin{pmatrix}2&&1\\ &\times& \\ 4&&6\end{pmatrix} \times 10 - \begin{pmatrix}1\\ \uparrow\end{pmatrix} \times 1$$

= 6000 – 2000 – 160 – 6. = 6000 – 3066 = 2934

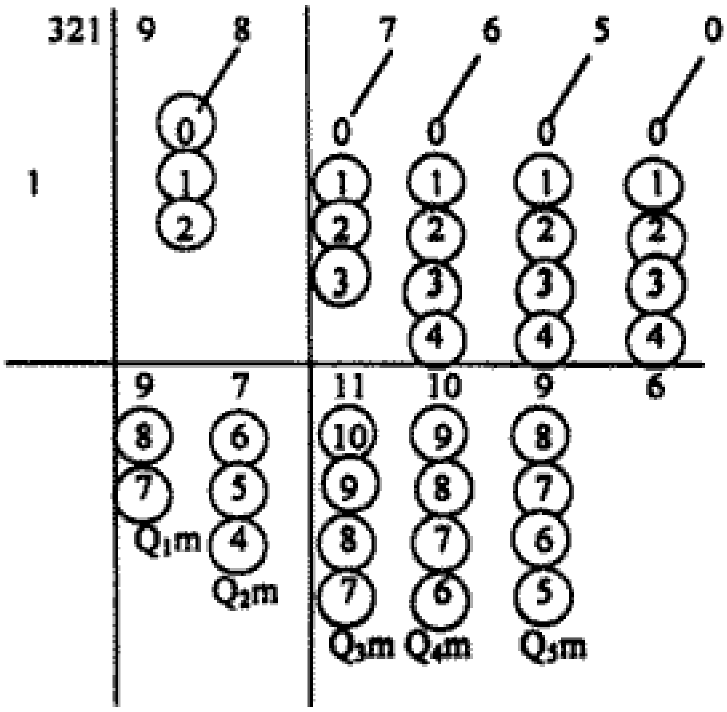
These remainders are well comparable with those obtained from Current Method

__Example 4__ : 98765 ÷ 1321

Current Method

```
1321 ) 98765 ( 74.765
       9247
      ------
        6295
        5284
        10110 R1
         9247
      ------
          8630 R2
          7926
      ------
           7040 R3
           6605
      ------
            435 R4
```

Vedic Method



1. $9 \div 1 = \underset{Q_1}{9}\ ,\ \underset{R_1}{0}$

2. $08 - \begin{pmatrix}3\\ \uparrow\\ 9\end{pmatrix} = 08 - 27 = -19$ (–ve) Q_1 is to be modified.

$18 - \begin{pmatrix}3\\ \uparrow\\ 8\end{pmatrix} = 18 - 24 = -6$ (–ve)

$28 - \begin{pmatrix}3\\ \uparrow\\ 7\end{pmatrix} = 28 - 21 = 7$ $\boxed{Q_1(m) = 7}$

$7 + 1 = \underset{Q_2}{7}\ ,\ \underset{R_2}{0}$

3 1st Decimal

$07 - \begin{pmatrix}3 & & 2\\ & \times & \\ 7 & & 7\end{pmatrix} = 07 - 35 = -19$ (–ve)

$17 - \begin{pmatrix}3 & & 2\\ & \times & \\ 7 & & 6\end{pmatrix} = 17 - 32 = -15$ (–ve)

$27 - \begin{pmatrix}3 & & 2\\ & \times & \\ 7 & & 5\end{pmatrix} = 27 - 29 = -2$ (–ve)

$37 - \begin{pmatrix}3 & & 2\\ & \times & \\ 7 & & 4\end{pmatrix} = 37 - 26 = 11$

$11 + 1 = \underset{Q_1}{11}\ ,\ \underset{R_1}{0}$ $\boxed{\therefore\ Q_2(m) = 4}$ and so on

Before entering into decimal points the remainder is

$$3765 - \begin{pmatrix}3 & 2 & 1\\ & \times & \\ 0 & 7 & 4\end{pmatrix} \times 100 - \begin{pmatrix}2 & & 1\\ & \times & \\ 7 & & 4\end{pmatrix} \times 10 - \begin{pmatrix}1\\ \uparrow\\ 4\end{pmatrix} \times 1$$

$= 3765 - 2600 - 150 - 4 = 1011\ (R_1)$

4. **2^{nd} Decimal**

$$06 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 11 \end{pmatrix} = -\text{ve}$$

$$16 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 10 \end{pmatrix} = -\text{ve}$$

$$26 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 9 \end{pmatrix} = -\text{ve}$$

$$36 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 8 \end{pmatrix} = -\text{ve}$$

$$46 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 7 \end{pmatrix} = 46 - 36 = \underset{Q_3(m)}{10} \qquad \boxed{\therefore\ Q_3(m) = 7}$$

$$10 - 1 = \underset{Q_4}{10},\ \underset{R_4}{1}$$

Remainder after second decimal point is

$$4650 - \begin{pmatrix} 3 & 2 & 1 \\ 0 & 7 & 4 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \\ 4 & 7 \end{pmatrix} \times 10 - \begin{pmatrix} 1 \\ \uparrow \\ 7 \end{pmatrix} \times 1$$

$$= 4650 - 2600 - 180 - 7 = 863\ (R_2)$$

5. 3rd decimal

$$05 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 10 \end{pmatrix} = -\text{ve}$$

$$15 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 9 \end{pmatrix} = -\text{ve}$$

$$25 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 8 \end{pmatrix} = -\text{ve}$$

$$35 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 7 \end{pmatrix} = -\text{ve}$$

$$45 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 6 \end{pmatrix} = 45 - 36 = 9 \qquad \boxed{\therefore\ Q_4(m) = 6}$$

$$9 \div 1 = \underset{Q_5}{9},\ \underset{R_5}{0}$$

Remainder after third decimal point is

$$4500 - \begin{pmatrix} 3 & 2 & 1 \\ 4 & 7 & 6 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \\ 7 & 6 \end{pmatrix} \times 10 - \begin{pmatrix} 1 \\ 6 \end{pmatrix} \times 1$$

$= 4500 - 3600 - 190 - 6 = 704\ (R_3)$

6. 4th decimal

$$0 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 9 \end{pmatrix} = -\text{ve}$$

$$10 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 8 \end{pmatrix} = -\text{ve}$$

$$20 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 7 \end{pmatrix} = -\text{ve}$$

$$30 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 6 \end{pmatrix} = -\text{ve}$$

$$40 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 5 \end{pmatrix} = 40 - 34 = 6 \qquad \boxed{\therefore\ Q_5(m) = 5}$$

$$\underset{Q_6\ R_6}{6 + 1 = 6,\ 0}$$

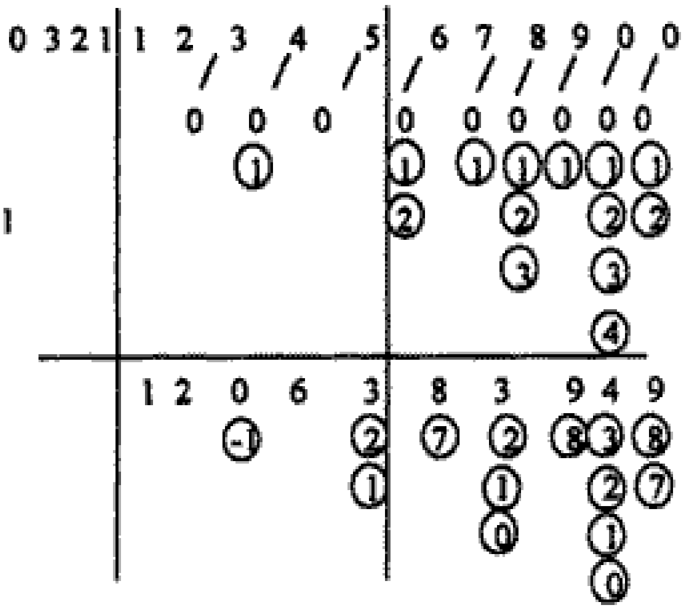
Remainder after fourth decimal point is

$$4000 - \begin{pmatrix} 3 & 2 & 1 \\ 7 & 6 & 5 \end{pmatrix} \times 100 - \begin{pmatrix} 2 & 1 \\ 6 & 5 \end{pmatrix} \times 10 \begin{pmatrix} 1 \\ \uparrow \\ 5 \end{pmatrix} - \times 1 = 4000 - 3400 - 160 - 5 = 435\ R_4 \text{ and so on}$$

Final Quotient =74.7656

Example 5 : 123456789 ÷ 10321

Vedic Method :



Quotient = 11961.70807

Current Method :

```
10321 ) 123456789 ( 11961.70807
        10321
         20246
         10321
           99257
           92889
            63688
            61926
              17629
              10321
               73080
               72247
                83300
                82568
                 73200
                 72147
                  1153
```

C) WORKING DETAILS OF A FEW OF EXAMPLES ALREADY GIVEN IN THIS LECTRURE NOTES

1) These consist of details of Vinculum Method for some examples already given in the text
2) Include decimal working
3) Details of reduction method
4) Include working details of 16 decimals using vinculum method for one problem.

Problem 1 : a) Consider 98765 ÷ 399

```
D1 D2

9 9    9    8    7 | 6    5
           /    /
          0    1̄

     3    6̄    8 |
     Q1   Q2   Q3 |
              -1 | 212
```

Quotient =3 6̄ 7 = 247

$$\text{Remainder} = 65 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} \bar{6} & & 8 \\ & \times & \\ 9 & & 9 \end{pmatrix} \\ Q_2 \quad Q_3 \end{array} 10 - \begin{array}{c} D_1 \\ \begin{pmatrix} 9 \\ \uparrow \\ 8 \end{pmatrix} \\ Q_3 \end{array} \qquad 65 - (2\ \bar{2}\ 0) - 72 = 65 + \bar{2}\ 2\ 0 + \bar{7}\ \bar{2}$$

$$\bar{2}13 + 399 = 212$$

When the remainder is negative one has to add n times the divisor where n= 1,2,3,…….to get positive value and finally one has to deduct n from the quotient
Q = 247; R = 212

(b) Using vinculum in both divisor and dividend

Divisor = 399 = $40\bar{1}$

Dividend = 98765 = $10\bar{1}\,\bar{2}\,\bar{3}\,\bar{5}$

$$\begin{array}{r|cccccc|c}
D_1 & & & & & & & \\
\bar{1} & 1 & 0 & \bar{1} & & \bar{2} & \bar{3} & \bar{5} \\
 & & / & / & & / & / & / \\
40 & & 1 & 10 & & 19 & 30 & 21 \\
\hline
 & 0 & 0 & 2 & & 4 & 7 & 212 \\
 & Q_1 & Q_2 & Q_3 & & Q_4 & Q_5 &
\end{array}$$

Q = 247

R = 212

Problem 2 : 124 + 2122

$$\begin{array}{r|cccccccc}
D_1\,D_2 & & & & & & & & \\
2\ 2 & 1 & : & 2 & 4 & 0 & 0 & 0 & 0 \\
 & & & / & / & / & / & / & / \\
21 & & & 1 & 12 & 19 & 12 & 10 & 13 \\
\hline
 & 0 & : & 0 & 5 & 8 & 4 & 3 & \\
 & Q_1 & & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 &
\end{array} \qquad Q = 0.05843$$

(1) $1 + 21 = \underset{Q_1}{0}, \quad \underset{R_1}{1}$

$$(2)\quad 12 - \begin{pmatrix} 0 \\ \uparrow \\ 2 \end{pmatrix} = 12 - 0 = 12 + 21 = \underset{Q_2}{0}, \quad \underset{R_2}{12}$$

D_1

$$(3)\quad 124 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \\ Q_1 \quad Q_2 \end{array} = 124$$

$$124 + 21 = \underset{Q_3}{5}, \quad \underset{R_3}{19}$$

$$(4)\quad 190 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 2 & 2 \\ 0 & 5 \end{pmatrix} \\ Q_2 \quad Q_3 \end{array} = 190 - 10 = 180$$

$$180 + 21 = \underset{Q_4}{8}, \quad \underset{R_4}{12}$$

$$(5)\quad 120 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 2 & 2 \\ 5 & 8 \end{pmatrix} \\ Q_3 \quad Q_4 \end{array} = 120 - 26 = 94$$

$$94 + 21 = \underset{Q_5}{4}, \quad \underset{R_5}{10}$$

$$(6)\quad 100 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 2 & 2 \\ 8 & 4 \end{pmatrix} \\ Q_4 \quad Q_5 \end{array} = 100 - 24 = 76$$

$$76 + 21 = \underset{Q_6}{3}, \quad \underset{R_6}{13}$$

Quotient = 0.05843

Problem 3(a) : 12.4 + 2122 a) Decimal in dividend considered

Current Method

$$\frac{12.4}{2122} = \frac{124}{21220}$$

$$\begin{array}{r} 21220\ )\ 124000\ (\ 0\ 005843 \\ \underline{106100} \\ 179000 \\ \underline{169760} \\ 92400 \\ \underline{84880} \\ 75200 \\ \underline{63660} \\ \underline{11540} \end{array}$$

Vedic Method

$$\begin{array}{r|cccccccc} 22 & 1 & 2\,. & 4 & 0 & 0 & 0 & 0 \\ & & / & / & / & / & / & / \\ 21 & . & 1 & 12 & 19 & 12 & 10 & 13 \\ \hline & . & 0 & 0 & 5 & 8 & 4 & 3 \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 \end{array}$$

Quotient = 0.005843

Answer is same as that of Problem 2 but with one decimal shifted to left

(1) $1 + 21 = \underset{Q_1}{0},\quad \underset{R_1}{1}$

(2) $12 - \begin{array}{c} D_1 \\ \begin{pmatrix} 2 \\ \uparrow \\ 0 \end{pmatrix} \\ Q_1 \end{array} = 12 - 0 = 12\ ,\ 12 + 21 = \underset{Q_2}{0},\ \underset{R_2}{12}$

(3) $124 - \begin{array}{c} D_1\quad D_2 \\ \begin{pmatrix} 2 & \nwarrow\!\nearrow & 2 \\ 0 & \times & 0 \end{pmatrix} \\ Q_2\quad Q_3 \end{array} = 124\ ,\ 124 + 21 = \underset{Q_3}{5},\quad \underset{R_3}{19}$

(4) $190 - \begin{array}{c} D_1\quad D_2 \\ \begin{pmatrix} 2 & \nwarrow\!\nearrow & 2 \\ 0 & \times & 5 \end{pmatrix} \\ Q_3\quad Q_4 \end{array} = 180\ ,\ 180 + 21 = \underset{Q_4}{8},\quad \underset{R_4}{12}$

(5) $120 - \begin{array}{c} D_1\quad D_2 \\ \begin{pmatrix} 2 & \nwarrow\!\nearrow & 2 \\ 5 & \times & 8 \end{pmatrix} \\ Q_4\quad Q_5 \end{array} = 120 - 26 = 94\ ,\ 94 + 21 = \underset{Q_5}{4},\ \underset{R_5}{10}$

(6) $100 - \begin{array}{c} D_1\quad D_2 \\ \begin{pmatrix} 2 & \nwarrow\!\nearrow & 2 \\ 8 & \times & 4 \end{pmatrix} \\ Q_5\quad Q_6 \end{array} = 100 - 24 = 76 + 21 = \underset{Q_6}{3},\ \underset{R_6}{13}$

The answer is 0.0 0 5 8 4 3

Problem: 3 (b) 1.24 ÷2122 = 124 + 212200

The answer is same as that in Problem 2 but with the decimal shifted to left by two more digit.

i.e., Quotient = 0 0 0 0 0 0 5 8 4 3

Problem 3 (c) 0.124÷2122 answer is same as that in Problem 2 but with decimal shifted to left by three more digits I.e quotient = 0.00005843

Problem 3 (d) 0.0124÷2122 = 124 + 2122000

Vedic Method

$$\begin{array}{ccccccccccc}
 & 22 & & 0 & 1 & 2 & 4 & 0 & 0 & 0 & 0\\
\underline{21} & & & 0 & 1 & 12 & 19 & 12 & 10 & 13 & \\ \hline
 & :0 & 0 & 0 & 0 & 0 & 5 & 8 & 4 & 3 &
\end{array}$$

(Answer is same as that in problem 2 but with decimal shifted to left by four digits)

Answer is 0.000005843

Problem : 4 (Decimal in the divisor)

Consider 124 + 212 2

Vedic Method

$$\begin{array}{c|cccccccc}
D_1 D_2 & & & & & & & & \\
2\ 2 & 1: & 2 & 4 & 0 & 0 & 0 & 0 & 0\\
21 & :1 & 12 & 19 & 12 & 10 & 13 & 11 & \\ \hline
 & 0. & 0 & 5 & 8 & 4 & 3 & 5 & \\
 & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 &
\end{array}$$

Current Method

1240 + 2122

$$\begin{array}{r}
2122)12400(.584344\\
\underline{10610}\\
17900\\
\underline{16976}\\
9240\\
\underline{8488}\\
7520\\
\underline{6366}\\
11540\\
\underline{10610}\\
9300\\
\underline{8488}\\
\underline{9120}
\end{array}$$

(1) $1 \div 21 = \underset{Q_1 R_1}{0,1}$

(2) $12 \div 21 = \underset{Q_2 R_2}{0,12}$

(3) $124 \div 21 = \underset{Q_3 R_3}{5,\ 19}$

$$\begin{array}{l} \quad\ D_1\ D_2 \\ 190 - \begin{pmatrix} 2 & 2 \\ 0 & 5 \end{pmatrix} = 180,\ 180 \div 21 = \underset{Q_4 R_4}{8,\ 12} \\ \quad\ Q_2\ \ Q_3 \end{array}$$

$$\begin{array}{l} \qquad\quad D_1\ \ D_2 \\ (4)\quad 120 - \begin{pmatrix} 2 & 2 \\ 5 & 8 \end{pmatrix} = 120 - 26 = 94 \div 21 = \underset{Q_5 R_5}{4,\ 10} \\ \qquad\quad Q_3\ Q_4 \end{array}$$

$$\begin{array}{l} \qquad\quad D_1\ \ D_2 \\ (5)\quad 100 - \begin{pmatrix} 2 & 2 \\ 8 & 4 \end{pmatrix} = 100 - 24 = 76 \div 21 = \underset{Q_6 R_6}{3,\ 13} \\ \qquad\quad Q_4\ \ Q_5 \end{array}$$

$$\begin{array}{l} \qquad\quad D_1\ \ D_2 \\ (6)\quad 130 - \begin{pmatrix} \text{[illegible]} & \\ 4 & 3 \end{pmatrix} = 116 \div 21 = \underset{Q_7\ R_7}{5,\ 11} \\ \qquad\quad Q_5\ \ Q_6 \end{array}$$

Ans = 0.05843

Due to the decimal in the divisor (212.2) the decimal is shifted by one digit to its right to get final Quotient as 0.5843

Problem 5: (Decimals in both divisor and dividend)

Vedic Method

12.4 ÷ 2.122

$$\begin{array}{lccccccccc}
 & D_1 D_2 & & & & & & & & \\
2\ 2\ 1 & 2 & . & 4 & 0 & 0 & 0 & 0 & 0 & \\
21 & 1 & 12 & & 19 & 12 & 10 & 13 & 11 & \\
 & 0 & 0 & 5 & 8 & 4 & 3 & 5 & & \\
 & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & &
\end{array}$$

(1) $1 \div 21 = 0,1$
Q_1R_1

(2) $12 - \overset{(2)}{} = 12,\ 13 + 21 = 0,12$
Q_2R_2

(3) $124 - \begin{array}{cc} D_1 & D_2 \\ 2 & 2 \\ \multicolumn{2}{c}{\times} \\ 0 & 0 \\ Q_1 & Q_2 \end{array} = 124,\ 124 + 21 = 5, 19$
$Q_3\,R_3$

(4) $190 - \begin{array}{cc} D_1 & D_2 \\ 2 & 2 \\ \multicolumn{2}{c}{\times} \\ 0 & 5 \\ Q_2 & Q_3 \end{array} = 180,\ 180 + 21 = 8, 12$
$Q_4\,R_4$

(5) $120 - \begin{array}{cc} D_1 & D_2 \\ 2 & 2 \\ \multicolumn{2}{c}{\times} \\ 5 & 8 \\ Q_3 & Q_4 \end{array} = 120 - 26 = 94,\ 94 + 21 = 4, 10$
$Q_5\,R_5$

Current Method :

$$\frac{12.4}{2.122} = \frac{124}{21.22} = \frac{12400}{2122}$$

```
2122)12400(5.84354
     10610
      17900
      16976
        9240
        8488
         7520
         6366
         11540
         10610
           930
```

Final answer 5.8435 as the divisor has 3 decimal points.

$$\text{(6)}\quad 100 - \overset{\begin{matrix}D_1 & & D_2\end{matrix}}{\begin{pmatrix} 2 & & 2 \\ & \times & \\ 0 & & 4 \end{pmatrix}} \qquad 100 - 24 = 76\,,\ 76 + 21 = \underset{Q_6\,R_6}{3,\ 13}$$

Q_5

$$\text{(7)}\quad 130 - (4 \nearrow \nwarrow 3) = 116 + 21 = \underset{Q_7\,R_7}{5,\ 11}$$

$$\text{(8)}\quad 110 - \begin{pmatrix} 2 & & 2 \\ & \times & \\ 3 & & 5 \end{pmatrix} = 94 + 21 = \underset{Q_8\,R_8}{4,\ 10}$$

Ans = .00584354

As there is a decimal in the divisor with three digits after the decimal, the decimal in the answer is to be shifted by three digits towards right

Quotient = 5 84354

Problem 6 : (Decimals in both divisor and dividend)

Vedic Method

1.24 + 21.22

$D_1\,D_2$

$$\begin{array}{ll|llllllll}
 & 2\ 2 & : & 1.2 & 4 & 0 & 0 & 0 & 0 & 0 \\
21 & & : & 1 & 12 & 19 & 12 & 10 & 13 & 11 \\
\hline
 & & .00 & 0 & 5 & 8 & 4 & 3 & 5 & \\
 & & Q_1Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 &
\end{array}$$

$$\text{(1)}\quad 1 + 21 = \underset{Q_2\ R_2}{0,\ 1}$$

$$\text{(2)}\quad 12 + 21 = \underset{Q_3\ R_3}{0,\ 12}$$

$$\text{(3)}\quad 124 + 21 = \underset{Q_4\ R_4}{5,\ 19}$$

$$\text{(5)}\quad 190 \overset{\begin{matrix}D_1 & D_2\end{matrix}}{\underset{\begin{matrix}Q_3 & Q_4\end{matrix}}{(0 \nearrow \nwarrow 5}} \cdot 180\,,\ 180 + 21 : \underset{Q_5\ R_5}{8,\ 12}$$

Current Method

$$\frac{1.24}{21.22} = \frac{124}{2122}$$

```
2122)12400(.058435
     10610
      17900
      16976
       9240
       8488
        7520
        6366
        11540
        10610
```

$$\begin{array}{lcl} & D_1 \quad D_2 & \\ (6)\quad 120 - & \begin{pmatrix}2 & & 2\\ & \times & \\ 5 & & 8\end{pmatrix} & = 94,\ 94 + 21 = 4,\ 10 \\ & & \qquad\qquad\qquad Q_6\ \ R_6 \\ & Q_4 \quad Q_5 & \end{array}$$

$$\begin{array}{lcl} & D_1 \quad D_2 & \\ (7)\quad 100 - & \begin{pmatrix}2 & & 2\\ & \times & \\ 8 & & 4\end{pmatrix} & = 76,\ 76 + 21 = 3,\ \ 13 \\ & & \qquad\qquad\qquad Q_7\ \ R_7 \\ & Q_5 \quad Q_6 & \end{array}$$

$$\begin{array}{lcl} & D_1\ D_2 & \\ (8)\quad 130 & \begin{pmatrix}2 & & 2\\ & \times & \\ 4 & & 3\end{pmatrix} & = 116,\ 116 + 21 = 5,\ \ 11 \\ & & \qquad\qquad\qquad Q_8\ \ R_8 \\ & Q_6\ Q_7 & \end{array}$$

Ans = 0.00058435

Due to two decimals in the divisor, the decimal in the answer is shifted by two digits.

The final answer = 0.058435

Problem 7: (Decimal in both divisor and dividend consider)

Vedic Method

0.124 + 0.2122

$$\begin{array}{l cccccccccccc} 122 & . & 1 & 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & / & / & / & / & / & / & / & / \\ & & & \underline{1} & \underline{0} & \underline{0} & \underline{\bar{1}} & \underline{\bar{1}} & \underline{1} & \underline{0} & 1 \\ & . & 0 & 0 & 0 & 0 & 6 & \bar{1} & \bar{5} & \bar{7} & 4 & 15 & \bar{4} \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} \end{array}$$

0000584355

Answer is 0.584355

Inview of the decimal in the divisor, it is shifted by three digits to the right. (Refer partition rules page:)

Current Method

$$\frac{0.124}{0.2122} - \frac{1240}{2122}$$

$$\begin{array}{r} 2122)12400(.584354 \\ \underline{10610} \\ 17900 \\ \underline{16976} \\ 9240 \\ \underline{8488} \\ 7520 \\ \underline{6366} \\ 11540 \\ \underline{10610} \\ \underline{9300} \end{array}$$

$Q_1 = Q_2 = Q_3 = 0$

(1) $1 \div 2 = \underset{Q_4}{0}, \underset{R_4}{1}$

(2) $\underset{D_1}{12} \div 2 = \underset{Q_5}{6}, \underset{R_5}{0}$

(3) $4 - \overset{D_1}{\underset{Q_5}{(1)}}\ 6 = -2,\ 2 + 2 = \underset{Q_6}{1}, \underset{R_6}{0}$

(4) $0 - \begin{pmatrix} \overset{D_1}{1} & \times & \overset{D_2}{2} \\ \underset{Q_5}{6} & & \underset{Q_6}{\bar{1}} \end{pmatrix} - 11, \bar{1}1 \div 2 = \underset{Q_7}{5}, \underset{R_7}{1}$

(5) $10 - \begin{pmatrix} \overset{D_1}{1} & \overset{D_2}{2} & \overset{D_3}{2} \\ \underset{Q_5}{6} & \underset{Q_6}{1} & \underset{Q_7}{\bar{5}} \end{pmatrix} \quad \bar{1}0 - 5 = \bar{1}\bar{5}, 1\bar{5} \div 2 = \underset{Q_8}{7}, \underset{R_8}{1}$

(6) $10 - \begin{pmatrix} \overset{D_1}{1} & \overset{D_2}{2} & \overset{D_3}{2} \\ \underset{Q_6}{\bar{1}} & \underset{Q_7}{5} & \underset{Q_8}{\bar{7}} \end{pmatrix} = 10 - 1\bar{9} = 10 + 19 = 9 \div 2 = \underset{Q_9}{4}, \underset{R_9}{1}$

(7) $10 - \begin{pmatrix} \overset{D_1}{1} & \overset{D_2}{2} & \overset{D_3}{2} \\ \underset{Q_7}{\bar{5}} & \underset{Q_8}{7} & \underset{Q_9}{4} \end{pmatrix} = 10 - \bar{2}0 = 10 + 20 = 30 \div 2 = \underset{Q_{10}}{15}, \underset{R_{10}}{0}$

Ans $= 0.00006\bar{1}\,\bar{5}\,\bar{7}\,4\ 15\,\bar{4}$

$0.00006\bar{1}\,\bar{5}\,\bar{7}\,5\,46$

$0.0000584\ \ 354$

As there are four digits after the decimal in the divisor the decimal has to be shifted by 4 digits.

∴ Final Quotient = 0.584354

Problem 8: 7896456 + 34 (Vinculum details for example 3 in page:)

$$\begin{array}{c|cccccccc}
D_1 & & & & & & & & \\
4 & 7 & 8 & 9 & 6 & 4 & 5 & : & 6 \\
 & & 1 & 1 & 1 & 2 & 1 & : & 2 \\
 & & R_1 & R_2 & R_3 & R_4 & R_5 & & R_6 \\
\hline
 & 2 & 3 & 2 & 2 & 5 & \bar{1} & : & 24 \\
 & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & & \text{Remainder}
\end{array}$$

Step 1 : $3)\ 7\ (\ 2\ (Q_1)$

$\underline{6}$

$\underline{1}\ (R_1)$

Step 2 : $$18 - \overset{D_1}{\underset{Q_1}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 18 - 8 = 10$$

$3\ )\ 10\ (\ 3\quad (Q_2)$

$\underline{9}$

$\underline{1}\ (R_2)$

Step 3 : $$19 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix}4\\ \uparrow\\ 3\end{pmatrix}}} = 19 - 12 = 7$$

$3\ )\ 7\ (\ 2\ (Q_3)$

$\underline{6}$

$\underline{1}\ (R_3)$

Step 4 : $$16 - \overset{D_1}{\underset{Q_3}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 6 - 8 = 8$$

$3\ )\ 8\ (\ 2\ (Q_4)$

$\underline{6}$

$\underline{2}\ (R_4)$

Step 5 : $$24 - \overset{D_1}{\underset{Q_4}{\begin{pmatrix}4\\ \uparrow\\ 2\end{pmatrix}}} = 24 - 8 = 16$$

$3\ )16\ (\ 5\quad (Q_5)$

$\underline{15}$

$\underline{1}\ (R_5)$

Step6: $$15 - \overset{D_1}{\underset{Q_5}{\begin{pmatrix}4\\ \uparrow\\ 5\end{pmatrix}}} = 15 - 20 = \bar{5}$$

$3\ )\ \bar{5}\ (\bar{1}\quad (Q_6)$

$\underline{\bar{3}}$

$\underline{\bar{2}}\quad (R_6)$

Step 7 : D₁ Remainder

$$\bar{2}6 - \quad = 26 - 4 = 26 + 4 = 10$$

$\bar{1}0 + 34 = 24$ ($\because \bar{1}0$ is negative, one has to add (n times) the divisor to it until it becomes positive) n =1 and hence 1 is to be subtracted from the quotient.

$\therefore$ Quotient = $23225\bar{1} - 1 = 23225\bar{2}$ = 232248

Remainder = 24

Problem 9: 897356 ÷ 721 (Vinculum details of example · 7 in page)

$$\begin{array}{cc|ccccccccccccc}
D_1 & D_2 & & & & & & & & & & & & & \\
2 & 1 & 8 & 9 & 7 & 3 & : & 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & / & / & / & & / & / & / & / & / & / & / & / & / \\
 & & & 1 & 3 & 4 & : & 5 & 1 & 0 & 1 & 5 & 0 & \bar{1} & 1 & 2 \\
7 & & & R_1 & R_2 & R_3\ R_4 & & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} & R_{11} & R_{12} \\
\hline
 & & 1 & 2 & 4 & 4 & : & 6 & 0 & \bar{1} & 1 & 7 & \bar{2} & \bar{2} & 2 & \\
 & & Q_1 & Q_2 & Q_3 & Q_4 & & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} & Q_{12} &
\end{array}$$

Step 1 :

$$\begin{array}{l} 7\,)\,8\,(\,1\ (Q_1) \\ \quad\ \underline{7} \\ \quad\ 1\ (R_1) \end{array}$$

Step 2 :

$$19 - \overset{D_1}{\underset{Q_1}{\begin{pmatrix} 2 \\ \uparrow \\ 1 \end{pmatrix}}} = 17 \qquad \begin{array}{l} 7\,)\,17\,(\,2\ (Q_2) \\ \quad\ \underline{14} \\ \quad\ \ \underline{3}\ (R_2) \end{array}$$

Step 3 :

$$37 - \overset{D_1\ D_2}{\underset{Q_1\ Q_2}{\begin{pmatrix} 2 & & 1 \\ & \times & \\ 1 & & 2 \end{pmatrix}}} = 32 \qquad \begin{array}{l} 7\,)\,32\,(\,4\ (Q_3) \\ \quad\ \underline{28} \\ \quad\ \ \underline{4}\ (R_3) \end{array}$$

Step 4 :

$$\begin{array}{c} D_1 \quad D_2 \\ \\ Q_2 \quad Q_3 \end{array}$$

$$\begin{array}{l} 7\,)\,33\,(\,4\ \ (Q_4) \\ \quad \underline{28} \\ \quad \underline{\ 5}\ \ (R_4) \end{array}$$

Step 5 :

$$\begin{array}{c} D_1 \quad D_2 \\ 55 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ 4 & & 4 \end{pmatrix} = 43 \\ Q_3 \quad Q_4 \end{array}$$

$$\begin{array}{l} 7\,)\,43\,(\,6\ \ (Q_5) \\ \quad \underline{42} \\ \quad \underline{\ 1}\ \ (R_5) \end{array}$$

Step 6 :

$$\begin{array}{c} D_1 \quad D_2 \\ 16 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ 4 & & 6 \end{pmatrix} = 0 \\ Q_4 \quad Q_5 \end{array}$$

$$\begin{array}{l} 7\,)\,0\,(\,0\ \ (Q_6) \\ \quad \underline{0} \\ \quad \underline{0}\ \ (R_6) \end{array}$$

Step 7 :

$$\begin{array}{c} D_1 \quad D_2 \\ 00 - \left(\begin{matrix} 2 & & 1 \\ & \times & \\ 6 & & 0 \end{matrix}\right. \\ Q_5 \quad Q_6 \end{array}$$

$$\begin{array}{l} 7\,)\,6\,(\,1\ \ (Q_7) \\ \quad \underline{\bar{7}} \\ \quad \underline{1}\ \ (R_7) \end{array}$$

Step 8 :

$$\begin{array}{c} D_1 \quad D_2 \\ 10 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ 0 & & \bar{1} \end{pmatrix} = 12 \\ Q_6 \quad Q_7 \end{array}$$

$$\begin{array}{l} 7\,)\,12\,(\,1.\ \ (Q_8) \\ \quad \underline{7} \\ \quad \underline{5}\ \ (R_8) \end{array}$$

Step 9 :

$$\begin{array}{c} D_1 \quad D_2 \\ 50 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ \bar{1} & & 1 \end{pmatrix} = 49 \\ Q_7 Q_8 \end{array}$$

$$\begin{array}{l} 7\,)\,49\,(\,7\ \ (Q_9) \\ \quad \underline{49} \\ \quad \underline{\ 0}\ \ R_9 \end{array}$$

Step 10:

$$\begin{array}{c} D_1 \quad D_2 \\ 0 - \begin{pmatrix} 2 & & 1 \\ & \times & \\ 1 & & 7 \end{pmatrix} = \overline{15} \\ Q_8 \quad Q_9 \end{array}$$

$$\begin{array}{l} 7\,)\,\overline{15}\,(\,\bar{2}\ \ (Q_{10}) \\ \quad \underline{\bar{1}\bar{4}} \\ \quad \underline{\ \ \bar{1}}\ \ (R_{10}) \end{array}$$

Step 11:

$$7)\overline{13}\,(\bar{2}\ (Q_{11})$$
$$\underline{\bar{1}4}$$
$$1\ (R_{11})$$

$\bar{1}0$ -

Step 12:

$$10 - \left(\cancel{\bar{3}}\ {}_{2} \right) = 16$$

$$7)16(2(Q_{12})$$
$$\underline{\bar{1}4}$$
$$2\ (R_{12})$$

Quotient = 1244.6011722 = 1244.5991682

<u>**Problem 10 :**</u> 7652 + 23 (Vinculum details of example 5 in page)

$$
\begin{array}{cccccccccccccc}
 & D_1 & & & & & & & & & & & & \\
 & 3 & 7 & 6 & 5 & : & 2 & 0 & 0 & 0 & 0 & 0 & 0 & \\
 & & & / & / & & / & / & / & / & / & / & / & \\
2 & & & 1 & 1 & & 0 & \bar{1} & \bar{1} & 0 & 1 & \bar{1} & 1 & \\
 & & & R_1 & R_2 & & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & \\
 & & 3 & 3 & 3 & : & \bar{3} & 0 & 5 & 7 & 5 & 2 & & \\
 & & Q_1 & Q_2 & Q_3 & & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & &
\end{array}
$$

Step 1 :

$$2\,)\,7\,(\,3\quad (Q_1)$$
$$\underline{6}$$
$$\underline{1}\quad (R_1)$$

Step 2 :

$$16 - \overset{D_1}{\underset{Q_1}{\begin{pmatrix} 3 \\ \uparrow \\ 3 \end{pmatrix}}} = 16 - 9 = 7$$

$$2\,)\,7\,(\,3\ (Q_2)$$
$$6$$
$$\underline{1}\quad (R_2)$$

Step 3 :

$$15 - \underset{Q_2}{(3)} = 15 - 9 = 6$$

$$2\,)\,6\,(\,3\ (Q_3)$$
$$\underline{6}$$
$$\underline{0}\quad (R_3)$$

Step 4 : $$02 - \overset{D_1}{(2)} = 2 + 9 = 7$$

$$\begin{array}{l} 2\,)\ 7\ (\ \dot{3}\ \ (Q_4) \\ \quad\ \ \bar{6} \\ \quad\ \ \underline{\bar{1}}\ \ (R_4) \end{array}$$

Step 5 : $$10 - \qquad 10 - \bar{9} = 19 = 1$$

$$\begin{array}{l} 2\,)\ 1\ (\ 0\ (Q_5) \\ \quad\ \ \underline{0} \\ \quad\ \ \underline{\bar{1}}\ \ (R_5) \end{array}$$

Step 6 : $$10 - \overset{D_1}{(2)} = 10 - 0 = 10$$

$$\begin{array}{l} 2\,)\ 10\ (\ 5\ \ (Q_6) \\ \quad\ \ \underline{\bar{1}0} \\ \quad\ \ \underline{\ \ 0}\ \ (R_6) \end{array}$$

Step 7 : $$0 - \qquad = 0 - 15 = 15$$

$$\begin{array}{l} 2\,)\ 15\ (7\ (Q_7) \\ \quad\ \ \underline{14} \\ \quad\ \ \underline{\ \ 1}\ (R_7) \end{array}$$

Step 8 : $$10 - \qquad = 10 - 2\,1 = 11$$

$$\begin{array}{l} 2\,)\ \bar{1}1\ (\bar{5}\ (Q_8) \\ \quad\ \ \underline{\bar{1}0} \\ \quad\ \ \underline{1\ \ }\ (R_8) \end{array}$$

Step 9 : $$\bar{1}0 - \underset{Q_8}{\overset{\overset{Q_7}{D_1}}{\left(\begin{array}{c} 3 \\ \uparrow \\ \bar{5} \end{array}\right)}} = \bar{1}0 - \bar{1}\,\bar{5} = 5$$

$$\begin{array}{l} 2\,)\ 5\ (2\ \ (Q_9) \\ \quad\ \ \underline{4} \\ \quad\ \ 1\ \ (R_9) \end{array}$$

Quotient $= 333.\bar{3}0\bar{5}7\bar{5}2 = 332.695652$

Problem 11: 8954 ÷ 89 (Vinculum details for example : 6 in Page)

$$
\begin{array}{c|ccccccccccccccc}
D_1 & & & & & & & & & & & & & & \\
9 & 8 & 9 & 5 & : & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & / & / & & / & / & / & / & / & / & / & / & / \\
8 & & 0 & 0 & : & \bar{3} & \bar{3} & \bar{2} & 3 & 2 & \bar{4} & 6 & 3 & 0 \\
 & & R_1 & R_2 & & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} & R_{11} \\
\hline
 & 1 & 0 & 1 & : & \bar{4} & 1 & \bar{4} & 8 & \bar{6} & 1 & 6 & \bar{3} & \\
 & Q_1 & Q_2 & Q_3 & & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} &
\end{array}
$$

Step 1 : 8) 8 (1 (Q_1)

$\underline{8}$

$\underline{0}$ (R_1)

Step 2 : $9 - \underset{Q_1}{\overset{D_1}{\begin{pmatrix}9\\ \uparrow \\ 1\end{pmatrix}}} = 0$

8) 0 (0 (Q_2)

$\underline{0}$

$\underline{0}$ (R_2)

Step 3 : $5 - \underset{Q_2}{\overset{D_1}{\begin{pmatrix}9\\ \uparrow \\ 0\end{pmatrix}}} = 5$

8) 5 (1 (Q_3)

$\underline{8}$

$\underline{\bar{3}}$ (R_3)

Step 4 : $\bar{3}4 - \underset{Q_3}{\overset{D_1}{\begin{pmatrix}9\\ \uparrow \\ 1\end{pmatrix}}} = \bar{3}\,\bar{5}$

8) $\bar{3}\,\bar{5}$ ($\bar{4}$ (Q_4)

$\underline{\bar{3}\,\bar{2}}$

$\underline{\bar{3}}$ (R_4)

Step 5 : $\bar{3}0 - \underset{Q_4}{\overset{D_1}{\begin{pmatrix}9\\ \uparrow \\ \bar{4}\end{pmatrix}}} = 6$

8) 6 (1 (Q_5)

$\underline{8}$

$\underline{\bar{2}}$ (R_5)

Step 6 : $$\bar{2}0 - \overset{D_1}{\underset{Q_5}{\begin{pmatrix}9\\ \uparrow\\ 1\end{pmatrix}}} = \overline{29}$$

$$\begin{array}{l} 8\,)\,\bar{2}\,\bar{9}\,(\,\bar{4}\quad (Q_6)\\ \quad\ \underline{\bar{3}\,\bar{2}}\\ \quad\ 1\underline{7} = 3\ (R_6)\end{array}$$

Step 7 : $$30 - \overset{D_1}{\underset{Q_6}{\begin{pmatrix}9\\ \uparrow\\ \bar{4}\end{pmatrix}}} = 66$$

$$\begin{array}{l} 8\,)\,66\,(\,8\ (Q_7)\\ \quad\ \underline{64}\\ \quad\ \underline{2}\ (R_7)\end{array}$$

Step 8 : $$20 - \overset{D_1}{\underset{Q_7}{\begin{pmatrix}9\\ \uparrow\\ 8\end{pmatrix}}} = \bar{5}\,\bar{2}$$

$$\begin{array}{l} 8\,)\,\overline{52}\,(\,\bar{6}\ (Q_8)\\ \quad\ \underline{\overline{48}}\\ \quad\ \underline{\bar{4}}\ (R_8)\end{array}$$

Step 9 : $$\bar{4}0 - \overset{D_1}{\underset{Q_8}{\begin{pmatrix}9\\ \uparrow\\ \bar{6}\end{pmatrix}}} = \bar{4}0 - \bar{5}\,\bar{4} = 14$$

$$\begin{array}{l} 8\,)\,14\,(\,1\ (Q_9)\\ \quad\ \underline{8}\\ \quad\ \underline{6}\ (R_9)\end{array}$$

Step 10 : $$60 - \overset{D_1}{\underset{Q_9}{\begin{pmatrix}9\\ \uparrow\\ 1\end{pmatrix}}} \cdot 51$$

$$\begin{array}{l} 8\,)\,51\,(\,6\ (Q_{10})\\ \quad\ 48\\ \quad\ \underline{3}\ (R_{10})\end{array}$$

Step 11 : $30-54 = \overline{24}$

$$\begin{array}{l} 8)\,\overline{24}\,(\,\bar{3}\ (Q_{11})\\ \quad\underline{\overline{24}}\\ \quad\ 0\ (R_{11})\end{array}$$

Quotient $= 101\ \bar{4}\,1\,\bar{4}\,8\,\bar{6}\,16\bar{3} = 100\ 60674157$

Problem 12: 89124 ÷ 5378 (Vinculum details of Example 10 in Page No)

$D_1\ D_2\ D_3$

$$\begin{array}{cccccccccc}
8 & 9 & : & 1 & 2 & 4 & 0 & 0 & 0 \\
 & / & & / & / & / & / & / & / \\
 & 3 & : & 1 & 2 & 1 & 2 & 0 & 2 \\
 & R_1 & : & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 \\
 & & & & & & & & \\
1 & 7 & : & 3 & 13 & 0 & 19 & 9 & \\
Q_1 & Q_2 & & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 &
\end{array}$$

(1) $5\,)\,8\,(\,1$ (Q_1)

$\underline{5}$

$\underline{3}$ (R_1)

D_1

(2) $39 - \begin{pmatrix}3\\ \uparrow\\ 1\end{pmatrix} = 36$

Q_1

$5\,)\,36\,(\,7$ (Q_2)

$\underline{35}$

$\underline{1}$ (R_2)

$D_1\ D_2$

(3) $11 - \begin{pmatrix}3 & & 7\\ & \times & \\ 1 & & 7\end{pmatrix} = \bar{1}\,\bar{7}$

$Q_1\ \ Q_2$

$5\,)\,\bar{1}\,\bar{7}\ (\,\bar{3}$ (Q_3)

$\underline{\bar{1}\,\bar{5}}$

$\underline{\bar{2}}$ (R_1)

$D_1\ D_2\ D_3$

(4) $\bar{2}\,2 - \begin{pmatrix}3 & 7 & 8\\ & \times & \\ 1 & 7 & \bar{3}\end{pmatrix} = \overline{66}$

$Q_1\ Q_2\ Q_3$

$5\,)\,\overline{66}\ (\,\overline{13}$ (Q_4)

$\underline{\bar{6}\,\bar{5}}$

$\underline{\bar{1}}$ (R_4)

$D_1\ D_2\ D_3$

(5) $\bar{1}4 - \begin{pmatrix}3 & 7 & 8\\ & \times & \\ 7 & \bar{3} & \overline{13}\end{pmatrix} = 0\bar{2}$

$Q_2\ Q_1\ Q_4$

$5\,)\,\bar{2}\ (\,0$ (Q_5)

$\underline{0}$

$\underline{\bar{2}}$ (R_5)

$$(6)\quad \bar{2}0 - \begin{array}{c} D_1 D_2 D_1 \\ \begin{pmatrix} 3 & 7 & 8 \\ \nwarrow & \uparrow & \nearrow \\ \bar{3} & \overline{13} & 0 \end{pmatrix} \\ Q_2 Q_3 Q_4 \end{array} = 1\bar{1}5 = 95$$

5) 95 (19 (Q_6)
$\underline{95}$
$\underline{0}$ (R_6)

$$(7)\quad 00 - \begin{array}{c} D_1 D_2 D_3 \\ \begin{pmatrix} 3 & 7 & 8 \\ \nwarrow & \uparrow & \nearrow \\ \overline{13} & 0 & 19 \end{pmatrix} \\ Q_3 Q_4 Q_5 \end{array} = 1\bar{5}\bar{3} = 47$$

5) 47 (9 (Q_7)
$\underline{45}$
2 (R_7)

Quotient = $17.\bar{3}\ \ \bar{1}\bar{3}\ \ 1\,9\,9 = 17.\bar{4}\ \ \bar{3}199 = 16.57199$

Problem 13 : 6543 + 89798 (Vinculum details of Example · 12 in Page No)

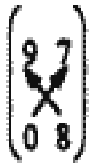
$$\begin{array}{c|ccccccc} D_1D_2D_3D_4 & & & & & & & \\ 9\ 7\ 9\ 8 & : & 6 & 5 & 4 & 3 & 0 & 0 \\ 8 & : & & 6 & 1 & \bar{2} & \bar{2} & \bar{2} \\ & & & R_1 & R_2 & R_3 & R_4 & R_5 \\ \hline & : & 0 & 8 & \bar{7} & \bar{1} & \bar{4} & \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & \end{array}$$

(1) 8) 6 (0 $(Q_1$
$\underline{0}$
$\underline{6}$

$$(2)\quad 65 - \begin{array}{c} D_1 \\ \begin{pmatrix} 9 \\ \uparrow \\ 0 \end{pmatrix} \\ Q_1 \end{array} = 65$$

8) 65 (8 (Q_2)
$\underline{64}$
$\underline{1}$ (R_2)

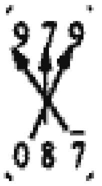
$D_1 D_2$

(3) 14 -  $= \bar{5}8$

$Q_1 Q_2$

$8\,)\,\overline{58}\ (\ \bar{7}\ (Q_3)$
$\quad \overline{56}$
$\quad \bar{2}\ (R_2)$

$D_1 D_2 D_3$

(4) 2 3 -  = 10

$Q_1\ Q_2\ Q_3$

$8\,)\,10\ (1\ (Q_4)$
$\quad \bar{8}$
$\quad \bar{1}\ 8 = \bar{2}\ (R_4)$

$D_1 D_2 D_3 D_4$

(5) 20-  = 46 = 34

$Q_1 Q_2 Q_3 Q_4$

$8\,)\,\overline{34}\ (\ 4\ (Q_5)$
$\quad \overline{32}$
$\quad \bar{2}\ (R_5)$

Quotient = 0 08 714
= 0.07286

Problem 14: 78 + 21345 (Vinculum details of example 13 Page No up to 6 decimals places)

$D_1 D_2 D_3 D_4$

1 3 4 5 7 8 0 0 0 0 0 0 0 0 0 0 0 0 0

1 1 0 0 $\bar{1}$ 1 0 0 $\bar{1}$ 0 0 0 1 $\bar{1}$

0 0 3 7 $\bar{3}$ $\bar{1}\bar{5}$ $\bar{9}$ 10 51 15 $\overline{81}$ $\bar{1}\bar{1}\bar{4}$ 21 285 256 $\bar{3}0\bar{7}$

$Q_1 Q_2 Q_3\ Q_4\ Q_5\ Q_6\ Q_7\ Q_8\ Q_9\ Q_{10}\ Q_{11}\ Q_{12}\ Q_{13}\ Q_{14}\ Q_{15}\ Q_{16}$

As the Dhwajanka has 4 digits the quotient starts with two zeros after the decimal (shifting the partition by two digits into the left of the dividend) $Q_1\ Q_2$ can be considered as 00 which are passive in the calculations, One can start writing with the digit 7 in the dividend. But while writing the quotient the decimal computed should be considered

Step 1 : 00 includes Q_1, Q_2

Step 2 : $2\,)\,7\,(\,3 \quad (Q_3)$

$$\begin{array}{l} 2\,)\,7\,(\,3 \quad (Q_3) \\ \quad \underline{6} \\ \quad \underline{1}\ (R_3) \end{array}$$

Step 3 : $$18 - \begin{array}{c} D_1D_2D_3 \\ \begin{pmatrix} 1 & 3 & 4 \\ 0 & 0 & 3 \end{pmatrix} \\ Q_1\,Q_2Q_3 \end{array} = 18 - 3 = 15$$

$$\begin{array}{l} 2\,)\,15\,(\,7\,(Q_4) \\ \quad \underline{14} \\ \quad 1 \quad (R_4) \end{array}$$

Step 4 : $$10 - \begin{array}{c} D_1D_2D_3D_4 \\ \begin{pmatrix} 1 & 3 & 4 & 5 \\ 0 & 0 & 3 & 7 \end{pmatrix} \\ Q_1\,Q_2Q_3Q_4 \end{array} = 10 - 16 = 6$$

$$\begin{array}{l} 2\,)\,\underline{6}\,(\,\bar{3}\ (Q_5) \\ \quad \underline{6} \\ \quad 0 \quad (R_5) \end{array}$$

Step 5 : $$0 - \begin{array}{c} D_1D_2D_3D_4 \\ \begin{pmatrix} 1 & 3 & 4 & 5 \\ 0 & 3 & 7 & \bar{3} \end{pmatrix} \\ Q_2Q_3Q_4Q_5 \end{array} = \bar{3}0$$

$$\begin{array}{l} 2\,)\,\bar{3}\,0\,(\,\overline{15} \quad (Q_6) \\ \quad \bar{3}\,0 \\ \quad \underline{\ 0\ } \quad (R_6) \end{array}$$

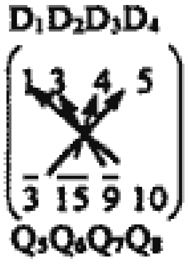
Step 6 : $$0 - \begin{array}{c} D_1D_2D_3\,D_4 \\ \begin{pmatrix} 1 & 3 & 4 & 5 \\ 3 & 7 & \bar{3} & \overline{15} \end{pmatrix} \\ Q_3Q_4Q_5Q_6 \end{array} = \overline{19}$$

$$\begin{array}{l} 2\,)\,\overline{19}\ (\,\bar{9} \quad (Q_7) \\ \quad \underline{\overline{18}} \\ \quad\ \bar{1}\ (R_7) \end{array}$$

Step 7 : $$\bar{1}0 - \begin{array}{c} D_1D_2D_3D_4 \\ \begin{pmatrix} 1 & 3 & 4 & 5 \\ 7 & \bar{3} & \overline{15} & \bar{9} \end{pmatrix} \\ Q_4\,Q_5\,Q_6\,Q_7 \end{array} = 21$$

$$\begin{array}{l} 2\,)\,21\,(\,10\ (Q_8) \\ \quad \underline{20} \\ \quad\ \underline{1}\ (R_8) \end{array}$$

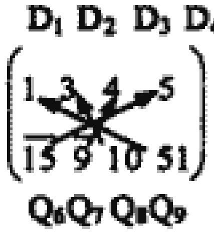
Step 8 : 10 -



= 102

$2\,)\,102\,(\,51$ (Q_9)
$\underline{102}$
$\underline{0}$ (R_9)

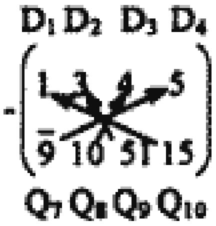
Step 9 : 0 -



$= 1\bar{7}0 = 30$

$2\,)\,30\,(\,15$ (Q_{10})
$\underline{30}$
$\underline{0}(R_{10})$

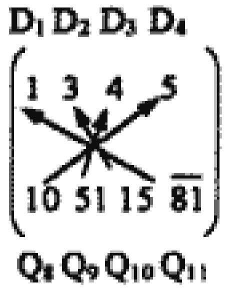
Step 10 : 10 -



$= \bar{2}4\bar{3} = \bar{1}\,\bar{6}\,\bar{3}$

$2\,)\,\bar{1}\bar{6}\bar{3}\;(\overline{81}$ (Q_{11})
$\underline{\bar{1}\bar{6}\bar{2}}$
$\underline{\bar{1}}$ (R_{11})

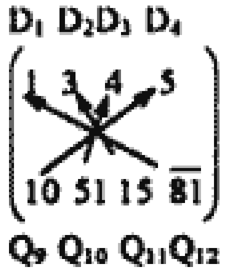
Step 11 : 0 -



$= \bar{2}\,\bar{2}\,\bar{8}$

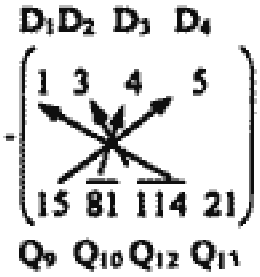
$2\,)\,\bar{2}\bar{2}\bar{8}\;(\,\bar{1}\bar{1}\bar{4}$ (Q_{12})
$\underline{\bar{2}\bar{2}\bar{8}}$
$\underline{0}$ (R_{12})

Step 12 : 0 -



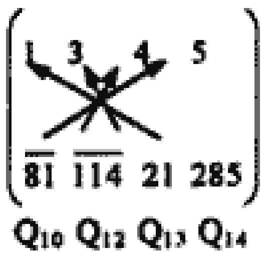
= 42

Step 13 : 0 -



= 570

Step 14 : 0 -



= 513

$$\textbf{Step 15 : } 10 - \begin{array}{c} D_1\,D_2\,D_3\;\,D_4 \\ \begin{pmatrix} 1 & 3 & 4 & 5 \\ \overline{114} & 21 & 285 & 256 \end{pmatrix} \\ Q_{12}\,Q_{13}\,Q_{14}\,Q_{15} \end{array} = \bar{6}\,\bar{1}\,\bar{5}$$

$$= \quad 0\,0\,3\,7\,\bar{3}\ \overline{15}\ \bar{9}\ 10\,51\ 15\ \bar{8}\ \bar{1}\ \ \bar{1}\ \bar{1}\ \bar{4}\ 21\ 285\ 256\ \overline{307}$$

$$= \quad .0\,0\,3\,7\ \bar{4}\ \bar{5}\ \bar{8}\ 5\,2\,\bar{4}\,\bar{2}\ 1\,1\,7\,6\,\bar{7}$$

$$= \quad .00\,3\,6\,5\,4\,2\,5\,1\,5\,8\,1\,1\,7\,5\,3$$

Problem 15 : 15628 ÷ 23 4 (Vinculum of Example 16 Page No.)

$$\begin{array}{ccc|cccccccccccc}
 & D1 & D2 & & & & & & & & & & & & \\
 & 3 & 4 & 1 & 5 & 6 & : & 2 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & & & & & 1 & & \bar{1} & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
 & & & & & R_1 & & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 \\
\hline
 & & & 7 & \bar{2} & & : & \bar{1}\bar{5} & 30 & \bar{1}0 & \overline{45} & 87 & \overline{35} & \overline{126} & 254 \\
 & & & Q_1 & Q_2 & & & Q_1 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10}
\end{array}$$

Step1 : 2) 15 (7 (Q_1)

$\underline{14}$

$\underline{1}$ (R_1)

$$16 - \begin{pmatrix} 3 \\ \uparrow \\ 7 \end{pmatrix} = 16 - 21 = \bar{5}$$

Step 2 : 2) $\bar{5}$ ($\bar{2}$ (Q_2)

$\underline{\bar{4}}$

$\underline{\bar{1}}$ (R_2)

$$\bar{1}2 - \begin{array}{c} D_1\,D_2 \\ \begin{pmatrix} 3 & 4 \\ 7 & \bar{2} \end{pmatrix} \\ Q_1\,Q_2 \end{array} = \bar{1}2 - \left(\bar{6} + 28\right)$$

$$\bar{1}2 - (22) = \bar{1}2 + \overline{22} = \bar{3}0$$

Step 3 : 2) $\bar{3}0$ ($\bar{1}\,\bar{5}$ (Q_3)

$\underline{\bar{3}0}$

$\underline{0}$ (R_3)

$$= 08 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ \bar{2} & & \overline{15} \end{pmatrix} \\ Q_1 \quad Q_2 \end{array} = 8 - (\overline{45} + \bar{8}) = 8 - (\overline{53}) = 8 + 53 = 61$$

Step 4 : 2) 61 (30 (Q_4)
60
1 (R_4)

$$= 10 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ \overline{15} & & 30 \end{pmatrix} \\ Q_3 \quad Q_4 \end{array} = 10 - [\,90 + \overline{60}\,] = 10 - 30 = \overline{20}$$

Step 5 : 2) $\bar{2}$ 0($\bar{1}$0 (Q_5)
$\bar{2}$ 0
0 (R_5)

$$= 0 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ 30 & & \bar{1}0 \end{pmatrix} \\ Q_4 \quad Q_5 \end{array} = 0 - \left[\overline{30} + 120\right] = 0 - 90 = \overline{90}$$

Step 6 : 2) $\bar{9}$ 0($\bar{4}$ $\bar{5}$ Q_5
$\bar{9}$ 0
0 R_5

$$= 0 - \begin{array}{c} D_1 \quad D_2 \\ \begin{pmatrix} 3 & & 4 \\ & \times & \\ \overline{10} & & \overline{45} \end{pmatrix} \\ Q_5 \quad Q_6 \end{array} = 0 - \left[\overline{40} + \overline{135}\right] = 0 - \left[\overline{175}\right] = 175$$

Step 7 : 2) 175(87 (Q_6)
174
1 (R_6)

$$10 - \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ \overline{45} & 87 \end{array}\right) \\ Q_6 \quad Q_7 \end{array} = 10 - \left(261 + \overline{180}\right) = 10 - 81 = \bar{7}\,\bar{1}$$

Step 8 : $2\,)\ \bar{7}\,\bar{1}\ (\ \bar{3}\,\bar{5}\quad (Q_8)$

$$\begin{array}{l} \bar{7}\,\bar{0} \\ \hline \quad\bar{1}\quad (R_8) \end{array}$$

$$\bar{1}0 = \begin{array}{c} D_1 \quad D_2 \\ \left(\begin{array}{cc} 3 & 4 \\ \multicolumn{2}{c}{\times} \\ 87 & \overline{35} \end{array}\right) \\ Q_7 \quad Q_8 \end{array} = \bar{6}\,\overline{10} - \left(\overline{105} \; + \; 348\right) = \overline{10} - (243) = \overline{253}$$

Step 9 : $2)\ \overline{253}\ (\ \bar{1}\,\bar{2}\,\bar{6}\quad (Q_6)$

$$\begin{array}{l} \overline{252} \\ \hline \quad\bar{1}\quad (R_6) \end{array}$$

To get the final answer the decimal in the Quotient is shifted towards Right by 2 digits.

$Q = 7\,\bar{2}\ ;\ \overline{15}\ 30\ \overline{10}\ \overline{45}\ 87\ \overline{35}\ \overline{126}$

$= 7\,\bar{3}\,\bar{3}\,9\,\bar{4}\,3\,3\,\bar{7}\,\bar{6}$

$=$ 6 6 7. 8 6 3 2 2 4 As the divisors has one digit after decimal.

Problem 16: (Vinculum of example 19 Page No)

0 461397 ÷ 123.4

(a) Vinculum details of example 19 Page 81

$$\begin{array}{r|lcccccccccccc} D_1D_2 & & & & & & & & & & & & & \\ 3\ \ 4 & : & .4 & 6 & 1 & 3 & 9 & 7 & 0 & 0 & 0 & 0 & 0 & \\ & : & & 4 & 10 & 8 & 2 & 1 & 4 & 8 & 11 & 11 & 0 & 6 \\ 12 & & & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 & R_7 & R_8 & R_9 & R_{10} & R_{11} \\ \hline & :000 & 0 & 3 & 7 & 4 & \bar{1} & 0 & 3 & 5 & 6 & 6 & \bar{4} & \\ & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 & Q_8 & Q_9 & Q_{10} & Q_{11} & \end{array}$$

Step 1 : 12) 4 (0 (Q_1)
<u>0</u>
<u>4</u> (R_1)

$$46 - \underset{Q_1}{\overset{D_1}{\begin{pmatrix}3\\ \uparrow\\ 0\end{pmatrix}}} = 46$$

Step 2 : 12) 46 (3 (Q_2)
<u>36</u>
<u>10</u> (R_2)

$$101 - \underset{Q_1Q_2}{\overset{D_1D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ 0 & & 3\end{pmatrix}}} = 92$$

Step 3 : 12) 92 (7 (Q_3)
<u>84</u>
<u>8</u> (R_3)

$$83 - \underset{Q_2Q_3}{\overset{D_1D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ 3 & & 7\end{pmatrix}}} = 50$$

Step 4 : 12) 50 (4 (Q_4)
<u>48</u>
<u>2</u> (R_4)

$$29 - \underset{Q_3Q_4}{\overset{D_1D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ 7 & & 4\end{pmatrix}}} = \bar{1}\bar{1}$$

Step 5 : 12) $\bar{1}\bar{1}$ ($\bar{1}$ (Q_5)
<u>$\bar{1}\bar{2}$</u>
<u>1</u> (R_5)

$$17 - \underset{Q_4Q_5}{\overset{D_1D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ 4 & & \bar{1}\end{pmatrix}}} = 17 - \left[\bar{3} + 16\right]$$

$$= 17 - 13 = 4$$

Step 6 : 12) 4 (0 (Q_6)
<u>0</u>
<u>4</u> (R_6)

$$40 - \underset{Q_5Q_6}{\overset{D_1\,D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ \bar{1} & & 0\end{pmatrix}}} = 40 - [\bar{4}] = 44$$

Step 7 : 12) 44 (3 (Q_7)
<u>36</u>
<u>8</u> (R_7)

$$80 - \underset{Q_6\,Q_7}{\overset{D_1\,D_2}{\begin{pmatrix}3 & & 4\\ & \times & \\ 0 & & 3\end{pmatrix}}} = 80 - 9 = 71$$

Step 8 : $12\,)\,71\,(\,5 \quad (Q_8)$

$$\begin{array}{l}\underline{60}\\ \underline{11} \quad (R_8)\end{array}$$

$D_1 D_2$

$$= 110 - \left.\begin{array}{cc}3 & 4\\ \multicolumn{2}{c}{\times}\\ 3 & 5\end{array}\right) \cdot 110 - 27 = 83$$

$Q_7 Q_8$

Step 9 : $12\,)\,83\,(\,6 \quad (Q_9)$

$$\begin{array}{l}\underline{72}\\ \underline{11} \quad (R_9)\end{array}$$

$D_1 D_2$

$$110 - \begin{array}{cc}3 & 4\\ \multicolumn{2}{c}{\times}\\ 5 & 6\end{array} = 72$$

$Q_8\ Q_9$

Step 10 : $12\,)\,72\,(\,6 \quad (Q_{10})$

$$\begin{array}{l}\underline{72}\\ \underline{\ 0} \quad (R_{10})\end{array}$$

$D_1 D_2$

$$0 - \begin{array}{cc}3 & 4\\ \multicolumn{2}{c}{\times}\\ 5 & 6\end{array} = 42$$

$Q_{10} Q_1$

Step 11 : $12)\bar{4}\ \bar{2}(\bar{4} \quad (Q_{11})$

$$\begin{array}{l}\underline{\bar{4}\ \bar{8}}\\ \underline{\quad 6} \quad (R_{11})\end{array}$$

Quotient = 0.000 3 7 4 1 0 3 5 6 6 4
= 0.000 3 7 3 9 0 3 5 6 5 6

For obtaining the final answer one has to consider the decimal in divisor and shift decimal point by on division towards right The final answer thus becomes = 0 003739035656

(b) O.461397 ÷ 123 4 (Reduction method of example : 19 Page:)

$D_1 D_2$										
3 4	.4	6	1	3	9	7	0	0	0	
		/	/	/	/	/	/	/	/	
12		4	10	8	14	4	8	8	9	
		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	
	000	3	7	3	9	0	3	6		
	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8		

(m)

Step 1 : 12) 4 (0 (Q_1)

$\underline{0}$

$\underline{4}$ (R_1)

$$46 - \underset{Q_1}{\overset{D_1}{\begin{pmatrix} 3 \\ \uparrow \\ 0 \end{pmatrix}}} = 46$$

Step 2 : 12) 46 (3 (Q_2)

$\underline{36}$

$\underline{10}$ (R_2)

$$= 101 - \underset{Q_1\,Q_2}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 0 & & 3 \end{pmatrix}}} = 101 - 9 = 92$$

Step 3 : 12) 92 (7 (Q_3)

$\underline{84}$

$\underline{8}$ (R_3)

$$= 83 - \underset{Q_2\,Q_3}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 3 & & 7 \end{pmatrix}}} = 83 \quad 33 = 50$$

83 - 33   50

Step 4 : 12) 50 (4 (Q_4)

$\underline{48}$

$\underline{2}$ (R_4)

$$= 29 - \underset{Q_3\,Q_4}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 7 & & 4 \end{pmatrix}}} = 29 - [\,12 + 28\,] = -\text{ve}$$

∴ Reduce Q_4 by 1

12) 50 (3 Q_4(m)

$\underline{36}$

$\underline{14}$ R_4

$$= 149 - \underset{Q_3\,Q_4(m)}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 7 & & 3 \end{pmatrix}}} = 149 - [\,9 + 28\,] = 149 - 37 = 112$$

Step 5 : 12) 112 (9 (Q_5)

$\underline{108}$

$\underline{4}$ (R_5)

$$= 47 - \underset{Q_3(m)\,Q_4}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 3 & & 9 \end{pmatrix}}} = 47 - [\,27 + 12\,] = 47 - 39 = 8$$

Step 6 : 12) 8 (0 (Q_6)

$\underline{0}$

$\underline{8}$ (R_6)

$$80 - \underset{Q_5\,Q_6}{\overset{D_1\,D_2}{\begin{pmatrix} 3 & & 4 \\ & \times & \\ 9 & & 0 \end{pmatrix}}} = 80 - 36 = 44$$

Step 7 : 12) 44 (3 (Q_7)

$\underline{36}$

$\underline{8}$ (R_7)

= 0.0003739036

The final answer is obtained by shifting the decimal towards right by one digit Thus he final answer is 0 003739036

Problem 17 : (Vinculum details for example 14 Pajic)

89 69 ÷ 243

$$\begin{array}{cccccccccccc}
D1D2 & & & & & & & & & & & \\
4\ 3 & :8 & 9 & . & 6 & 9 & 0 & 0 & 0 & 0 & 0 \\
 & & / & & / & / & / & / & / & / & / \\
 & :0 & & 1 & 0 & 0 & 0 & 1 & 1 & 0 & \\
2 & R_1 & & R_2 & R_3 & R_4 & R_5 & R_6 & K_7 & R_8 & \\
\hline
 & .4 & 3 & 2 & 13 & 23 & 26 & 12 & 20 & & \\
 & Q_1 & Q_2 & Q_1 & Q_4 & Q_5 & Q_6 & Q_7 & Q_4 & &
\end{array}$$

As the Dhwajanka has two digits the decimal in the dividend is shifted by two digits to the left of the dividend The dividend is now 0 8969

Step 1 :

$$\begin{array}{l} 2\ )\ 8\ (\ 4 \quad (Q_1) \\ \quad\ \ \underline{8} \\ \quad\ \ \underline{0}\ (R_1) \end{array}$$

$$= 9 - \begin{array}{c} D_1 \\ \begin{pmatrix} 4 \\ \uparrow \\ 4 \end{pmatrix} \\ Q_1 \end{array} = 9 - 16 = \bar{7}$$

Step 2 :

$$\begin{array}{l} 2\ )\ \bar{7}\ (\ \bar{3} \quad (Q_2) \\ \quad\ \ \underline{\bar{6}} \\ \quad\ \ \underline{\bar{1}}\ (R_2) \end{array}$$

$$16 - \begin{array}{c} D_1\ D_2 \\ \begin{pmatrix} 4 & 3 \\ & \times \\ 4 & 3 \end{pmatrix} \\ Q_1\ Q_2 \end{array} = \bar{1}6 - (\bar{1}\bar{2} + 12) = \bar{1}6 = \bar{4}$$

Step 3 :

$$\begin{array}{l} 2\ )\ \bar{4}\ (\ \bar{2} \quad (Q_3) \\ \quad\ \ \underline{\bar{4}} \\ \quad\ \ \underline{0}\ (R_3) \end{array}$$

$$9 - \left(\begin{array}{cc} D_1 & D_2 \\ 4 & 3 \\ \multicolumn{2}{c}{\times} \\ 3 & 2 \\ Q_1 & Q_2 \end{array}\right) = 9 - (\bar{8} + \bar{9}) = 9 - (\bar{1}\ \bar{7}) = 26$$

Step 4 :

$$\begin{array}{l} 2\ )\ 26\ (\ 13 \quad (Q_4) \\ \quad\ \underline{26} \\ \quad\ \underline{\ 0\ }\ (R_4) \end{array}$$

$$= 0 - \left(\begin{array}{cc} D_1 & D_2 \\ 4 & 3 \\ \multicolumn{2}{c}{\times} \\ 2 & 13 \\ Q_1 & Q_2 \end{array}\right) = 0 - [52 + \bar{6}] = -(46) = \overline{46}$$

Step 5 :

$$\begin{array}{l} 2\ )\ \overline{46}\ (\ \bar{2}\ \bar{3}\ (Q_5) \\ \quad\ \underline{\overline{46}} \\ \quad\ \underline{\ 0\ }\ (R_5) \end{array}$$

$$0 - \left(\begin{array}{cc} D_1 & D_2 \\ 4 & 3 \\ \multicolumn{2}{c}{\times} \\ \overline{13} & \overline{23} \\ Q_4 & Q_5 \end{array}\right) = -[\overline{92} + 39] = -(\bar{5}\ \bar{3}) = 53$$

Step 6 :

$$\begin{array}{l} 2\ )\ 53\ (\ 26 \quad Q_6 \\ \quad\ \underline{52} \\ \quad\ \underline{\ 1\ } \qquad R_6 \end{array}$$

$$10 - \left(\begin{array}{cc} D_1 & D_2 \\ 4 & 3 \\ \multicolumn{2}{c}{\times} \\ 2\bar{3} & 26 \\ Q_5 & Q_6 \end{array}\right) = 10 - [104 + \bar{6}\ \bar{9}] = 10 - 35 = 2\ \bar{5}$$

Step 7 :

$$\begin{array}{l} 2\ )\ \bar{2}\ \bar{5}\ (\ \overline{12}\ (Q_7) \\ \quad\ \bar{2}\ \bar{4} \\ \\ \quad\ \underline{\ 1\ }\ (R_7) \end{array}$$

$$10 - \left| \begin{array}{c} D_1D_2 \\ 4\ \ 3 \\ \times \\ 26\ \overline{12} \\ Q_6\,Q_7 \end{array} \right. = 10 - [\ 4\ 8 - 78\ ] = 10 - 30 = 10\ +\ \bar{3}0\ =\ \bar{4}0$$

Step 8 : 2) $\bar{4}0$ ($\bar{2}$ 0 (Q_8)

$\underline{\bar{4}0}$

$\underline{0}$ (R_8)

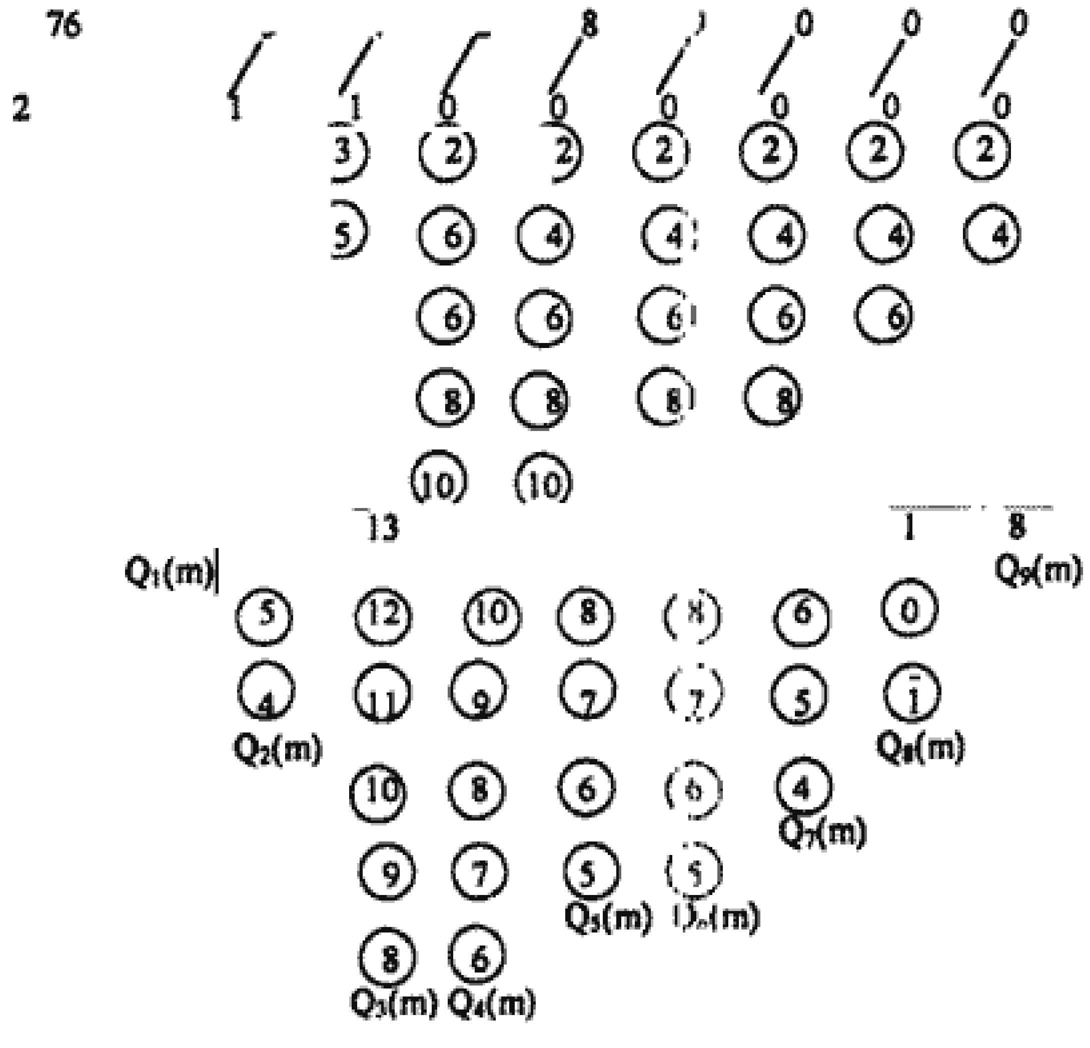
$\cdot\ 0\,4\ \bar{3}\,\bar{2}\ \ 13\ \overline{23}\ \ 26\ \bar{1}\,\bar{2}\ \ \bar{2}\,0$

$0.4\ \bar{3}\ \bar{1}\ 1\ \bar{1}\ 5\ \bar{4}\ 0$

0 36909460

∴ Quotient = 0.36909460

Problem 18 : 134 289 ÷ 2 76 (Reduction Method of Example 17 Page No)



(1) 2) 1 (0 (Q_1) $\boxed{\therefore Q_1 = 0}$

$\underline{0}$

$\underline{1}$ (R_1)

(2) $13 - \begin{pmatrix} 7 \\ \uparrow \\ 0 \end{pmatrix} = 13$

2) 13 (6 (Q_2)

$\underline{12}$

$\underline{1}$ (R_2)

(3) $14 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 0 & & 6 \end{pmatrix} = -28$ (negative)

Q_2 is reduced by 1 $\Rightarrow Q_2(m) = 5$

R_2 is raised by 2 $\Rightarrow R_2(m) = 3$

$34 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 0 & & 5 \end{pmatrix} = -1$ (still negative)

Q_2 is further reduced to 4 and R_2 is modified to 5 $\boxed{Q_2(m) = 4}$

$54 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 0 & & 4 \end{pmatrix} = 26$

2) 26 (13 (Q_3)

$\underline{26}$

$\underline{0}$ (R_3)

(4) $2 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 13 \end{pmatrix} =$ negative

$22 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 12 \end{pmatrix}$ negative

$42 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 11 \end{pmatrix}$ negative

$62 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 10 \end{pmatrix} =$ negative

$82 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 9 \end{pmatrix} =$ negative $\boxed{\therefore Q_1(m) = 8}$

$102 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 8 \end{pmatrix} = 22$

2) 22 (11 (Q_4)

$\underline{22}$

$\underline{0}$ (R_4)

(5) $08 - \left(\begin{array}{ccc} & & \\ 8 & \times & 11 \end{array}\right) = \text{negative}$

$28 - \left(\begin{array}{ccc} 7 & & 1 \\ 8 & \times & 10 \end{array}\right) = \text{negative}$

$- \left(\begin{array}{ccc} 7 & & 6 \\ 8 & \times & 9 \end{array}\right) = \text{negative}$

$68 - \left(\begin{array}{ccc} & & \\ 8 & \times & 8 \end{array}\right) = \text{negative}$

$88 - \left(\begin{array}{ccc} 7 & & 6 \\ 8 & \times & 7 \end{array}\right) = \text{negative}$

$\boxed{Q_5(m) = 6}$

$108 - \left(\begin{array}{ccc} & & \\ & \times & 6 \end{array}\right) = 18$

2) 18 (9 (Q_5)
18
0 (R_5)

(6) $09 - \left(\begin{array}{ccc} & & \\ & \times & 9 \end{array}\right) = \text{negative}$

$29 - \left(\begin{array}{ccc} & & \\ & \times & 8 \end{array}\right) = \text{negative}$

$49 - \left(\begin{array}{ccc} 7 & & 6 \\ 6 & \times & 7 \end{array}\right) = \text{negative}$

$69 - \left(\begin{array}{ccc} & & \\ & \times & 6 \end{array}\right) = \text{negative}$

$\boxed{Q_5(m) = 5}$

$89 - \left(\begin{array}{ccc} 7 & & 6 \\ 6 & \times & 5 \end{array}\right) = 18$

2) 18 (9 (Q_6)
18
0 (R_6)

(7) $00 - \begin{pmatrix} & & \\ & \times & \\ & & \end{pmatrix} = \text{negative}$

$20 - \begin{pmatrix} & & \\ & \times & \\ & & 8 \end{pmatrix} = \text{negative}$

$40 - \begin{pmatrix} & & \\ & \times & \\ & & 7 \end{pmatrix} = \text{negative}$

$60 - \begin{pmatrix} & & \\ & \times & \\ 5 & & 6 \end{pmatrix} = \text{negative}$

$\boxed{Q_6(m) = 5}$

$80 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 5 & & 5 \end{pmatrix} = 15$

2) 15 (7 (Q_7)
<u>14</u>
<u>1</u> (R_7)

(8) $0 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 5 & & 7 \end{pmatrix} = \text{negative}$

$20 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 5 & & 6 \end{pmatrix} = \text{negative}$

$40 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 5 & & 5 \end{pmatrix} = \text{negative}$

$\boxed{Q_7(m) = 4}$

$60 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 5 & & 4 \end{pmatrix} = 2$

2) 2 (1 (Q_8)
<u>2</u>
<u>0</u> (R_8)

(9) $0 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 1 \end{pmatrix} = \text{negative}$

$20 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & 0 \end{pmatrix} = \text{negative}$

$\boxed{Q_8(m) = \bar{1}}$

$40 - \begin{pmatrix} 7 & & 6 \\ & \times & \\ 4 & & \bar{1} \end{pmatrix} = 23$

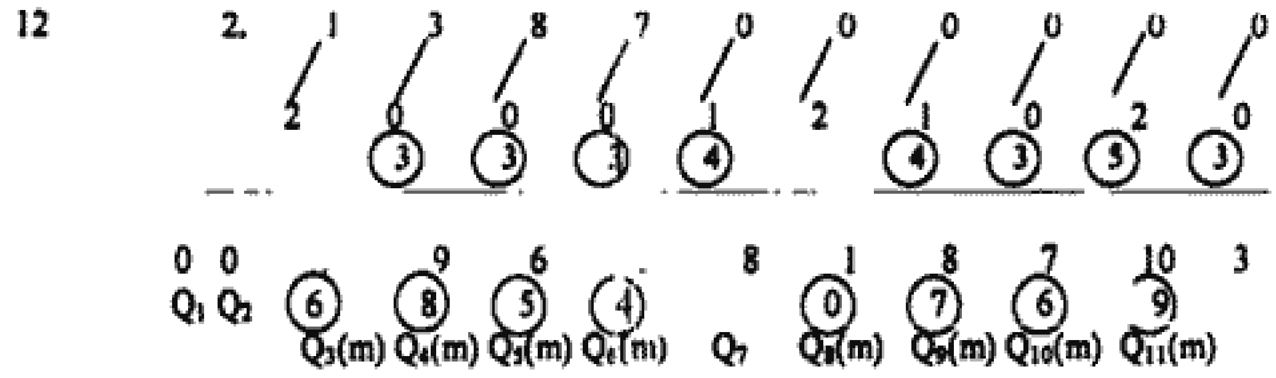
2).23 (11 (Q_9)
<u>22</u>
<u>1</u> (R_9)

Ans = 0. 4 8 6 5 5 4 $\bar{1}$ 8

(As these are two digits after decimal in the divisor, the decimal is shifted to right by two numbers)

Quotient = 48. 6 5 5 4 $\bar{1}$ 8
= 48 6 5 5 3 9 8

Problem 19 : 2.1387 + 3 12 (Reduction Method of Example 18 Page No)



(1) $Q_1 = 0$ (Due to partition rule Page:)

(2) $$2 - \begin{pmatrix} 1 \\ \uparrow \\ 0 \end{pmatrix} = 2$$

3) 2 (0 (Q_2)
<u>0</u>
2 (R_2)

(3) $21 - \left| \text{[illegible]}_{0} \right| = 21$

3) 21 (7 (Q_3)
<u>21</u>
<u>0</u> (R_3)

(4) $03 - \begin{pmatrix} 1 & 2 \\ 0 & 7 \end{pmatrix} = \text{negative}$

$\boxed{Q_3(m) = 6}$

$33 - \begin{vmatrix} & \\ & 6 \end{vmatrix} = 27$

3) 27 (9 (Q_4)
$\underline{27}$
$\underline{0}$ (R_4)

(5) $08 - \begin{vmatrix} 1 & 2 \\ 6 & 9 \end{vmatrix} = \text{negative}$

$\boxed{Q_4(m) = 8}$

38 - 18

3) 18 (6 (Q_5)
$\underline{18}$
$\underline{0}$ (R_5)

(6) $07 - \begin{vmatrix} & \\ 8 & 6 \end{vmatrix} = \text{negative}$

$\boxed{Q_5(m) = 5}$

$37 - \begin{pmatrix} 1 & 2 \\ 8 & 5 \end{pmatrix} = 16$

3) 16 (5 (Q_6)
$\underline{15}$
$\underline{1}$ (R_6)

(7) $10 - \begin{pmatrix} 1 & 2 \\ 5 & 5 \end{pmatrix} = \text{negative}$

$\boxed{Q_6(m) = 4}$

$40 - \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$ 26

3) 26 (8 (Q_7)
$\underline{24}$
2 (R_7)

(8) $20 - \begin{pmatrix} 1 & 2 \\ 4 & 8 \end{pmatrix}$ 4

$\boxed{Q_7(m) = 8}$

3) 4 (1 (Q_8)
$\underline{3}$
$\underline{1}$ (R_8)

(9) $10 - \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix} = \text{negative}$

$\boxed{Q_8(m) = 0}$

3) 24 (8 (Q_9)
$\underline{24}$
$\underline{0}$ (R_9)

$40 - \begin{vmatrix} & \\ 8 & 0 \end{vmatrix} = 24$

(10) $0 - \begin{pmatrix} & \\ & 8 \end{pmatrix}$ = negative

$\boxed{Q_9(m) = 7}$

$0 - \begin{pmatrix} 1 & 2 \\ 0 & 7 \end{pmatrix} = 23$

3) 23 (7 (Q_{10})
21
2 (R_{10})

(11) $20 - \begin{pmatrix} 1 & 2 \\ 7 & 7 \end{pmatrix}$ = negative

$\boxed{Q_{10}(m) = 6}$

$50 - \begin{pmatrix} 1 & 2 \\ 7 & 6 \end{pmatrix} = 30$

3) 30 (10 (Q_{11})
30
0 (R_{11})

(12) $0 - \begin{pmatrix} 1 & 2 \\ 6 & 10 \end{pmatrix}$ = negative

$\boxed{Q_{11}(m) = 9}$

$30 - \begin{pmatrix} 1 & 2 \\ 6 & 9 \end{pmatrix}$

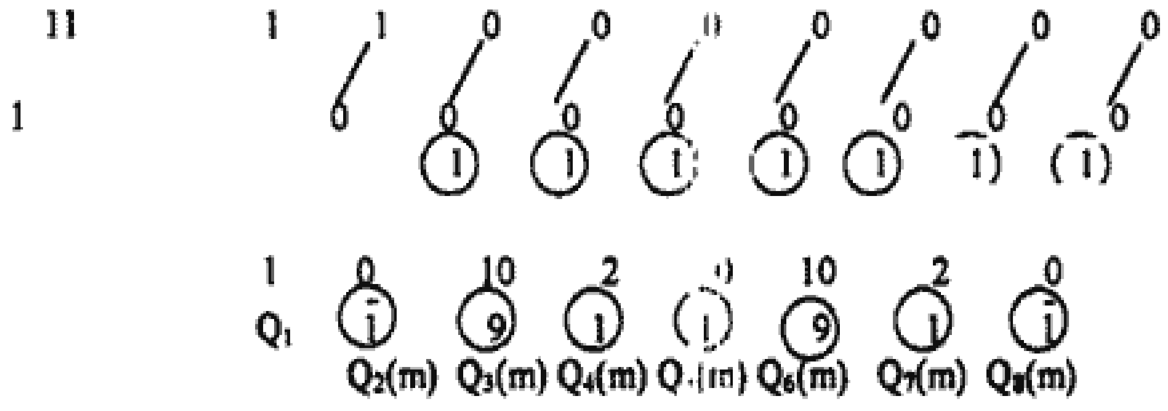
3) 9 (3 (Q_{12})
9
0 (R_{12})

Ans = 0 0 0 6 8 5 4 8 0 7 6 9 3

Since divisor has two digits after decimal, the decimal in the answer has to be shifted by two digits

∴ Quotient 0 6 8 5 4 8 0 7 6 9 3

Problem 20: 11 ÷ 111 (Reduction)



(1) $1)\ 1\ (1\ (Q_1)$
$\underline{1}$
$\underline{1}\ (R_1)$

$\boxed{Q_1 = 1}$

(2) $01 - \left|\begin{smallmatrix}1\\ \uparrow\\ 1\end{smallmatrix}\right. = 0$

$1)\ 0\ (0\ (Q_2)$
0
$\underline{0}\ (R_2)$

(3) $00 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 0\end{smallmatrix}\right) = \text{negative}$

$|Q_2(m) = 1|$

$10 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 1\end{smallmatrix}\right) = 10$

$1)\ 10\ (10\ (Q_3)$
$\underline{10}$
$\underline{0}\ (R_3)$

(4) $00 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 10\end{smallmatrix}\right) = \text{negative}$

$10 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 9\end{smallmatrix}\right) = 2$

$1)\ 2\ (2\ (Q_4)$
$\underline{2}$
$0\ (R_4)$

(5) $00 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 9 & & 2\end{smallmatrix}\right) = \text{negative}$

$|Q_4(m) = 1|$

$10 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 9 & & 1\end{smallmatrix}\right) = 0$

$1)\ 0\ (0\ (Q_5)$
$\underline{0}$
$\underline{0}\ (R_5)$

(6) $00 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 0\end{smallmatrix}\right) = \text{negative}$

$10 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 1\end{smallmatrix}\right) = 10$

$1)\ 10\ (10\ (Q_6)$
$\underline{10}$
$\underline{0}\ (R_6)$

$\boxed{Q_6(m) = 9}$

(7) $00 - \left(\begin{smallmatrix}1 & & 1\\ & \times & \\ 1 & & 10\end{smallmatrix}\right) = \text{negative}$

$10 - \left|\begin{smallmatrix} & & \\ & \times & \\ & & 9\end{smallmatrix}\right| = 2$

$1)\ 2\ (2\ (Q_7)$
$\underline{2}$
$\underline{0}\ (R_7)$

(8) $00 - \begin{pmatrix} 1 & & 1 \\ & \times & \\ 9 & & 0 \end{pmatrix} = \text{negative}$ $\boxed{[C_8(m) = 1]}$

$10 - \begin{pmatrix} 1 & & 1 \\ & \times & \\ 9 & & 1 \end{pmatrix} = 0$

1) 0 (0 (Q_8)
  0
  0 (R_8)

(9) $00 - \begin{pmatrix} 1 & & 1 \\ & \times & \\ 1 & & 0 \end{pmatrix} = \text{negative}$ $\boxed{[C_9(m) = 1]}$

$10 - \begin{pmatrix} 1 & & 1 \\ & \times & \\ 1 & & 1 \end{pmatrix} = 10$

Quotient = 0 1 1 9 1 1 9 1 1
  = 0 0 9 9 0 9 9 0 9

Current Method

```
111 ) 1100 ( 0.09909909
      999
      1010
       999
        1100
         999
         1010

            1100
             999
```

Problem 21. (a) 1 + 11 (Reduction Method)

1 : 1 0 0 0 0 0 0 0

1 0 0 0 0 0 0 0

: ① ① ① ① ① ① 1

:① 10 1 10 1 10 1

0 ⑨ ⓪ ⑨ ⓪ ⑨ ⓪

(1) 1) 1 (1 (Q_1)
1
0 (R_1)

(2) $0 - \begin{pmatrix}1\\ \uparrow\\ 1\end{pmatrix} = -1$ (negative)

Reducing Q_1 by 1 we get $Q_1(m) = 0$
Adding 1 to R_1 we get $R_1(m) = 1$

$10 - \begin{pmatrix}1\\ \uparrow\\ 0\end{pmatrix} = 10$ 1) 10 (10 (Q_2)
10
0 (R_2)

(3) $0 - \begin{pmatrix}1\\ \uparrow\\ 10\end{pmatrix} = -10$ (negative)

Reducing Q_2 by 1 we get $Q_2(m) - 9$
Adding 1 to R_2 we get $R_2(m) = 1$

$10 - \begin{pmatrix}1\\ \uparrow\\ 9\end{pmatrix} = 1$ 1) 1 (1 (Q_3)
1
0 (R_3)

(4) $0 - \begin{pmatrix}1\\ \uparrow\\ 1\end{pmatrix} = -1$ (negative)

Reducing Q_3 we get $Q_3(m) = 0$

$10 - \begin{pmatrix}1\\ \uparrow\\ 0\end{pmatrix} = 10$ 1) 10 (10 (Q_4)
10
0(R_4)

(5) $0 - \begin{pmatrix} 1 \\ \uparrow \\ 10 \end{pmatrix} = -10$ (negative)

Reducing Q_4, we get $Q_4(m) = 9$

$10 - \begin{pmatrix} 1 \\ \uparrow \\ 9 \end{pmatrix} = 1$

1) 1 (1 (Q_5)
$\underline{1}$
$\underline{0}$(R_5)

(6) $0 - \begin{pmatrix} 1 \\ \uparrow \\ 1 \end{pmatrix} = -1$ (negative)

Reducing Q_5, we get $Q_5(m) = 0$

$10 - \begin{pmatrix} 1 \\ \uparrow \\ 0 \end{pmatrix} = 10$

1) 10 (10 (Q_6)
$\underline{10}$
$\underline{0}$(R_6)

(7) $0 - \begin{pmatrix} 1 \\ \uparrow \\ 10 \end{pmatrix} = -10$ (negative)

Reducing Q_6, we get $Q_6(m) = 9$

$10 - \begin{pmatrix} 1 \\ \uparrow \\ 9 \end{pmatrix} = 1$

1) 1 (1 (Q_7)
$\underline{1}$
$\underline{0}$(R_7)

(8) $0 - \begin{pmatrix} 1 \\ \uparrow \\ 1 \end{pmatrix} = -1$ (negative)

Reducing Q_7, we get $Q_7(m) = 0$

Ouotient = 0.090909 .. ……

Problem 21: (b) 1 ÷ 11 (Vinculum Method)

$$\begin{array}{ccccccccc}
 & 1 & 1 & & 0 & 0 & 0 & 0 & \bar{} \\
 & & & / & / & / & / & / & / \\
1 & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & R_1 & R_2 & R_3 & R_4 & R_5 & R_6 \\
 & & : & 1 & \bar{1} & 1 & 1 & 1 & 1 \\
 & & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6
\end{array}$$

(1) $\begin{array}{l} 1)\ 1\ (1(Q_1) \\ \quad \underline{1} \\ \quad \underline{0}\ (R_1) \end{array}$

(2) $00 - \begin{pmatrix} 1 \\ \uparrow \\ 1 \end{pmatrix} = \bar{1}$ $\begin{array}{l} 1)\ 1\ (1(Q_2) \\ \quad 1 \\ \quad \underline{0}\ (R_2) \end{array}$

(3) $00 - \begin{pmatrix} 1 \\ \uparrow \\ \bar{1} \end{pmatrix} = 1$ $\begin{array}{l} 1)\ 1\ (1\ (Q_3) \\ \quad 1 \\ \quad 0\ (R_3) \end{array}$

(4) $00 - \begin{pmatrix} 1 \\ \uparrow \\ 1 \end{pmatrix} = \bar{1}$ $\begin{array}{l} 1)\ 1\ (\bar{1}(Q_4) \\ \quad \underline{1} \\ \quad 0\ (R_4) \end{array}$

(5) $00 - \begin{pmatrix} 1 \\ \uparrow \\ \bar{1} \end{pmatrix} = 1$ $\begin{array}{l} 1)\ 1\ (1\ (Q_5) \\ \quad \underline{1} \\ \quad 0\ (R_5) \end{array}$

(6) $00 - \begin{pmatrix} 1 \\ \uparrow \\ 1 \end{pmatrix} = \bar{1}$ $\begin{array}{l} 1)\ \bar{1}\ (\bar{1}(Q_6) \\ \quad \underline{\bar{1}} \\ \quad \underline{0}\ (R_6) \end{array}$

Quotient $= 0\ 1\bar{1}1\bar{1}1\bar{1}1 = 0\ 090909$ . .

Problem 22 : 1 ÷ 111 (Reduction)

11 : 1 0 0 0 0 0 0

1 0 0 0 0 0 0

: ① ① ① ① ① ①

0 1 10

⓪ ⑨

(1) 1) 1 (1 Q_1

1

0 (R_1)

(2) $0 - \begin{pmatrix}1\\ \uparrow\\ 1\end{pmatrix} = -1$ (negative)

$Q_1(m) = 0$, $R_1(m) = 1$

$10 - \begin{pmatrix}1\\ \uparrow\end{pmatrix} = 10$

1) 10 (10 (Q_2)

10

0 (R_2)

(3) 0 - -10 (negative)

$Q_2(m) = 9$, $R_2(m) = 1$

$10 - \begin{pmatrix}1 & & 1\\ & \times & \\ 0 & & 9\end{pmatrix} = 1$

1) 1 (1 (Q_3)

1

0 (R_3)

(4) $0 - \begin{pmatrix}1 & & 1\\ & \times & \\ 9 & & 1\end{pmatrix} = -10$ (negative)

$Q_1(m) = 0$, $R_3(m) = 1$

$10 - \begin{pmatrix}1 & & 1\\ & \times & \\ 9 & & 0\end{pmatrix} = 1$

1) 1 (1 (Q_4)

1

0 (R_4)

(5) $0 - \begin{pmatrix}1 & & 1\\ & \times & \\ 0 & & 1\end{pmatrix} = -1$ (negative)

1) 10 (10 (Q_5)

10

0 (R_5)

$Q_4(m) = 0$, $R_4(m) = 1$

$10 - \begin{pmatrix}1 & & 1\\ & \times & \\ 0 & & 0\end{pmatrix} = 10$

(6) $0 - \begin{pmatrix}1 & & 1\\ & \times & \\ 0 & & 10\end{pmatrix} = -10$(negative)

$Q_5(m) = 9$ $R_5(m) = 1$

$10 - \begin{pmatrix}1 & & 1\\ & \times & \\ 0 & & 9\end{pmatrix} = 1$

1) 1 (1 (Q_6)

1

0 (R_6)

(7) $0 - \begin{pmatrix}1 & & 1\\ & \times & \\ 9 & & 1\end{pmatrix} = -10$ (negative)

$Q_6(m) = 0$ $R_6(m) = 1$

Quotient = 0 0090090

Chapter III

Combined Division and Multiplication:

a)

Combined Division and Multiplication can be worked out by applying the principles that are enumerated under division.

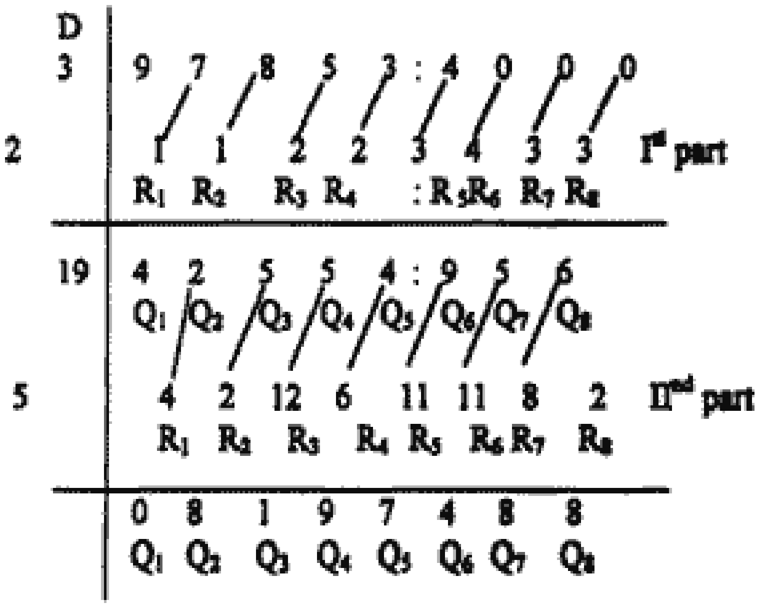
1) 978534 + (23 × 519)

Current Method

```
       519
       x23
    1557
   1038
   11937

11937)978534(81.9748
       95496
        23574
        11937
         116370
         107433
            89370
            83559
             58110
             47748
             103620
             95496
              81240
```

Vedic Method



D
3 | 9 7 8 5 3 : 4 0 0 0
2 | 1 1 2 2 3 4 3 3 Ist part
R1 R2 R3 R4 : R5 R6 R7 R8

19 | 4 2 5 5 4 : 9 5 6
Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8
5 | 4 2 12 6 11 11 8 2 IInd part
R1 R2 R3 R4 R5 R6 R7 R8

0 8 1 9 7 4 8 8
Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8

Step 1 : $2\,)\,9\,(\,4 \quad (Q_1)$

$\underline{8}$

$\underline{1} \quad (R_1)$

D

$$17 - \begin{pmatrix}3\\ \uparrow\\ 4\end{pmatrix} = 5$$

Q_2

Step 2 : $2\,)\,5\,(\,2 \quad (Q_2)$

$\underline{4}$

$\underline{1} \quad (R_2)$

D

$$18 - \begin{pmatrix}3\\ \uparrow\\ 2\end{pmatrix} = 12$$

Q_2

Step 3 : $2\,)\,12\,(\,6 \quad (Q_3)$

$\underline{12}$

$\underline{0} \quad (R_3)$

D

$$1 - \begin{pmatrix}3\\ \uparrow\end{pmatrix} = -18 \text{ (negative)}$$

Q_3

$\therefore Q_3$ is reduced by 1

$2\,)\,12\,(\,5 \quad Q_3\ (m)$

$\underline{10}$

$\underline{2} \quad R_3\ (m)$

D

$$25 - \begin{pmatrix}3\\ \uparrow\\ 5\end{pmatrix} = 10$$

$Q_3(m)$

Step 4 : $2\,)\,10\,(\,5 \quad (Q_4)$

$\underline{10}$

$\underline{0} \quad (R_4)$

D

$$03 - \begin{pmatrix}3\\ \uparrow\\ 5\end{pmatrix} = -12\ (-\ell)$$

Q_5

Reducing Q_4 by 1

$2\,)\,10\,(\,4 \quad (Q_4(m))$

$\underline{8}$

$\underline{2} \quad (R_4(m))$

D

$$23 - \begin{pmatrix}3\\ \uparrow\\ 4\end{pmatrix} = 11$$

$Q_5(m)$

Step 5 : $2\,)\,11\,(\,5 \quad (Q_5)$

$\underline{10}$

$\underline{1} \quad (R_5)$

D

$$14 - \begin{bmatrix}5\\ \uparrow\\ 3\end{bmatrix} = -1\text{(negative)}$$

Q_6

Reducing Q_5 by 1

$2\,)\,11\,(\,4 \quad (Q_5(m))$

$\underline{8}$

$\underline{3} \quad (R_5)$

D

$$34 - \begin{pmatrix}3\\ \uparrow\\ 4\end{pmatrix} = 22$$

$Q_5(m)$

Step 6 :

2) 22 (11 (Q_6)
22
0 (R_6)
D

$$0 - \begin{pmatrix}3\\ \uparrow\\ 11\end{pmatrix} = -33 \text{ (negative)}$$

Q_6
Reducing Q_6 by 1

2) 22 (10 ($Q_6(m)$)
20
2 ($R_6(m)$)
D

$$20 - \begin{pmatrix}3\\ \uparrow\\ 10\end{pmatrix} = -10 \text{ (negative)}$$

$Q_6(m)$
Reducing $Q_6(m)$ by 1

2) 22 (9 $Q_6(m)$
18
4
18
4 ($R_6(m2)$)
D

$$40 - \begin{pmatrix}3\\ \uparrow\\ 9\end{pmatrix} = 13$$

$Q_6(m)$

Step 7 :

2) 13 (6 (Q_7)
12
1 (R_7)
D

$$10 - \begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix} = -8 \text{ (negative)}$$

Q_7
Reducing Q_7 by 1

2) 13 (5 ($Q_7(m)$)
10
3 ($R_7(m)$)
D

$$30 - \begin{pmatrix}3\\ \uparrow\\ 5\end{pmatrix} = 15$$

$Q_7(m)$

Step 8 :

2) 15 (7 (Q_8)
14
1 ($R_8(m)$)
D

$$10 - \begin{pmatrix}3\\ \uparrow\\ 7\end{pmatrix} = 11\text{(negative)}$$

Q_8

Reducing Q_8 by 1
2) 15 (6 ($Q_8(m)$)
12
3 ($R_8(m)$)
D

$$30 - \begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix} = 12$$

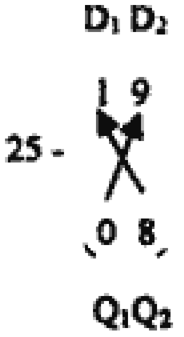
$Q_8(m)$

II **Part division : e, 42544 956 ÷ 519**

Step 1 : 5) 4 (0 (Q_1)
0
4 (R_1)

42 - | ↑

Step 2 : 5) 42 (8 (Q_2)
40
2 (R_2)



Step 3 : 5) 17 (3 (Q_3)
15
2 (R_3)

D_1 D_2
9
24 - = 2 4- (3 + 72) = - ve
3
Q_1 Q_2

Reducing Q_3 by 1

5) 17 (2 $(Q_3(m))$
10
7 (R_3)

Reducing $Q_3(m)$ by 1

5) 17 (1 $(Q_3(m))$
5
12 (R_3)

$$74 - \left(\begin{array}{ccc}1 & & 9\\ & \times & \\ -8 & & 2\end{array}\right) = ($$

$$04 - \left(\begin{array}{ccc} & \times & \\ 2 & & 0\end{array}\right) = \text{negative}$$

2) 0 (0 (Q_4)
$\underline{0}$
$\underline{0}$ (R_4)
$D_1\,D_2$

124 - $\quad$: 124 – 73 = 51

8 1

$Q_2\,Q_3$

Step 4 : 5) 51 (10 (Q_4)
$\underline{50}$
$\underline{\ 1}$ (R_4)
$D_1\,D_2$

$$14 - \begin{array}{ccc}1 & & 9\\ & \times & \\ 1 & & 10\end{array} = -5 \text{ (-ve)}$$

$Q_3\,Q_4$

Reducing Q_4 by 1

5) 51 (9 $(Q_4(m))$
$\underline{45}$
$\underline{\ 6}$ $(R_4(m))$
$D_1\,D_2$

$$64 - \begin{array}{ccc}1 & & 9\\ & \times & \\ 1 & & 9\end{array} = 46$$

$Q_3\,Q_4(m)$

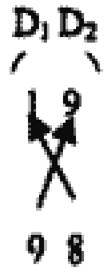
5) 46 (9 (Q_5)
$\underline{45}$
$\underline{\ 1}$ (R_5)

19 - 

= negative

Q_5 is reduced by 1

Step 5 : 5) 46 (8 (Q_5)
40
6 (R_5)

69 - 

= 69 – (81 + 8) = (- ve)

$Q_4(m)Q_5$

Reducing Q_5 by 1

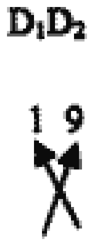
5) 46 (7 ($Q_5(m)$)
35
11 ($R_5(m)$)

119 - 

=119 – (88) =31

$Q_4(m)$ $Q_5(m)$

Step 6 : 5) 31 (6 (Q_6)
30
1 (R_6)

15 - 

= 5 – (6 + 63) = -ve

Q_5 Q_6

Reducing Q_6 by 1

$5\,)\,31\,(\,5\quad (Q_6(m))$
$\underline{25}$
$\underline{6}\quad (R_6(m))$

$$\begin{array}{c} D_1\,D_2 \\ \\ 65 - \begin{array}{cc} 1 & 9 \\ \multicolumn{2}{c}{\times} \\ 7 & 5 \end{array} \qquad 65 - (63 + 5) = \text{negative} \\ \\ Q_5\,Q_6(m) \end{array}$$

Reducing $Q_6(m)$ by 1

$5\,)\,30\,(\,4\quad (Q_6(m))$
$\underline{20}$
$\underline{11}\quad (R_6)$

$$\begin{array}{c} D_1\,D_2 \\ \\ 115 - \begin{array}{cc} 1 & 9 \\ \multicolumn{2}{c}{\times} \\ 7 & 4 \end{array} \qquad 115 - (4 + 63) = 48 \\ \\ Q_5Q_6(m) \end{array}$$

Step 7 : $5\,)\,48\,(\,9\quad (Q_7)$
$\underline{45}$
$\underline{3}\quad (R_7)$

$$\begin{array}{c} D_1\ \ D_2 \\ 36 - \left| \begin{array}{cc} & 9 \\ \multicolumn{2}{c}{\times} \\ & 9 \end{array} \right) = \text{negative} \\ Q_6\ \ Q_7 \\ (m) \end{array}$$

Reducing Q_7 by 1

$5\,)\,48\,(\,8\quad (Q_7(m))$
$\underline{40}$
$\underline{8}\quad (R_7)$

$$\begin{array}{c} D_1\ \ D_2 \\ 86 - \left(\begin{array}{cc} 1 & 9 \\ \multicolumn{2}{c}{\times} \\ 4 & 8 \end{array} \right) = 42 \\ Q_6\,Q_7 \\ (m)(m) \end{array}$$

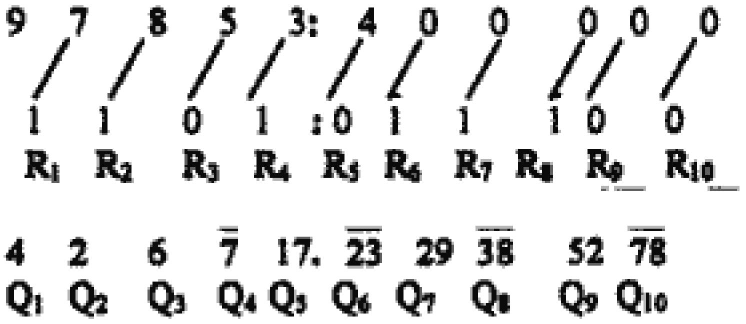
5) 42 (8 (Q_8)

<u>40</u>

2(R_8)

Final Answer = 81.97488

I Part (Vinculum)



Step 1 : 2) 9 (4 (Q_1)

<u>8</u>

1 (R_1)

17 -

Step 2 : 2) 5 (2 (Q_2)

<u>4</u>

1 (R_2)

18 - | ↑ = 12

Step 3 : 2) 12 (6 (Q_3)

<u>12</u>

0 (R_3)

$$5 - \overset{D_1}{\underset{Q_3}{\begin{pmatrix}3\\ \uparrow\\ 6\end{pmatrix}}} = \overline{1}\,\overline{3}$$

Step 4 : 2) $\overline{1}\,\overline{3}$ ($\overline{7}$ (Q_4)

$\underline{\overline{1}\,\overline{4}}$

$\underline{1}$ (R_4)

Step 5 :

$$13 - \overset{D_1}{\underset{Q_4}{\begin{bmatrix}3\\ \uparrow\\ \overline{7}\end{bmatrix}}} = 13 - (\overline{2}\,\overline{1})$$

$= 13 + 21 = 34$

2) 34 (17 (Q_5)

$\underline{34}$

$\underline{0}$ (R_5)

Step 6 :

$$4 - \overset{D_1}{\underset{Q_5}{\begin{bmatrix}3\\ \uparrow\\ 17\end{bmatrix}}} = 4 - 51 = \overline{47}$$

2) $\overline{47}$ ($\overline{2}\,\overline{3}$ (Q_6)

$\underline{\overline{46}}$

$\underline{\overline{1}}$ (R_6)

Step 7 :

$$\overline{1}0 - \overset{D_1}{\underset{Q_6}{\begin{bmatrix}3\\ \uparrow\\ \overline{23}\end{bmatrix}}} = \overline{1}0 - (\overline{69})$$

$= +69 = 59$

2) 59 (29 (Q_7)

$\underline{58}$

$\underline{1}$ (R_7)

Step 8 :

$$10 - \underset{Q_7}{\begin{bmatrix}3\\ \uparrow\\ 29\end{bmatrix}} = 10 - 87 = \overline{77}$$

$$\begin{array}{l} 2\,)\ \overline{77}\ (\ \overline{3}\,\overline{8}\quad (Q_8) \\ \quad\ \ \underline{\overline{76}} \\ \quad\ \ \ \underline{\ \overline{1}\ }\ (R_8) \end{array}$$

Step 9 :

$$\overline{1}0-\underset{Q_8}{\overset{D_1}{\begin{bmatrix} 3 \\ 4 \\ \updownarrow \\ \overline{3}\,\overline{8} \end{bmatrix}}}=\overline{1}0-\left(\overline{114}\right)$$

$$=\ \overline{10}+114=104$$

$$\begin{array}{l} 2\,)\ 104\ (\ 52\quad (Q_9) \\ \quad\ \ \underline{104} \\ \quad\ \ \ \underline{\ 0\ }\quad (R_9) \end{array}$$

Step 10 :

$$0-\underset{Q_9}{\overset{D_1}{\begin{matrix} 3 \\ 4 \\ \updownarrow \\ 52 \end{matrix}}}\ =\overline{1}\,\overline{56}$$

$$\begin{array}{l} 2\,)\ \overline{1}\,\overline{56}\ (\ \overline{78}\qquad (Q_{10}) \\ \quad\ \ \underline{\overline{1}\,\overline{56}} \\ \qquad\qquad\quad (R_{10}) \end{array}$$

$$\begin{aligned} Q &= 4\,2\,6\,\overline{7}\ \ 17.\overline{23}\ 2938\ \ 52\ 78 \\ &= 4\,2\,6\,\overline{6}\ \ 5.\overline{1}\,6\,\overline{4}\ \ 5\,\overline{8} \\ &= 42544.95642 \end{aligned}$$

II Part (using Vinculum)

$$\begin{array}{c|ccccccccccccc} & 4 & 2 & 5 & : & 4 & 4 & . & 9 & 5 & 6 & 4 & 2 & 0 \\ 19 & & / & / & & / & / & & / & / & / & / & / & / \\ 5 & & 4 & 2 & : & 2 & 1 & & \bar{3} & 3 & 2 & \bar{1} & \bar{3} & 0 \\ \hline & 0 & \dot{8} & 3 & . & \bar{1}0 & \bar{4} & & 14 & 11 & \bar{2}\bar{2} & \bar{1}\bar{6} & 37 & \\ & Q_1 & Q_2 & Q_3 & & Q_4 & Q_5 & & Q_6 & Q_7 & Q_8 & Q & Q_{10} & \end{array}$$

Step 1:
$$\begin{array}{l} 5)\,4\,(0 \quad (Q_1) \\ \ \ \underline{0} \\ \ \ \underline{4} \quad (R_1) \end{array}$$

Step 2:
$$42 - \begin{pmatrix} 1 \\ \uparrow \\ 0 \end{pmatrix} = 42$$
$$\begin{array}{l} 5)\,42\,(\,8 \quad (Q_2) \\ \ \ \underline{40} \\ \ \ \underline{\ 2} \quad (R_2) \end{array}$$

Step 3:
$$25 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ 0 & & 8 \end{pmatrix} = 17$$
$$\begin{array}{l} 5)\,17\,(\,3 \quad (Q_3) \\ \ \ \underline{15} \\ \ \ \underline{\ 2} \quad (R_3) \end{array}$$

Step 4:
$$24 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ 8 & & 3 \end{pmatrix} = \bar{5}\,\bar{1}$$
$$\begin{array}{l} 5)\,\bar{5}\,\bar{1}\,(\,\bar{1}\,\bar{0} \quad (Q_4) \\ \ \ \underline{\bar{5}\,\bar{0}} \\ \ \ \ \ \bar{1} \quad (R_4) \end{array}$$

Step 5:
$$\bar{1}4 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ 3 & & \bar{1}0 \end{pmatrix} = \bar{2}\,\bar{3}$$
$$\begin{array}{l} 5)\,\bar{2}\,\bar{3}\,(\,\bar{4} \quad (Q_5) \\ \ \ \underline{\bar{2}\,\bar{0}} \\ \ \ \ \ \ \bar{3} \quad (R_5) \end{array}$$

Step 6:
$$\dot{3}9 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ \bar{10} & & \bar{4} \end{pmatrix} = 73$$
$$\begin{array}{l} 5)\,73\,(\,14 \quad (Q_6) \\ \ \ \underline{70} \\ \ \ \underline{\ 3} \quad (R_6) \end{array}$$

Step 7:
$$35 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ \bar{4} & & 14 \end{pmatrix} = 57$$
$$\begin{array}{l} 5)\,57\,(\,11 \quad (Q_7) \\ \ \ \underline{55} \\ \ \ \underline{\ 2} \quad (R_7) \end{array}$$

Step 8:
$$26 - \begin{pmatrix} 1 & & 9 \\ & \times & \\ 14 & & 9 \end{pmatrix} = \bar{1}\,\bar{1}\,\bar{1}$$
$$= \bar{1}\,\bar{1}\,\bar{1}$$
$$\begin{array}{l} 5)\,\bar{1}\,\bar{1}\,\bar{1}\,(\bar{2}\,\bar{2} \quad (Q_8) \\ \ \ \underline{\bar{1}\,\bar{1}\,\bar{0}} \\ \ \ \ \ \underline{\ \ \ 1} \quad (R_8) \end{array}$$

Step 9: $\bar{1}4 - \left(\frac{1}{11} \times \frac{9}{22}\right) = 8\,3$

$$5)\ \bar{8}\,\bar{3}\ (1\,\bar{6} \quad (Q_9)$$
$$\underline{\bar{8}\,\bar{0}}$$
$$\underline{\bar{3}} \quad (R_9)$$

Step 10: $\bar{3}2 - \left(\frac{1}{22} \times \frac{9}{16}\right) = 186$

$$5)\ 186\ (\,37 \quad (Q_{10})$$
$$\underline{185}$$
$$\underline{1} \quad (R_{10})$$

$$\text{Final Answer} = 0\,8\,3\,.\,\bar{1}0\ \bar{4}\ 14\ 11\ \bar{2}\,\bar{2}\ \bar{1}\bar{6}\ \ 37$$
$$= 8\,2\,.\ 0\ \bar{3}\ 5\ \ \bar{1}\ \ \bar{3}\ \ \bar{3}\ \ 7$$
$$= 8\,1\ \ 9\ 7\ 4\ \ 8\ \ 6\ \ 7\ \ 7$$

2 $6345.12 \div (232 \times 67 \times 13)$

Current Method

232 × 67 × 13

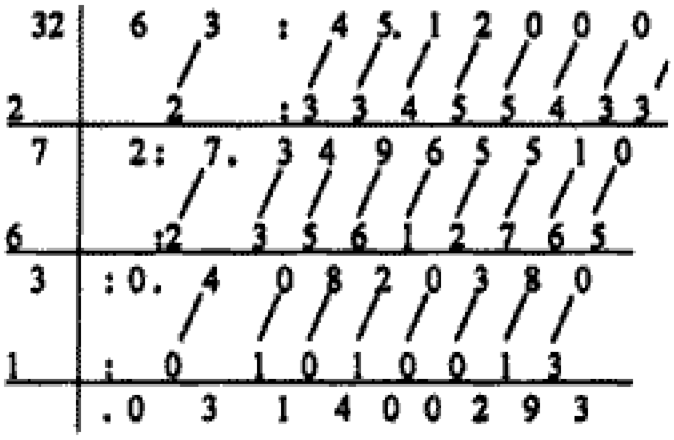
```
   232
 × 67
  1624
 1392
 15544

 46632
15544
202072
6345.12     634512
202072      20207200
20207200) 63451200 (0.03140029
          606216000
           28296000

           20207200
             80888000

                59200000
                40414400
                18785600
               181864800
                  5991200
```

Vedic Method



Final Answer = 0.031400293

3) **210678 ÷ (1.98 × 0.267)**

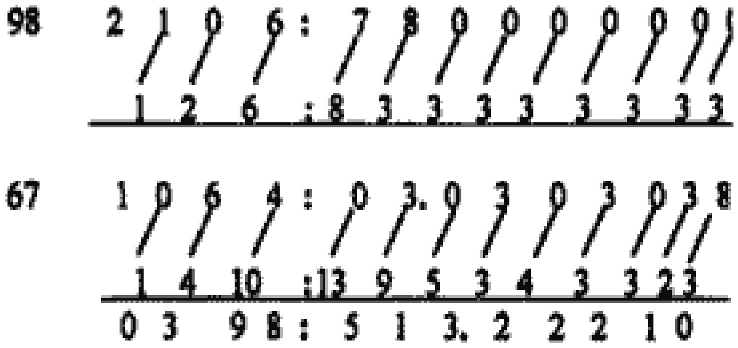
Current Method

0.267
× 1.98
2136
2403
267
0.52866

$$\frac{210678}{0.52866} = \frac{21067800000}{52866}$$

52866) 21067800000 (398513.22210
158598
520800
475794
450060
422928
271320
264330
69900
528866
170340
158598
117420
105732
116880
105732
111480
105732
57480
52866
461400
422928
39472

Vedic Method



Final Answer = 398513.22210

(b) Combined Addition and Division (Left to Right Operation)(V.M.):

Combined operation of two or more individual mathematical operations such as addition, division, multiplication in general can also be carried out using Vedic principles.

Here we are giving a few examples wherein a combined addition and division is demonstrated both in Current and Vedic Method.

Examples:

Example 1. (132 + 255 + 273 + 891) + 5

In Current Method the numbers are first added up and then the result is divided by 5 showing the quotient and the remainder.

In the Vedic Method, this has a difference in operation in the sense that addition is carried out from left to right and division is simultaneously carried out as detailed in examples.

Step 1:

The addition is carried out from left to right and from top to bottom keeping the numbers vertically. While doing so, if one gets a value greater than or equal to the divisor, the result is divided at that stage, and quotient is shown on the left, remainder is carried out to the next value in that column.

The division is carried out in such a way that the quotient is adjusted to have the modulus of the remainder least. For example, if 8 is to be divided by 5, the quotient 1 and remainder 3 is not preferred to the quotient 2 and remainder $\bar{2}$. This is to be followed throughout.

After the first column is over, all the quotients so obtained are added which shows the corresponding digit in the final result under that column. At the end of the first column, the remainder is carried out to the beginning of the next column. Same procedure of addition and simultaneous division is carried to the rest of the columns representing the addition. The addition of the remainder in each column, which is brought into it from the previous column, is necessarily multiplied by 10 and then proceeded.

Current Method

$$\begin{array}{ccc} 1 & 3 & 2 \\ 2 & 5 & 5 \\ 2 & 7 & 3 \\ 8 & 9 & 1 \\ \hline 15 & 5 & 1 \\ \hline \end{array}$$

5) 1551 (310

$\underline{15}$

05

$\underline{5}$

01

Quotient = 310
Remainder = 1

Vedic Method

$$5\left|\begin{array}{ccc} & \bar{2} \quad & \bar{1} \\ \hline 1 & 3 & 2 \\ 2 & 5 & 5 \\ {}_{1}2 & 7 & 3 \\ {}_{2}8 & {}_{1}9 & {}_{0}1 \\ \hline 3 & 1 & 0 \end{array}\right. \quad R = 1$$

Quotient = 310
Remainder = 1

First Column:

Step 1:

$1 + 2 + 2 = 5$
$5 + 5 = 1$
$R = 0$

1 is kept as quotient to the left of 2

Step 2:

$$\begin{array}{c|c|c} 5 & 8 & 2 \\ & \underline{10} & \\ & -2 & \end{array}$$

$0 + 8 = 8$

$8 + 5 = 2\frac{\bar{2}}{5}$ instead of $1\frac{3}{5}$. Quotient is 2 and Remainder is $\bar{2}$. Quotient is kept to the left of 8 and $\bar{2}$ is carried to second column as $\bar{2} \times 10 = \overline{20}$

$\bar{2}$ is carried to second column as $\bar{2} \times 10 = \overline{20}$

Two quotients are added to give the result under the first column, i.e., 1 + 2 = 3

Second Column:

Step 1:

$\overline{20} + 3 = \overline{17}$
$\overline{17} + 5 = \overline{12}$
$\overline{12} + 7 = \bar{5}$
$\bar{5} + 9 = 4$
$4 + 5 = 1\frac{\bar{1}}{5}$. Quotient is 1 and remainder is $\bar{1}$.

$$\begin{array}{l} 5)\,4\,(1 \\ \;\;\underline{5\;\;} \\ \;\;\bar{1} \end{array}$$

$\bar{1}$ is carried to third column as $\overline{10}$.
Quotient is 1, which is the result in the second column, and is kept to the left of 9

Third Column:

Step 1:

$\overline{10} + 2 = \bar{8}$
$\bar{8} + 5 = \bar{3}$
$\bar{3} + 3 = 0$
$0 + 1 = 1$

$$\begin{array}{l} 5)\,1\,(0 \\ \;\;\underline{0\;\;} \\ \;\;1 \end{array}$$

When 1 is divided by 5, quotient is 0 and is kept to the left of 1. The remainder is 1 This remainder represents the final remainder.
∴ Quotient = 310
Remainder = 1

Combined Addition followed by Division by two digits in the Divisor (V.M.):

The divisor is partitioned into two with the Dhwajanka process, where the lower one is used for division and upper one is used for multiplication, as is described in straight division.

The provision for the digits in the remainder is shown accordingly as determined by

1. The number of digits that are given in the Dhwajanka,
2. Accordingly the partition in the addition is shown as the number of columns in the addition.

While all the other procedure is common as described for the divisor having one digit, one has to take into consideration of the Dhwajanka multiplication, which is applied to the value resulting in each column by the addition of corresponding quotients. From this it is followed by the usual division method. (Refer Straight division). The difference between single digit divisor (5) in ex.1 and multiple digit divisor (54) in ex.2 is that from 2nd column onwards R subtraction similar to that carried out in straight division is also applied here ie formation of ID's and ND's. If –ve value results as the ND, that is carried out as Vinculum.

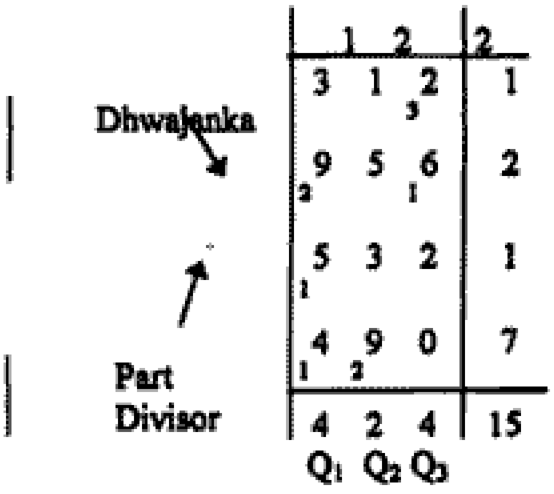
All other steps are just the same as in the previous case.

Example 2 : (3121 + 9562 + 5321 + 4907) + 54

Current Method

$$\begin{array}{r} 3121 \\ 9562 \\ 5321 \\ \underline{4907} \\ 22911 \end{array}$$

54) 22911 (424
$\underline{216}$
131
$\underline{108}$
231
$\underline{216}$
15



Vedic Method

Partition divisor 54 into two parts as 5 and 4, where 4 is (Dhwajanka) which is used for multiplication and 5 is part divisor used in Division as in the case of straight divisor.

First Column:

Step 1:

$3 + 9 = 12$

$12 + 5 = 2\frac{2}{5}$

2 is kept as quotient to the left of 9.

Step 2:

(Remainder) $2 + 5 = 7$

$7 + 5 = 1\frac{2}{5}$

1 is kept at the left of 5 in the column.

Step 3:

$2 + 4 = 6$

$6 + 5 = 1\frac{1}{5}$

1 is kept at the left of 4. Remainder 1 is carried to the next column as 10.
Quotient in the first column = 2 + 1 + 1 = 4

Second Column:

Step 1:

$$10 - \begin{bmatrix} 4 \\ \uparrow \\ 4 \end{bmatrix} \begin{matrix} \longrightarrow \text{(Dhwajanka)} \\ \\ \longrightarrow \text{(Quotient digit in the first column)} \end{matrix}$$

$10 - 16 = \bar{6}$

Step 2:

$\bar{6} + 1 = \bar{5}$
$\bar{5} + 5 = 0$
$0 + 3 = 3$
$3 + 9 = 12$

$12 + 5 = 2\frac{2}{5}$

2 is quotient and remainder 2 is carried out to the next column as 20 Quotient in the second column = 2.

Third Column:

Step 1:

$$20 - \begin{bmatrix} 4 \\ \uparrow \\ 2 \end{bmatrix} \begin{matrix} \longrightarrow \text{(Dhwajanka)} \\ = 20 - 8 = 12 \\ \longrightarrow \text{(Quotient digit in the second column)} \end{matrix}$$

$12 + 2 = 14$

$14 + 5 = 3\frac{\bar{1}}{5}$

$\bar{1} + 6 = 5$
$5 + 5 = 1$
2 is carried as 20 to the next column, i.e., remainder column.
Quotient in the third column = 3 + 1 = 4

Remainder Column:

In the remainder column add all the digits of the column along with the modified remainder obtained in third column to get the total remainder

Step 1:
(Dhwajanka)
(4)
20 – = 4 (modified remainder)

(Quotient digit in the third column)

Step 2:

4 + 1 + 2 + 1 + 7 = 15
∴ Remainder = 15
Quotient = 424

Example 3 (54563 + 92821 + 76543 + 24095) + 321 (In Dhwajanka there are two digits)

Current Method

$$\begin{array}{rrrrr}
5 & 4 & 5 & 6 & 3\\
9 & 2 & 8 & 2 & 1\\
7 & 6 & 5 & 4 & 3\\
\underline{2} & \underline{4} & \underline{0} & \underline{9} & \underline{5}\\
24 & 8 & 0 & 2 & 2
\end{array}$$

321) 248022 (772
$\underline{2247}$
2332
$\underline{2247}$
852
$\underline{642}$
210

Quotient = 772
Remainder = 210

Vedic Method

$$\begin{array}{c|ccc|cc}
\text{D} & 2 & 1 & & 1 & 0\\ \hline
21 & {}_{1}5 & {}_{3}4 & 5 & 6 & 3\\
 & {}_{1}9 & {}_{1}2 & 8 & 2 & 1\\
 & {}_{3}7 & {}_{2}6 & {}_{2}5 & 4 & 3\\
3 & 2 & {}_{1}4 & {}_{0}0 & 9 & 5\\ \hline
\text{PD} & 7 & 7 & 2 & 20 & 10 = 210\\
 & Q_1 & Q_2 & Q_3 & &
\end{array}$$

∴ Quotient = 772
Remainder = 210

First Column:

Step 1:

$5 + 3 = 1\frac{2}{3}$

Step 2:

2 + 9 = 11

$11 + 3 = 3\frac{2}{3}$

Step 3:

2 + 7 = 9
9 + 3 = 3

Quotient in the first column is 7 Remainder is 2, which is carried to the next column as 20

Second Column:

Step 1:

$$20 - \overset{\text{Dhwajanka}}{\underset{Q_1}{\begin{bmatrix} 2 \\ \uparrow \\ 7 \end{bmatrix}}} = 6$$

Quotient digit in the first column

Step 2:

$6 + 4 = 10$

$10 \div 3 = 3\frac{1}{3}$

Step 3:

$1 + 2 = 3$

$3 \div 3 = 1$

Step 4:

$6 \div 3 = 2$

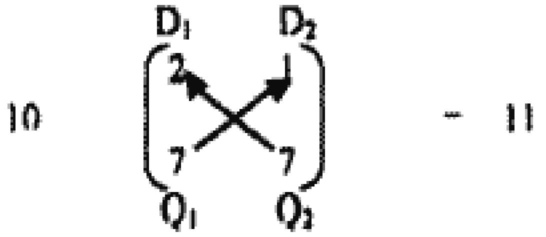
Step 5:

$4 \div 3 = 3\frac{1}{3}$

Quotient in the second column is 7 Remainder is 1, which is carried to the next column as 10

Third Column:

Step 1:



Quotient digit in the first column Q_1

Quotient digit in the second column Q_2

Step 2:

$-11 + 5 = -6$

$6 + 8 - 2$

$2 + 5 \quad 7$

[illegible]

Step 3:

$1 + 0 = 1$

$1 + 3 = 0\,(Q)$

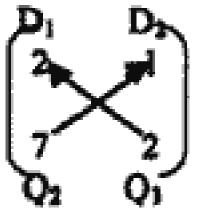
$0\frac{1}{3}$ Remainder 1

Quotient in the third column is 2 Remainder is 1, which is carried to the remainder column as 10

Fourth Column:

Step 1:

10 –



= –1

Step 2:

–1 + 6 + 2 + 4 + 9 = 20

We put 2 in the remainder

Quotient digit in the third column

Fifth Column:

Step 1:

0 – = – 2

Q3 Quotient digit in the third column

Step 2:

- 2 + 3 + 1 + 3 + 5 = 10

3 + 1+3 + 5 = 12 – 2 = 10

∴ Quotient = 772, Remainder = 210

Chapter IV

a) Division by Paravartya Method (Division of Polynomials) (V.M.):

Paravartya Yojayet sutram is applied for division The modus operandi is as follows

The application of Paravartya is by considering the opposite signs for all coefficients of x excepting for the highest power in the divisor x. The division is carried out with such a re–combination of the coefficients, which is shown below the dividend, after the first term in the quotient is worked out with the first term of the divisor

Case 1: The divisor having coefficient of highest power of x as 1:

Consider one example $8x^2 - 4x - 24 \div x - 2$

Examples:

1. Divide $8x^2 - 4x - 24$ by $x - 2$

Current Method

$$\begin{array}{l} x-2)\ 8x^2-\ 4x-24\ (8x+12 \\ \quad\ \underline{8x^2-16x} \\ \qquad\ +12x-24 \\ \qquad\ \underline{+12x-24} \\ \qquad\qquad \underline{0} \end{array}$$

Vedic Method

$$\begin{array}{rl|l} \underline{x-2} & 8x^2-\ 4x & -24 \\ +2 & \underline{\quad +16x} & \underline{+24} \\ \text{Quotient after} \rightarrow & 8x+12 & 0 \leftarrow \text{Remainder} \\ \text{dividing by x} & & \end{array}$$

First the dividend and the divisor are written in the decreasing orders of powers of x (zeroes supplemented if any terms of x are missing in the Dividend and the Divisor)

The dividend is partitioned from right end into two parts The second part is the remainder region, which may contain more than one term, but depends on the number of terms in the Paravartya.

The Paravartya form of $x - 2$ is $+2$. Division is carried out with 2 as follows.

Step 1:

The first term of the dividend is divided by the first term of divisor to get the first term in the quotient.

$8x^2 / x = 8x\ (Q_1)$

The Paravartya Division is effective from this step onwards

Step 2:

The quotient so obtained in the first step is multiplied with the Paravartya form. The result is placed under the next term of the dividend and the corresponding coefficients are suitably added to get the second term of the quotient

$8x \times 2 = 16x$; $16x - 4x = 12x$ This result is now divided by the highest power of x in the divisor

Hence = $12x + x = 12$

This is the second term of the quotient Q_2

Step 3:

Second term of the quotient Q_2 is multiplied with the Paravartya followed by addition to get the remainder.

$12 \times 2 = 24; 24 - 24 = 0$

Some more examples are given below when higher powers are considered for Dividend and Divisor:

2. Divide $9x^3 - 7x^2 + 5x + 3$ by $x + 3$

Current Method

$$\begin{array}{l} x+3)9x^3 - 7x^2 + 5x + 3\ (9x^2-34x+107 \\ \quad \underline{9x^3 + 27x^2} \\ \qquad -34x^2 + 5x \\ \qquad\quad [illegible] \\ \qquad\qquad 107x + 3 \\ \qquad\qquad \underline{107x + 321} \\ \qquad\qquad\qquad -318 \end{array}$$

Quotient = $9x^2 - 34x + 107$

Remainder = -318

Vedic Method

$$\begin{array}{c|l|l} \dfrac{x+3}{-3} & 9x^3 - 7x^2 + 5x & +\ 3 \\ & \underline{\quad - 27x^2 + 102x} & \underline{-321} \\ & 9x^2 - 34x\ \ +107 & -318 \end{array}$$

Step 1:$9x^3 / x = 9x^2$ (Q_1)

Step 2 $9x^2 (-3) = -27x^2$

$- 7x^2 - 27x^2 = -34x^2 + x = -34x$ (Q_2)

Step 3. $(-34x)(-3) = -102x$

$102x + 5x = 107x$; $107x + x = 107$

Step 4: $(107)(-3) = -321$

$+3 - 321 = -318$ (R)

Ans :$9x^2-34x+107$

R = -318

3. Divide $x^4 - 2x^3 + 5x^2 + x + 4$ by $x + 4$

Current Method

$$\begin{array}{l} x+4)x^4 - 2x^3 + 5x^2 + x + 4(x^3-6x^2+29x-115 \\ \quad \underline{x^4 + 4x^3} \\ \qquad -6x^3 + 5x^2 \\ \qquad -6x^3\ \ 24x^2 \\ \qquad\qquad + 29x^2 + x \\ \qquad\qquad \underline{+29x^2 + 116x} \\ \qquad\qquad\qquad -115x + 4 \\ \qquad\qquad\qquad \underline{-115x - 460} \\ \qquad\qquad\qquad\qquad +464 \end{array}$$

Vedic Method

$$\begin{array}{r|l|l} \underline{x+4} & x^4 - 2x^3 + 5x^2 + x & +\ 4 \\ -4 & \underline{\quad -4x^3 + 24x^2 - 116x} & \underline{+460} \\ & x^3 - 6x^2 + 29x - 115 & +464 \end{array}$$

Quotient = $x^3 - 6x^2 + 29x - 115$

Remainder = 464

4 Divide $x^5 - 2x^3 + 5x + 1$ by $x - 1$

Current Method

$$\begin{array}{l}
x-1)x^5+0x^4-2x^3+0x^2+5x+1(x^4+x^3-x^2-x+4\\
\quad\underline{x^5-x^4}\\
\qquad+x^4-2x^3\\
\qquad\underline{+x^4-x^3}\\
\qquad\quad-x^3+0x^2\\
\qquad\quad\underline{-x^3+x^2}\\
\qquad\qquad-x^2+5x\\
\qquad\qquad\underline{-x^2+x}\\
\qquad\qquad\quad 4x+1\\
\qquad\qquad\quad\underline{4x-4}\\
\qquad\qquad\qquad+5
\end{array}$$

Vedic Method

$$\begin{array}{c|l|l}
\underline{x-1} & x^5+0x^4-2x^3+0x^2+5x & +1\\
+1 & \underline{\quad 1x^4+1x^3-1x^2-1x} & \underline{+4}\\
 & x^4+x^3-x^2-x+4 & +5
\end{array}$$

Quotient $= x^4 + x^3 - x^2 - x + 4$

Remainder $= 5$

5 Divide $x^5 + 4x^4 + 5x^3 + 2x + 1$ by $x^2 + 3x + 2$

Current Method

$$\begin{array}{l}
x^2+3x+2)x^5+4x^4+5x^3+0x^2+2x+1(x^3+x^2-2\\
\qquad\underline{x^5+3x^4+2x^3}\\
\qquad\quad x^4+3x^3 \qquad +2x\\
\qquad\quad\underline{x^4+3x^3+2x^2}\\
\qquad\qquad-2x^2+2x+1\\
\qquad\qquad\underline{-2x^2-6x-4}\\
\qquad\qquad\qquad 8x+5
\end{array}$$

Vedic Method

$$\begin{array}{c|r|l}
x^2\underline{+3x+2} & x^5+4x^4+5x^3+0x^2 & +2x+1\\
-3x-2 & -3x^4-2x^3 & \\
 & -3x^3-2x^2 & \\
 & +0x^2 & +0x\\
 & & +6x+4\\
\hline
 & x^3+x^2+0x-2 & +8x+5
\end{array}$$

Quotient $= x^3 + x^2 - 2$

Remainder $= 8x + 5$

6 Divide $x^6 + x^4 + 3x^3 + 4x^2 + 5$ by $x^3 + x + 1$

Current Method

$$\begin{array}{l}
x^4+3x^3+4x^2+5(x^3+2\\
\underline{x^4+x^3}\\
\quad 2x^3+4x^2 \qquad +5\\
\quad\underline{2x^3 \qquad +2x+2}\\
\underline{\quad 4x^2 \qquad -2x+3}
\end{array}$$

Vedic Method

$$\begin{array}{c|r|l}
\underline{x^3+0x^2+x+1} & x^6+0x^5+x^4+3x^3 & +4x^2+0x+5\\
0x^2-1x-1 & 0x^5-1x^4-1x^3 & \\
 & 0x^4+0x^3 & 0x^2\\
 & 0x^3 & 0x^2 \quad 0x\\
 & & 0x^2 \quad -2x-2\\
\hline
 & x^3+0x^2+0x+2 & 4x^2-2x+3
\end{array}$$

Quotient $= x^3 + 0x^2 + 0x + 2$

Remainder $= 4x^2 - 2x + 3$

Case 2 : If the coefficient is not 1 for the highest power of x in the divisor.

The procedure is as follows.

Method I

1) To divide the first term of the dividend by the first term of the divisor as it is.
2) To divide each quotient term by the first term in the divisor and the result is used for the multiplication with the Paravartya form.
3) The remainder is left as it is.

Method II

one may obtain the unit coefficient for the highest power in the divisor by dividing it through out by that coefficient and taking the corresponding Paravarthya form. Only the quotients at the end are divided by the coefficient of the highest power of x in the divisor Both the methods are shown

Examples :

1. Divide $6x^3 - 12x^2 + 3x - 10$ by $2x - 5$

Current Method

$$x\text{-}5)6x^3 - 12x^2 + 3x - 10(3x^2 + \frac{3}{2}x + \frac{21}{4}$$

$$\begin{array}{l}
\underline{6x^3 - 15x^2} \\
\qquad 3x^2 + 3x \\
\qquad 3x^2 - \frac{15}{2}x \\
\qquad\qquad \frac{21}{2}x - 10 \\
\qquad\qquad \frac{21}{2}x - \frac{105}{4} \\
\\
\qquad\qquad\quad \frac{65}{4}
\end{array}$$

Vedic Method I

$$\begin{array}{c|c|c}
\underline{2x-5} & 6x^3 - 12x^2 + 3x & -10 \\
+5 & +15x^2 + \frac{15}{2}x & +\frac{105}{4} \\
\hline
 & \frac{6}{2}x^2 + \frac{3}{2}x + \frac{21}{4} & \frac{65}{4}
\end{array}$$

Quotient $= 3x^2 + \frac{3}{2}x + \frac{21}{4}$

Remainder $= \frac{65}{4}$

Vedic Method II

$$\begin{array}{c|c|c}
x - \frac{5}{2} & 6x^3 - 12x^2 + 3x & -10 \\
\hline
\frac{5}{2} & +15x^2 + \frac{15}{2}x & \frac{105}{4} \\
\hline
 & 6x^2 + 3x + \frac{21}{2} & \frac{65}{4}
\end{array}$$

Dividing each quotient by 2, the final quotient is

$3x^2 + \frac{3}{2}x + \frac{21}{4}$

Remainder $= \frac{65}{4}$

Working Details of Method I

Step 1 . $\frac{6x^3}{2x} = 3x^2$; (Q_1)

Step 2 · $(3x^2)(5) - 12x^2 = 3x^2$,

$\frac{3x^2}{2x} = \frac{3x}{2} Q_2$

[illegible]

$\frac{21}{4}x)\left(\frac{1}{2x}\right) = \frac{21}{4} Q_3$

Step 4: $\left(\frac{21}{4}\right)(5) = \frac{105}{4} - 10 = \frac{65}{4}$ (R)

(2) Divide $6x^5 + 2x^4 + 5x^3 + 1$ by $3x^2 - 2x + 1$

Current Method

$$\begin{array}{l}
3x^2-2x+1)6x^5+2x^4+5x^3+1(2x^3+2x^2+\frac{7}{3}x+\frac{8}{9} \\
\qquad\qquad \underline{6x^5-4x^4+2x^3} \\
\qquad\qquad 6x^4+3x^3 \qquad +1 \\
\qquad\qquad \underline{6x^4-4x^3+2x^2} \\
\qquad\qquad\qquad 7x^3-2x^2 \qquad +1 \\
\qquad\qquad\qquad \underline{7x^3-\frac{14}{3}x^2+\frac{7}{3}x} \\
\qquad\qquad\qquad\qquad \frac{8}{3}x^2-\frac{7}{3}x+1 \\
\qquad\qquad\qquad\qquad \underline{\frac{8}{3}x^2-\frac{16}{9}x+\frac{8}{9}} \\
\qquad\qquad\qquad\qquad\qquad -\frac{5}{9}x+\frac{1}{9}
\end{array}$$

Vedic Method I

$$\begin{array}{l|l|l}
\underline{3x^2-2x+1} & 6x^5+2x^4+5x^3+0.x^2 & +0x+1 \\
2x-1 & \qquad 4x^4-2x^3 & \\
 & \qquad\qquad 4x^3-2x^2 & \\
 & \qquad\qquad\qquad \frac{14}{3}x^2 & -\frac{7}{3}x \\
 & & \frac{16}{9}x-\frac{8}{9} \\
\hline
 & \frac{6}{3}x^3+\frac{6}{3}x^2+\frac{7}{3}x+\frac{8}{9} & -\frac{5}{9}x+\frac{1}{9}
\end{array}$$

Quotient = $2x^3 + 2x^2 + \frac{7}{3}x + \frac{8}{9}$

Remainder = $-\frac{5}{9}x + \frac{1}{9}$

Vedic Method II

$$\begin{array}{l|l|l}
x^2-\frac{2}{3}x+\frac{1}{3} & 6x^5+2x^4+5x^3+0.\,x^2 & 0x+1 \\
\hline
\frac{2}{3}x-\frac{1}{3} & \qquad 4x^4-2x^3 & \\
 & \qquad\qquad 4x^3-2x^2 & \\
 & \qquad\qquad\qquad \frac{14}{3}x^2 & -\frac{7}{3}x \\
 & & +\frac{16}{9}x-\frac{8}{9} \\
\hline
 & 6x^3+6x^2+7x+\frac{8}{3} & \frac{-5}{9}x+\frac{1}{9}
\end{array}$$

Final quotient $= 2x^3 + 2x^2 + \frac{7}{3}x + \frac{8}{9}$

Remainder = $-\frac{5}{9}x + \frac{1}{9}$

3 Divide $x^3 - 6x^2 + 11x - 6$ by $2x - 1$

Current Method

$$-6x^2 + 11x - 6(\frac{x^2}{2} - \frac{11}{4}x + \frac{33}{8}$$

$$\frac{\underline{-\frac{x^2}{2}}}{}$$

$$\frac{-11}{2}x^2 + 11x$$

$$\underline{\frac{-11}{2}x^2 + \frac{11}{4}x}$$

$$\frac{33}{4}x - 6$$

$$\underline{\frac{33}{4}x - \frac{33}{8}}$$

$$\frac{-15}{8}$$

Vedic Method I

$$\begin{array}{c|l|l} 2x-1 & x^3 - 6x^2 + 11x & -6 \\ \hline +1 & \quad + \frac{x^2}{2} - \frac{11}{4}x & \frac{33}{8} \\ \hline & \frac{x^2}{2} - \frac{11}{4}x + \frac{33}{8} & \frac{-15}{8} \end{array}$$

Final quotient $= \frac{x^2}{2} - \frac{11}{4}x + \frac{33}{8}$

Remainder $= \frac{-15}{8}$

Vedic Method II

$$\begin{array}{c|l|l} x - \frac{1}{2} & x^3 - 6x^2 + 11x & -6 \\ \hline \frac{1}{2} & \quad \frac{1}{2}x^2 - \frac{11}{4}x & +\frac{33}{8} \\ \hline & x^2 - \frac{11}{2}x + \frac{33}{4} & \frac{-15}{8} \end{array}$$

Quotient $= \dfrac{x^2 - \frac{11}{2}x + \frac{33}{4}}{2} = \frac{1}{2}x^2 - \frac{11}{4}x + \frac{33}{8}$

Remainder $= \frac{-15}{8}$

4 $x^2 - 2x + 1 + x^3 - 3x^2 + 2x + 1$

urrent Method

$$\begin{array}{r}
x^3-3x^2+2x+1\,\big)\,x^2-2x+1\,\Big(\frac{1}{x}+\frac{1}{x^2}+\frac{2}{x^3}+\frac{3}{x^4}\,..\\
x^2-3x+2+\frac{1}{x}\\
\hline
x-1-\frac{1}{x}\\
x-3+\frac{2}{x}+\frac{1}{x^2}\\
\hline
2-\frac{3}{x}-\frac{1}{x^2}\\
2-\frac{6}{x}+\frac{4}{x^2}+\frac{2}{x^3}\\
\hline
\frac{3}{x}-\frac{5}{x^2}-\frac{2}{x^3}\\
\frac{3}{x}-\frac{9}{x^2}+\frac{6}{x^3}+\frac{3}{x^4}\\
\hline
\frac{4}{x^2}-\frac{8}{x^3}-\frac{3}{x^4}
\end{array}$$

Vedic Method

$$\begin{array}{c|c|l}
\dfrac{x^3-3x^2+2x+1}{3x^2-2x-1} & 0x^3 & x^2-2x+1\\
 & & 0+0+0\\
 & & \quad +3x-2-\frac{1}{x}\\
 & & \qquad +3-\frac{2}{x}-\frac{1}{x^2}\\
 & & \qquad\quad +\frac{6}{x}-\frac{4}{x^2}-\frac{2}{x^3}\\
\hline
 & 0 & \frac{1}{x}+\frac{1}{x^2}+\frac{2}{x^3}+\frac{3}{x^4}-\frac{5}{x^5}-\frac{2}{x^6}.
\end{array}$$

Quotient $= \frac{1}{x}+\frac{1}{x^2}+\frac{2}{x^3}+\frac{3}{x^4}$ ....

(for $x \neq 0$)

The Problem on this type will be discussed more clearly in Power Series later in another Lecture Notes

5. $2x^3 + 4x^2 + 6x - 8 + 2x^4 - x^3 + 4x + 6$

Current Method

$$\begin{array}{l} x^4-x^3+ \\ x^2+4x+6 \end{array}\Big)\, 2x^3+4x^2+6x-8 \quad \left(\frac{1}{x}+\frac{5}{2x^2}+\frac{17}{4x^3}-\frac{31}{8x^4}\right.$$

$$\underline{2x^3-x^2+0x+4+\frac{6}{x}}$$

$$5x^2+6x-12-\frac{6}{x}$$

$$\underline{5x^2-\frac{5}{2}x+0+\frac{10}{x}+\frac{15}{x^2}}$$

$$\frac{17}{2}x-12-\frac{16}{x}-\frac{15}{x^2}$$

$$\underline{\frac{17}{2}x-\frac{17}{4}+\frac{0}{x}+\frac{17}{x^2}+\frac{51}{2x^3}}$$

$$-\frac{31}{4}-\frac{16}{x}-\frac{32}{x^2}-\frac{51}{2x^3}$$

$$\underline{-\frac{31}{4}+\frac{31}{8x}+\frac{0}{x^2}-\frac{31}{2x^3}-\frac{93}{4x^4}}$$

$$-\frac{159}{8x}-\frac{32}{}-\frac{10}{}+\frac{93}{4x^4}$$

Vedic Method I

$$\begin{array}{l|l|l} \underline{2x^4-x^3+0x^2+4x+6} & 0x^4 & 2x^3+4x^2+6x-8 \\ +x^3-0x^2-4x-6 & & \\ & & x^2+0x-4-\frac{6}{x} \\ & & \frac{5}{2}x+0-\frac{10}{x}-\frac{15}{x^2} \\ & & +\frac{17}{4}+\frac{0}{x}+\frac{17}{x^2}+\frac{51}{2x^3} \\ & & \frac{-31}{8x}+0+\frac{31}{x^3}+\frac{93}{4x^4} \\ \hline & 0 & \frac{1}{x}+\frac{5}{2x^2}+\frac{17}{4x^3}-\frac{31}{8x^4}\ldots\ldots \end{array}$$

$$\text{Quotient} = \frac{1}{x}+\frac{5}{2x^2}+\frac{17}{4x^3}-\frac{31}{8x^4}\ \ldots$$

Vedic Method II

$$\frac{2x^4-x^3+0x^2+4x+6}{x^4-\frac{x^3}{2}+\frac{0x^2}{2}+2x+3} \quad 2x^3+4x^2+6x-8$$

$$+\frac{x^3}{2}-\frac{0x^2}{2}-2x-3 \qquad x^2+0x-4-\frac{0}{x}$$

$$+\frac{5}{2}x+0-\frac{10}{x}-\frac{15}{x^2}$$

$$\underline{+\frac{17}{4}+\frac{0}{x}-\frac{17}{x^2}-\frac{51}{2x^3}}$$

$$\frac{2}{x}+\frac{5}{x^2}+\frac{17}{2x^3}-\frac{31}{4x^4}$$

Each term of the quotient is to be divided by 2 to get the final quotient

$$\text{Quotient} = \frac{1}{x}+\frac{5}{2x^2}+\frac{17}{4x^3}-\frac{31}{8x^4}\ldots$$

(b) Applying Paravartya Sutra to Numbers (V.M.)

The Paravartya form is obtained by taking all the digits with their opposite sign excepting the first digit in the divisor The procedure is same as explained for polynomials, having the coefficient of the highest power as 1.

Examples:

Example 1: 13653 +111

Current Method

111) 13653 (123
111
255
222
333
333
0

Vedic Method

1 1 1	1	3	6	5	3
-1 -1					
		-1	-1		
			-2	-2	
				-3	-3
	1	2	3	0	0

Quotient = 123
Remainder = 0

Example 2: 49897 + 121

Current Method

121) 49897 (412
484
149
121
287
242
45

Vedic Method

1 2 1	4	9	8	9	7
-2 -1					
		-8	-4		
			-2		
	4	1	2	4	5

Quotient = 412
Remainder = 45

Example 3: 159568 + 14312

Current Method

14312) 159568 (11
14312
16448
14312
2136

Vedic Method

1 4 3 1 2	1	5	9	5	6	8
–4 –3 –1 –2						
			–3	–1	–2	
			–4	–3	–1	–2
	1	1	2	1	3	6

Quotient = 11
Remainder = 2136

Example 4: 29721 + 142

Current Method

142) 29721 (209
2840
1321
1278
43

Vedic Method

1 4 2	2	9	7	2	1
–4 –2					
		–8	–4		
			–4	–2	
				4	2
	2	1	$\bar{1}$	4	3

Quotient = 2 1 $\bar{1}$ = 209
Remainder = 43

If the quotient or the remainder consists of Vinculum then it has to be devinculised to get it into the ordinary form. Even after this, if the remainder has a Vinculum form then add 'n' times the original divisor to the reminder to get into normal form (n which is an integer should be the required minimum value) This is followed by subtraction of 'n' from the previous quotient to obtain the final quotient. (refer Example Page No.) One can also further divide the Vinculum remainder.

Example 5: 98765 + 1321 a) Further division of the Remainder

Current Method

1321) 98765 (74
$\underline{9247}$
6295
$\underline{5284}$
1011

Vedic Method

$$\begin{array}{c|cc|ccc} \underline{1\ 3\ 2\ 1} & 9 & 8 & 7 & 6 & 5 \\ -3\ -2\ -1 & & & & & \\ & & -27 & -18 & -9 & \\ & & & +57 & +38 & +19 \\ \hline & 9 & \bar{1}\bar{9} & 46 & 35 & 24 \\ \hline Q_1 = & & 71 & 4974(R) & & \end{array}$$

Quotient = 9 $\bar{1}\bar{9}$ = 8 $\bar{9}$ = 71
Remainder = 4974 (R)

since remainder >Divisor R is to be further the divided by the Divisor and the quotient thus obtained is added to the previous quotient

(b) 4974 + 1321

$$\begin{array}{c|c|ccccc} \frac{1\,3\,2\,1}{-3-2-1} & 4 & 9 & 7 & 4 & & \\ & & -12 & -8 & -4 & & \\ \hline Q_2 = & 4 & \bar{3} & \bar{1} & 0 & = & \bar{1}690 \end{array}$$

(c) Further division of the remainder

$\bar{1}690 \div 1321$

$$\begin{array}{c|c|ccccc} \frac{1\,3\,2\,1}{-3-2-1} & \bar{1} & 6 & 9 & 0 & & \\ & & 3 & 2 & 1 & & \\ \hline Q_3 = & \bar{1} & 9 & 11 & 1 & = & 1011 \end{array}$$

Final Quotient = $Q_1 + Q_2 + Q_3 = 71 + 4 + \bar{1} = 74$

Final Remainder = 1011

Example 6: 29429 + 1463 a) Further Division

Current Method

1463) 29429 (20
$\underline{29260}$
169

Vedic Method

2 9 4 2 9
-4 -6 -3
-8 -12 -6

-4 -6 -3

12 10 6 = 1306

169

Quotient = 21 + $\dot{1}$ = 20

Final Remainder = $\bar{1}\bar{3}06$ + 1463 = 169 (n = 1)

If more than one digit results as a single unit in the quotient / remainder the normal Vedic addition holds good.

Example 7: 7967 + 1627

Current Method

1627) 17967 (11
$\underline{1627}$
1697
$\underline{1627}$
70

Vedic Method

$\underline{1\ 6\ 2\ 7}$ 1 7 6 7
-6 -2 -7
-2 -7
$\underline{-6\ -2\ -7}$

1 1 1 3 0

$1\ 1^{1}$ 70

Quotient = 11, Remainder = 70

In case the first digit of the divisor is not 1, then Vinculum is tried to see if it can be achieved. This is to see that an easy division with 1 is obtained. This facilitates the secondary multiplication easy. If it is not converted, then each digit of the quotient is to be divided by that number, which probably may result in fractions. These fractions are needed to be carried over properly.

Example 8: 32517 ÷ 987

Conversion to Vinculum followed by Paravartya

Current Method

987) 32517 (32
2961
2907
1974
933

Vedic Method

9 8 7	3	2	5	1	7
$10\ \bar{1}\ \bar{3}$					
0 1 3		0	3	9	
			0	2	6
	3	2	8	12	13
	3	2	933		

Quotient = 32
Remainder = 933

Divisor is converted into Vinculum form and then Paravarthya is applied

b) If the remainder is greater than the original divisor, subtract n times the divisor from the remainder until the resulting remainder is less than the divisor (n should be minimum) In this case one has to add 'n' to the previous quotient (see ex)

Example 9: 25935 + 829

Current Method

829) 25935 (31
2487
1065
829
236

Vedic Method

8 2 9 $1\bar{2}3\bar{1}$	2	5	9	3	5
2 −3 +1		4	−6	2	
			18	−27	9
	2	9	21	$\overline{22}$	14 = $2\bar{1}\bar{1}4$
Q_1	2	9	1894	R > the divisor	

Final Remainder = 1894 − 2 x 829 = 236(n = 2)

Final Quotient = Q_1 + 2 = 29 + 2 = 31

(or)

Dividing the remainder further

829 $1\bar{2}3\bar{1}$ $2\bar{3}1$	1	8	9	4
		2	$\bar{3}$	1
Q_2 =	1	10	6	5 > divisor

829 $1\bar{2}3\bar{1}$ $2\bar{3}1$	1	0	6	5
		2	$\bar{3}$	1
Q_3 =	1	2	3	6

Final Quotient = Q_1 + Q_2 + Q_3 = 29 + 1+1 = 31

Final Remainder = 236

or dividing $2\bar{1}\bar{1}4$ by 2 − 3 1

2 − 3 1	2	$\bar{1}$	$\bar{1}$	4
		4	6	2
	2	3	$\bar{7}$	6
	2	2	3	6

Q = 29 + 2 = 31
R = 236

Example 10: 12345 ÷ 7869

Current Method

$$\begin{array}{l}
7869)\ 12345\ (1 \\
\qquad\ \ \underline{7869} \\
\qquad\ \ 4476
\end{array}$$

Vedic Method

$$\begin{array}{l l}
\underline{7869} & 2\ \ 3\ \ 4\ \ 5 \\
1\overline{2131} & \\ \hline
2131 & 2\ \ 1\ \ 3\ \ 1 \\
 & \\
 & 4\ \ 4\ \ 7\ \ 6
\end{array}$$

Quotient = 1
Remainder = 4476

In order to get '1'as the first digit in the divisor, one may also divide (eg. 11, 12) or multiply (eg. 13) the divisor suitably followed or vice versa by Vinculum, if necessary, and then finally by Paravartya.

When the divisor is multiplied or divided suitably before the actual division is carried out, the final quotient is also multiplied or divided accordingly to obtain the final result.

In doing so, if one gets fractions (eg. 11) then that fraction is carried out to the remainder part of the divisor while retaining the integer part in the quotient .

Example 11: 4298 ÷ 273

Current Method

$$\begin{array}{l}
273)\ 4298\ (15 \\
\qquad\ \underline{273} \\
\qquad\ 1568 \\
\qquad\ \underline{1365} \\
\qquad\ \ \ 203
\end{array}$$

Vedic Method

$$\begin{array}{l r r|r r l}
\underline{2\ \ 7\ \ 3} & 4 & 2 & 9 & 8 & \\
\underline{3)3\ \ \overline{3}\ \ 3} & & & & & \\
\underline{1\ \ \overline{1}\ \ 1} & & 4 & -4 & & \\
1\ -1 & & & & & \\
 & & & 6 & -6 & \\ \cline{2-5}
 & 3)\ 4 & 6 & 11 & 2 & \text{----R < 273 (Divisor)} \\ \cline{2-5}
 & \multicolumn{2}{r|}{15\frac{1}{3}} & 11 & 2 & \\ \cline{2-5}
 & & 15 & \boxed{91} & +112 & \\ \cline{2-5}
 & & 15 & \multicolumn{2}{c}{203} & \frac{1}{3}\ \text{of the divisor 273}
\end{array}$$

∴ Quotient = 15
Remainder = 203

I Step for the Divisor – Vinculum
II Step for the Divisor – sub multiple of the vinculum
III Step for the Divisor –Paravartya

Example 12: 101100 +486

Current Method

486) 101100 (208
972
3900
3888
12

Vedic Method

6)486	1	0	1	1	0	0
81						
1 $\bar{2}$1		2	–1			
2 $\bar{1}$						
			4	–2		
Sub multiple				8	– 4	
Vinculum						
Paravartya					14	–7
	1	2	4	7	10	$\bar{7}$
	6)1247				93	
	$207\frac{4}{6}$				93	
	207				$\boxed{405}$ + 93	
	207 1				498 -486	
	208				12	

$\frac{5}{6}$ fraction part of the divisor 486

Quotient = 208, Remainder = 12
498 > 486 and 498 – 1x 486 = 12

∴ 1 is to be added to Q
∴ Q = 207 + 1 = 208 and Remainder = 12

Example 13: 16770 + 249

Current Method

249) 16770 (67
1494
1830
1743
87

If the remainder has Vinculum then add the original divisor once or twice or n times etc. as the case may be and remove that n where n = 1, 2, …. From the previous quotient.

Vedic Method

4x249	1	6	7	7	0
9 9 6					
100 $\bar{4}$		0	0	4	
0 0 4					
Multiple			0	0	24
Vinculum					
Paravartya	1	6	7	11	24
	4x16		834		
	64		834 R> Divisor 24		

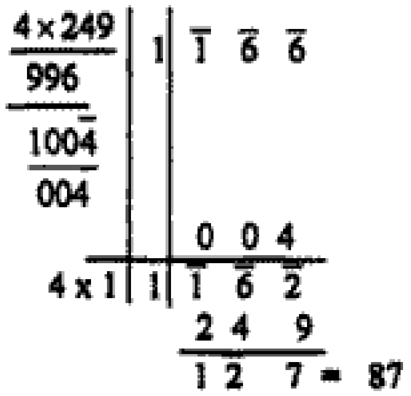
Final Remainder = 834 – 3 x 249 = 87
Final Quotient = 64 + 3 = 67

When the remainder > Original divisor then divide it further by the same method
For example R = .834

In order to have partition 3 digits, convert this to Vinculum form i.e

$834 = 1\bar{2}\ \ 1\bar{7}\ \ 1\bar{6} = 1\bar{1}\bar{6}\bar{6}$

$1\ \overline{166} + 249$



As the remainder is in the Vinculum add one time 249 which results in 87 This is less than the original divisor. Subtract 1 from previous value of the quotient

∴ From the Quotient subtract 1
∴ the final Quotient is 64 + 4 - 1 · 67

Comparison of different methods :

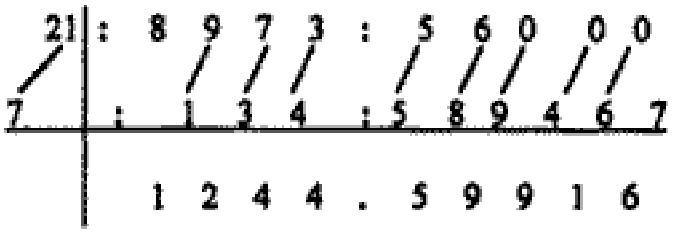
897356 + 721

Current Method

```
721) 897356 (1244 . 59916
     721
     1763
     1442
      3215
      2884
       3316
       2884
        4320
        3605
         7150
         6489
          6610
          6489
           1210
            721
            4890
            4326
             564
```

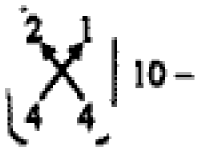
Quotient = 1244
Remainder = 432

Straight Division Method



Quotient = 1244

Remainder = 556 –



= 556 – 120 – 4
= 432

Quotient in decimals = 1244.59916

Paravartya Division Method

	721	8	9	7	3	5	6
Vinculum	$1\bar{3}21$						
Paravartya	$3\overline{21}$		24	$\overline{16}$	$\bar{8}$		
				99	$\overline{66}$	$\overline{33}$	
					270	$\overline{180}$	$\bar{9}0$
		8	33	90	$20\bar{1}$	$\overline{21}2$	$\bar{9}6$
Q_1			1220			$2\overline{227}6$	

= 17736 > divisor

17736 > 24 x 721
∴ 17736 - 24 x 721
17736 - 17304 - 432 - Remainder
24 is to be added to Q
∴ Q = 1220 + 24 = 1244
and R = 432

or

Dividing the remainder $2\bar{2}\ \bar{2}\ \bar{7}6$ similarly by treating the remainder as the dividend

721	2	$\bar{2}$	$\bar{2}$	$\bar{7}$	6
$1\bar{3}21$					
$3\overline{21}$		6	$\bar{4}$	$\bar{2}$	
			12	$\bar{8}$	$\bar{4}$
	2	4	$\bar{1}4$	$\overline{17}$	2
	2	4	$1\overline{57}2$		
Q_2	2 4		432		

∴ Quotient = $Q_1 + Q_2$ = 1220 + 24 = 1224
Remainder = 432

or

(a) 17736 + 721

3	$\bar{2}$	$\bar{1}$	1	7	7	3	6	
				3	$\bar{2}$	$\bar{1}$		
					30	$\overline{20}$	$\overline{10}$	
			1	10	35	$\overline{18}$	$\bar{4}$	= 3316 > 721
	Q_2 =			20				

(b) 3316 + 721

3	3	1	6	
	9	$\bar{6}$	$\bar{3}$	
Q_3 = 3	12	$\bar{5}$	3	> 721

(c) $12\bar{5}3$ + 721

1	2	$\bar{5}$	3	
	3	$\bar{2}$	$\bar{1}$	
Q_4 = 1	5	$\bar{7}$	2	= 432

Final Quotient (Q) = $Q_1 + Q_2 + Q_3 + Q_4$
= 1220 + 20 + 3 + 1 = 1244
∴ Quotient = 1244
Final Remainder = 432

For decimal points in the quotient, add 0 to the remainder and apply the division.

721	4	3	2	0	1^{st} decimal digit
$1\bar{3}21$					
$3\bar{2}\bar{1}$		12	$\bar{8}$	$\bar{4}$	
	4	15	$\bar{6}$	$\bar{4}$	
Q_1	4	14	3	6	> 721
	5	7	1	5	R_1

$Q = Q_1 + Q_2 = 4 + 1 = 5$

∴ further divide

3	2	1	4	3	6	
				$\bar{2}$	$\bar{1}$	
Q_2			7	1	5	R_1

or

1436
- 721 (1 x 721)
715 n = 1

$Q = 4 + 1 = 5$

$R = 715$

721	7	1	5	0	2^{nd} decimal digit
$1\bar{3}21$					
$3\bar{2}\bar{1}$		21	$1\bar{4}$	$\bar{\bar{7}}$	
	7	22	$\bar{1}1$	$\bar{\bar{7}}$	
	7	21	1	$\bar{\bar{7}}$	
Q_1	7	21	0	3	> 721 ∴ further division
	9	6	6	1	

$Q_1 + Q_2 - 7 + 2 = 9$

3	$\bar{2}$	$\bar{1}$	1	0	3	
			6	$\bar{4}$	$\bar{2}$	
Q_2			7	$\bar{4}$	1	
			6	6	1	

or
2103
1442 (2x 721)
n = 2
661

$Q = 7+2 = 9$
$R = 661$

$$\begin{array}{r|c|cccl}
\underline{721} & 6 & 6 & 1 & 0 & \text{3rd decimal digit} \\
\underline{1\bar{3}21} & & & & & \\
3\bar{2}\bar{1} & & 18 & \overline{12} & \bar{6} & \\ \hline
 & 6 & 24 & \overline{11} & \bar{6} & \\ \hline
 & 6 & 23 & \bar{1} & \bar{6} & \\ \hline
Q_1 & 6 & 22 & 8 & 4 & > R
\end{array}$$

$22384 - 3 \times 721 = 121$

$\therefore$ Quotient $= 6 + 3 = 9$

(or)

$$\begin{array}{ccccccc}
3 & \bar{2} & \bar{1} & & 2 & 8 & 4 \\
 & & & & 6 & \bar{4} & \bar{2}
\end{array}$$

$Q_2 \mid 2 \mid 8 \quad 4 \quad 2 \quad > 721$

$\therefore$ $Q_3 = 1$ by Vilokanam

Quotient $= Q_1 + Q_2 + Q_3 = 9$

$= 6 + 2 + 1 = 9$

$R = 121$

$$\begin{array}{r}
842 \\
-\underline{721} \\
\underline{121}
\end{array}$$

(or)

II) one can write Remainder part 842 into Vinculum form so that it can have 4 digits and then divide

$842 = 1\bar{2} \quad 1\bar{6} \quad 1\bar{8} \quad = \quad 1\bar{1}\bar{5}\bar{8}$

$$\begin{array}{ccc|c|ccc}
3 & \bar{2} & \bar{1} & 1 & \bar{1} & \bar{5} & \bar{8} \\
 & & & & 3 & \bar{2} & \bar{1} \\ \hline
 & & Q_3 & 1 & 2 & \bar{7} & \bar{9} \\ \hline
 & & & & 1 & 2 & 1
\end{array}$$

4th decimal digit

$$\begin{array}{r|c|ccc}
721 & 1 & 2 & 1 & 0 \\
\underline{1\bar{3}21} & & & & \\
3\bar{2}\bar{1} & & 3 & \bar{2} & \bar{1} \\ \hline
 & 1 & 5 & \bar{1} & \bar{1} \\ \hline
 & 1 & 4 & 8 & 9
\end{array}$$

Quotient in decimals = <u>1244.59916</u>

5th decimal digit

$$\begin{array}{r|c|cccl}
\underline{721} & 4 & 8 & 9 & 0 & \\
\underline{1\bar{3}2\bar{1}} & & & & & \\
3\bar{2}\bar{1} & & 12 & \bar{8} & \bar{4} & \\ \hline
 & 4 & 20 & 1 & \bar{4} & \\ \hline
Q_1 & 4 & 20 & 0 & 6 & > R \\ \hline
 & 6 & 5 & 6 & 4 &
\end{array}$$

$$\begin{array}{ccc|c|ccc}
3 & \bar{2} & \bar{1} & 2 & 0 & 0 & 6 \\
 & & & & 6 & \bar{4} & \bar{2} \\ \hline
 & & Q_2 & 2 & 6 & \bar{4} & 4 \\
 & & & & 5 & 6 & 4 \\
 & & & o & & &
\end{array}$$

$$\begin{array}{rl}
2006 & \\
-\underline{1442} & (-2x721) \\
\underline{564} & n = 1
\end{array}$$

$Q = Q_1 + Q_2 = 4 + 2$

$\therefore Q = 4 + 2$

Division by Nikhilam Method Applying the Nikhilam sutram to the Divisor

721) 279)	8	9	7	3	5	6
		16	56	72		
			50	175	225	
				226	791	1017
Q_1 =	8	25	113 |	476	1021	1023

58 833 (R_1) > 721
Hence further division

Q_1 = 1163

79)	5	8 |	8	3	3
		10 |	35	45	
		|	36	126	162
Q_2 = 68	5	18 |	79	174	165

= 9805 R_2 > 721 further division

279)	9 |	8	0	5
	|	18	63	81
Q_3 =	9 |	26	·63	86

= 3316 R_3 > 721
hence further division

or
9805
-9373 (13x721)
432 n = 13
Q = 1163 +68+13

279)	3 |	3	1	6
	|	6	21	27
Q_4 =	3 |	9	22	33

= 1153 R_4>721 hence further division

or
3316
-2884 4 x 721 n = 4
532 Q 1163+68+19+4

279)	1 |	1	5	3
	|	2	7	9
Q_5 =	1 |	3	12	12
	1 |	4	3	2

= 432 R_5 < 721

or
1153
- 721 1 x 721
432

$$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

∴ Quotient = 1163 + 68 + 9 + 3 + 1 = 1244
Remainder = 432

For decimal points in the quotient

1st decimal

$\underline{721}$ 279	4	3	2	0
		8	28	36
Q_1	4	11	30	36

R = 1436 > 721 hence further division

$\underline{721}$ 279	1	4	3	6
		2	7	9
Q_2	1	6	10	15

R = 715

or
1436
-$\underline{721}$ (1 x 721) n = 1
$\underline{715}$
Q = 4 + 1 = 5

Quotient = 4 + 1 = 5, Remainder = 715

2nd decimal

$\underline{721}$ 279	7	1	5	0
		14	49	63
Q_1	7	15	54	63

R = 2103 > 721 hence further division

or
2103
-$\underline{1442}$ 2 x 721
$\underline{661}$ n = 2
Q = 7+ 2 = 9

$\underline{721}$ 279	2	1	0	3
		14	14	18
Q_2	2	5	14	21

R = 661

Quotient = 7 + 2 = 9, Remainder = 661.

3rd decimal

$\underline{721}$ 279	6	6	1	0
		12	42	54
	6	18	43	54

R = 2284 > 721

or
2284
-$\underline{2163}$ = 3 x 721
$\underline{121}$

842 can be written as 1 $\bar{2}$ 4 2 to facilitate division

842 n = 1
-$\underline{721}$ 1x721
$\underline{121}$
Q = 8 + 1

$\underline{721}$ 279	2	2	8	4
		4	14	18
Q_2	2	6	22	22
		8	4	2 > 721
	$\boxed{1}$	1	2	1

hence further division

or we can simply subtract 721 from 842 once and then add 1 to the Quotient

279	1	$\bar{2}$	4	2
		2	7	9
	1	0	11	11

121

∴ Quotient = 6 + 2 + 1 = 9, Remainder = 121

4th decimal point

$$\begin{array}{l|l|lll} \underline{721} & 1 & 2 & 1 & 0 \\ 279 & & 2 & 7 & 9 \\ \hline & \boxed{1} & 4 & 8 & 9 \end{array}$$

Quotient = 1
Remainder = 489

5th decimal point

$$\begin{array}{ll|l|lll} \underline{721} & & 4 & 8 & 9 & 0 \\ 279 & & & 8 & 28 & 36 \\ \hline & Q_1 & \boxed{4} & 16 & 37 & 36 \\ \hline & & 2 & 0 & 0 & 6 \\ & & & 4 & 14 & 18 \\ \hline & Q_2 & \boxed{2} & 4 & 14 & 24 \\ \hline & & & 5 & 6 & 4 \end{array}$$

2006 > 721 hence further division

or
2006
$\underline{1442}$ 2 x 721
$\underline{564}$ n = 2
Q = 4+ 2 = 6

Quotient = 4 + 2 = 6
Remainder = 564

∴ Quotient in decimal points = 1244.59916

Chapter V

a) Argumental Division: (Significance of Left Hand to Right Hand Multiplication)(V.M.)

By extending a simple method of the application of Urdhva Tiryagbhyam, one can obtain the quotient and remainder by an argumentation, (in a converse manner, converting a division into multiplication).

Example: consider $2x^2 + 5x - 5 \div x + 3$

The procedure is as follows.

Write down the quotient as $Ax + B$ form and multiply it by the divisor $x + 3$, the value is compared with the given dividend to obtain the quotient and the remainder by the argumentation process. A and B are to be determined.

$$\begin{array}{ll} Ax + B & \text{Quotient} \\ \underline{\quad x + 3 \quad} & \\ 2x^2 + 5x - 5 & \text{(Given dividend)} \end{array}$$

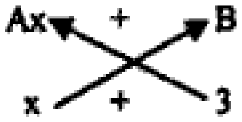
Step 1: $\begin{pmatrix} Ax \\ \uparrow \\ x \end{pmatrix} = Ax^2$ (Urdhva)

It is obvious that the division is now converted into multiplication

Starting from the left hand, the vertical multiplication is Ax^2, which gives value 2 for A when compared with $2x^2$

Now the Quotient is $2x + B$,

Step 2:



$= 3Ax + Bx = 6x + Bx \quad (\because A = 2)$

Applying the sutram Tiryak and comparing the x terms we get,

$6x + Bx = 5x$

$\therefore B = -1$

Step 3: Applying Urdhva to the last column

$3B = 3(-1) = -3$

On comparison, constant term -3 is different from the value -5 of the dividend and hence remainder resulting from this is $-5 - (-3) = -2$

$\therefore$ Quotient $= 2x - 1$

Remainder $= -2$

Example 2: $x^2 + 6x + 12 \div x + 2$

$$\begin{array}{l} Ax + B \\ \underline{\quad x + 2 \quad} \\ x^2 + 6x + 12 \end{array}$$

Step 1:

$Ax^2 = x^2$
$A = 1$

Step 2:

$2Ax + Bx = 6x$
$2x + Bx = 6x$
$\therefore B = 4$

Step 3:

$2B = 12$. But $B = 4 \Rightarrow 2B = 8$
$\therefore R = 12 - 8 = 4$ (on companions with constant of dividend)

$\therefore$ Quotient $= x + 4$
Remainder $= 4$

Example 3: $3x^3 + 6x^2 + 5x + 13 + x + 5$

$Ax^2 + Bx + C$
$\underline{\quad x + 5 \quad}$
$3x^3 + 6x^2 + 5x + 13$

Step 1:

$Ax^3 = 3x^3$
$\therefore A = 3$

Step 2:

$5Ax^2 + Bx^2 = 6x^2$
$15x^2 + Bx^2 = 6x^2$
$\therefore B = -9$

Step 3:

$5Bx + Cx = 5x$
$-45x + Cx = 5x$
$\therefore C = 50$

Step 4:

$5C = 250$
$R = 13 - 250 = -237$(on comparison with constant in the dividend)

Quotient $= 3x^2 - 9x + 50$
Remainder $= -237$

Example 4: $x^4 + 10x^3 + 35x^2 + 50x + 24 + x + 4$

$Ax^3 + Bx^2 + Cx + D$
$\underline{\quad x + \quad 4 \quad}$
$x^4 + 10x^3 + 35x^2 + 50x + 24$

$\therefore$ Quotient $= x^3 + 6x^2 + 11x + 6$
Remainder $= 0$

Step 1:

$Ax^4 = x^4$
$\therefore A = 1$

Step 2:

$4Ax^3 + Bx^3 = 10x^3$
$4x^3 + Bx^3 = 10x^3$
$Bx^3 = 6x^3$
$\therefore B = 6$

Step 3:

$4Bx^2 + Cx^2 = 35x^2$
$24x^2 + Cx^2 = 35x^2$
$\therefore C = 11$

Step 4:
$4Cx + Dx = 50x$
$44x + Dx = 50x$
$\therefore D = 6$

Step 5:
$4D = 24$
$\therefore R = 24 - 24 = 0$ (on comparison with constant in the dividend)

Example 5: $6x^2 + 5x + 10 + 2x + 1$

$$\begin{array}{l} Ax + B \\ \underline{2x + 1\quad\quad} \\ 6x^2 + 5x + 10 \end{array}$$

Step 1:

$2Ax^2 = 6x^2$
$\therefore A = 3$

Step 2:

$Ax + 2Bx = 5x$
$3x + 2Bx = 5x$
$\therefore B = 1$

Step 3:

$B = 1$
$\therefore R = 10 - 1 = 9$ (on comparison with the constant term in the dividend)

$\therefore$ Quotient $= 3x + 1$
Remainder $= 9$

Example 6: $24x^4 + 50x^3 + 35x^2 + 10x + 13 + 4x + 1$

$$\begin{array}{l} \quad Ax^3 + Bx^2 + Cx + D \\ \underline{\quad\quad 4x\ +\ 1\quad\quad\quad\quad} \\ 24x^4 + 50x^3 + 35x^2 + 10x + 13 \end{array}$$

Step 1:

$4Ax^4 = 24x^4$
$\therefore A = 6$

Step 2:

$Ax^3 + 4Bx^3 = 50x^3$
$6x^3 + 4Bx^3 = 50x^3$

Step 3:

$Bx^2 + 4Cx^2 = 35x^2$
$11x^2 + 4Cx^2 = 35x^2$
$\therefore C = 6$

Step 4:

$Cx + 4Dx = 10x$
$6x + 4Dx = 10x$
$\therefore D = 1$

Step 5:

$D = 13$
But $D = 1$

$\therefore R = 13 - 1 = 12$(on comparison with the constant in the dividend)

$\therefore$ Quotient $= 6x^3 + 11x^2 + 6x + 1$
Remainder $= 12$

Example 7: $10x^4 + 17x^3 + 20x^2 + 6x + 3 + 2x^2 + 3x + 3$

$Ax^2 + Bx + C$
$\underline{2x^2 + 3x + 3}$
$10x^4 + 17x^3 + 20x^2 + 6x + 3$

Step 1:

$2Ax^4 = 10x^4$
$\therefore A = 5$

Step 2:

$3Ax^3 + 2Bx^3 = 17x^3$
$15x^3 + 2Bx^3 = 17x^3$
$\therefore B = 1$

Step 3:

$3Ax^2 + 2Cx^2 + 3Bx^2 = 20x^2$
$15x^2 + 2Cx^2 + 3x^2 = 20x^2$
$\therefore C = 1$

Step 4:

$3Bx + 3Cx = 6x$
$\because B=1, C=1, R_1=0$

Step 5:

$3C = 3$
Also $C = 1$, $R_2=0$

$\therefore$ Remainder = 0 (on comparison with the x coeff and constant the remainders R_1 and R_2 are zero)
Quotient = $5x^2 + x + 1$

Example 8:

$(2x^{10} + 4x^9 + 9x^8 + 14x^7 + 17x^6 + 20x^5 + 15x^4 + 16x^3 + 16x^2 + 8x + 10)$
$+ (2x^5 + 2x^4 + 3x^3 + x^2 + 2x + 3)$

$Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$
$\underline{2x^5 + 2x^4 + 3x^3 + x^2 + 2x + 3}$
$2x^{10} + 4x^9 + 9x^8 + 14x^7 + 17x^6 + 20x^5 + 15x^4 + 16x^3 + 16x^2 + 8x + 10$

Step 1:

$2Ax^{10} = 2x^{10}$
$\therefore A = 1$

Step 2:

$2Ax^9 + 2Bx^9 = 4x^9$
$2x^9 + 2Bx^9 = 4x^9$
$\therefore B = 1$

Step 3:

$3Ax^8 + 2Cx^8 + 2x^8 = 9x^8$
$3x^8 + 2Cx^8 + 2x^8 = 9x^8$
$5x^8 + 2Cx^8 = 9x^8$
$\therefore C = 2$

Step 4:

$Ax^7 + 2Dx^7 + 3Bx^7 + 2Cx^7 = 14x^7$
$x^7 + 2Dx^7 + 3x^7 + 4x^7 = 14x^7$
$(2D + 8)\, x^7 = 14x^7$
$\Rightarrow 2D + 8 = 14$
$\therefore D = 3$

Step 5:

$2Ax^6 + 2Ex^6 + Bx^6 + 2Dx^6 + 3Cx^6 = 17x^6$
$2x^6 + 2Ex^6 + x^6 + 6x^6 + 6x^6 = 17x^6$
$\therefore E = 1$

Step 6:

$3Ax^5 + 2Fx^5 + 2Bx^5 + 2Ex^5 + Cx^5 + 3Dx^5$
$= 20x^5$
$3x^5 + 2Fx^5 + 2x^5 + 2x^5 + 2x^5 + 9x^5 = 20x^5$
$\therefore F = 1$

Step 7:

$3Bx^4 + 2Fx^4 + 2Cx^4 + 3Ex^4 + Dx^4$
$= 3x^4 + 2x^4 + 4x^4 + 3x^4 + 3x^4$
$= 15x^4$ Same as the dividend value
$\therefore R_1 = 0$

Step 8:

$3Cx^3 + 3Fx^3 + 2Dx^3 + Ex^3$
$= 6x^3 + 3x^3 + 6x^3 + x^3$
$= 16x^3$ Same as the dividend value
$R_2 = 0$

Step 9:

$3Dx^2 + Fx^2 + 2Ex^2$
$= 9x^2 + x^2 + 2x^2$
$= 12x^2$
$\therefore R_3 = 16x^2 - 12x^2 = 4x^2$ R_1 on comparison the coeff of x^2 in the dividend, the remainder $R_3 = 16x^2 - 12x^2 = 4x$

Step 10:

$3Ex + 2Fx = 8x$
$3Ex + 2Fx = 3x + 2x = 5x$
$R_4 = 8x - 5x = 3x \quad R_2$

Step 11:

$3F = 10$
But $3F = 3 \times 1 = 3$
$\therefore R_5 = 10 - 3 = 7 \quad R_3$

$\therefore$ Quotient $= x^5 + x^4 + 2x^3 + 3x^2 + x + 1$
Remainder $= 4x^2 + 3x + 7$

b) Argumental division as applied to numbers (V.M.):

Consider one example

Example 1 : 438 ÷ 23

438 is the dividend and 23 is the divisor
Obviously the divisor × quotient + remainder = dividend.

The problem is to find out the quotient considering the dividend as a result of the multiplication of the quotient with the divisor In this process the remainder, if any, can also be obtained. The procedure is to apply Urdhva Tiryak sutram between the quotient and the divisor followed by a comparison with the given dividend.

Considering the above examples, one can write down quotient AB as multiplicand (to be determined), the divisor as multiplier and the dividend as the result of multiplication, which includes the remainder if any.

Now the process is with the left-hand side multiplication using Urdhva Tiryak.

Step1:

A B Quotient

Divisor

4 3 8 Dividend (given)

2 2

The vertical multiplication(left to right multiplication)

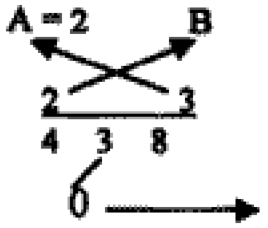
$2A = 4$

$A = 2$

Since $2A = 4$ on comparison with the given Dividend. The remainder is zero

$R_1 = 0$

Step 2: Substituting the value of $A = 2$ and the Tiryak multiplication



$3A + 2B = 3$

$6 + 2B = 3$

$2B = -3$

$B = -3/2$

To avoid –ve value we will reduce A by 1, i.e., $A = 1$. Now $2A = 2$. But on comparison $2A = 4$. ∴ Remainder is 2. The remainder R_1 is changed to 2 (modified) from 0.

Step2 : One can also continue the procedure by considering negative value in Vinculum.

$2B = \bar{3}$

$B = 1$. 1

A B

2 · 3

3 8

0 1

R_1 R_2

2 3

4 3 8

2 R_1 (modified)

Now $3A + 2B = 23$

$3 + 2B = 23$

$2B = 20$

$B = 10$ with $R_2 = 0$

Step 3:

$A = 1$ $B = 10$

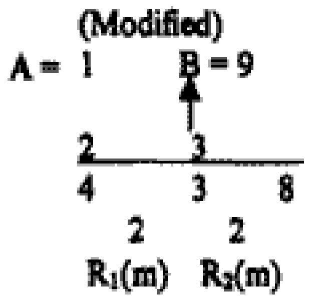
3

4 3 8

2 0

$R_1(m)$ R_2

On comparison with 8 on Urdhva multiplication
Excess will be 8 – 30 = -22
3B = 30, which is greater than 8.
By reducing the value of B by 1 to B = 9
we get a remainder R_2 (m) as 2



Now 3B = 3 × 9 = 27

On comparison with the value 28

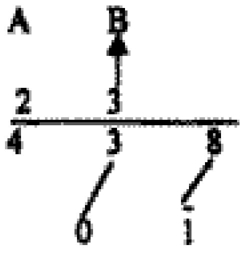
the remainder is 1.

∴ Quotient comes out as A = 1, B = 9,

i.e., AB Quotient is 19

Remainder = 1

Step 3:



$3B = \bar{1}8$

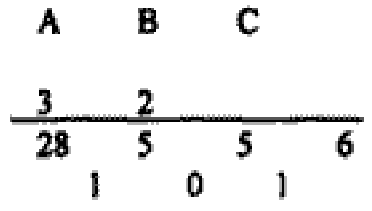
But $3B = \bar{3}$ (on substitution of $B = \bar{1}$)

$\therefore\ R_3 = \bar{1}8 + 3$
$= \bar{1}8 + \bar{3}$
$= 1$

∴ Final Quotient = AB = $2\bar{1}$ = 19
Final Remainder = 1

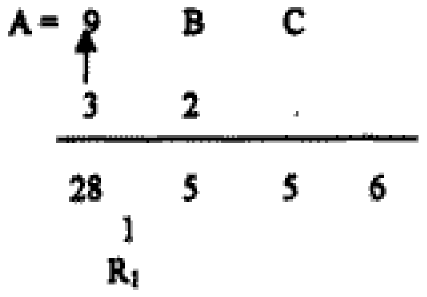
This procedure is called Argumental Division and can be applied to division involving any number of digits.

Example 2: 28556 ÷ 32

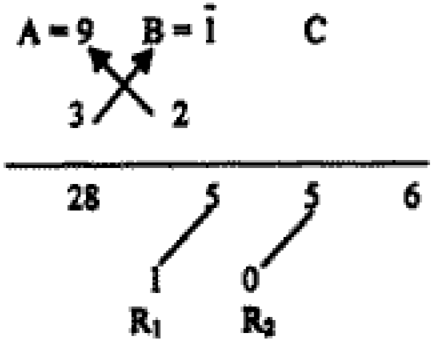


Step 1:

3A = 28
A is 9 with remainder R_1 as 1



Step 2:
2A + 3B = 15
18 + 3B = 15
3B = – 3
B = –1 = $\bar{1}$ one can continue the procedure by considering –ve value in Vinculum
Remainder R_2 = 0



Step 3:

2B + 3C = 5

$\bar{2}$ + 3C = 5

3C = 7

C is 2 with remainder R_3 as 1

28 $\quad {}_1 5 \quad {}_0 5 \quad {}_1 6$

Step 4:

$R_1 \quad R_2 \quad R_3$

2C = 16

Since C = 2

∴ Remainder is 16 – 4 = 12

Quotient = $9\bar{1}2$ = 892

Verification

32 x 892 + 12 = 28556

Example 3:

81420 + 236

A B C

$\underline{2 \quad 3 \quad 6}$

8 1 4 2 0

0 14

Step 1:

2A = 8

A = 4 with remainder R_1 as 0

A = 4 B C

3 6

8 1

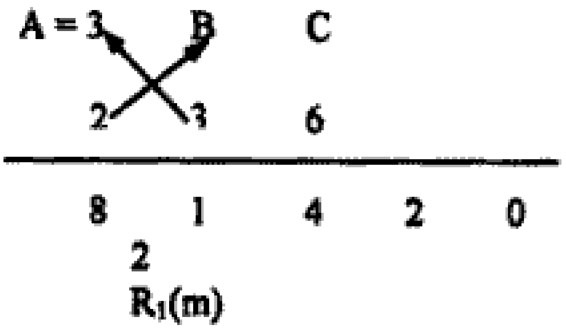
0

R_1

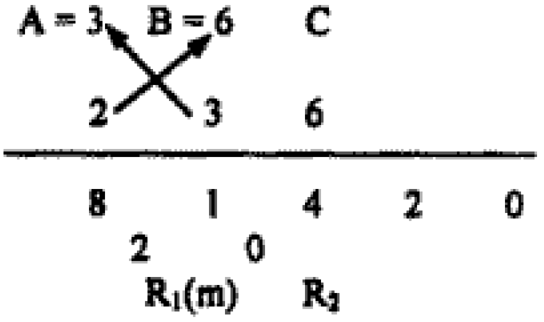
Step 2:

$3A + 2B = 1$
$12 + 2B = 1$
$2B = -11$

To avoid negative value we reduce A value by 1
A = 3 with remainder R_1 as 2



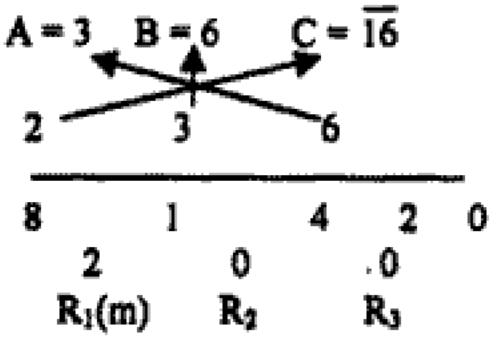
$3A + 2B = 21$
$9 + 2B = 21$
$2B = 12$
$B = 6$ with $R_2 = 0$



Step : 3

$6A + 2C + 3B = 4$
$18 + 2C + 18 = 4$
$2C = \overline{32}$
$C = \overline{16}$ $R_3 = 0$

One can keep the value in Vinculum

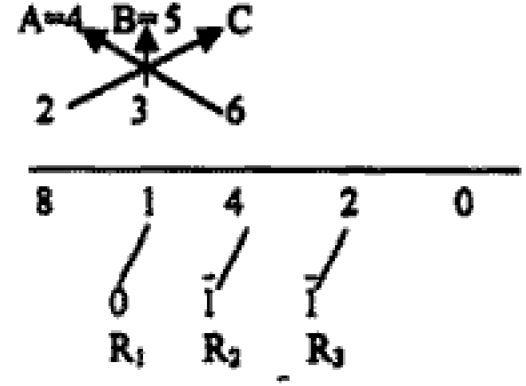


Using Vinculum

Step 2: $2B = \bar{1}\bar{1}$

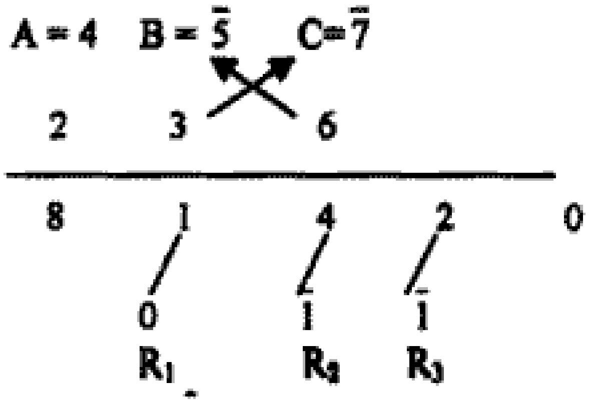
$B = \bar{5}$, $R_2 = \bar{1}$

Step 3:



$6A + 2C + 3B = \bar{1}4$
$24 + 2C + \bar{1}\bar{5} = \bar{1}4$
$2C + 9 = \bar{1}4$
$2C = \bar{1}\bar{5}$ ==>$C = \bar{7}$, $R_3 = \bar{1}$

Step : 4



$6B + 3C = \bar{1}2$
But $6B + 3C = 6(\bar{5}) + 3(\bar{7}) = \bar{5}\bar{1}$
$\therefore R_4 = \bar{1}2 - \bar{5}\bar{1} = 43$

Step 5:

$6C = -96$

$\therefore R_5 = 140 - (-96) = 236$ Or

Remainder is equal to divisor

$\therefore$ We can further divide remainder with divisor and get 1 as Quotient and 0 as final remainder

We add this 1 to the previous Quotient.

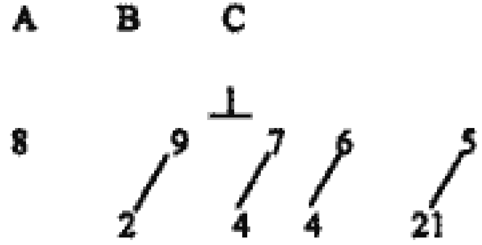
$A = 3, B = 6, C = \bar{1}\ \bar{6}$

$ABC = 36\ \overline{16} = 35\bar{6}$

Final Quotient $= 1 + 35\bar{6} = 35\bar{5} = 345$

Final Remainder $= 0$

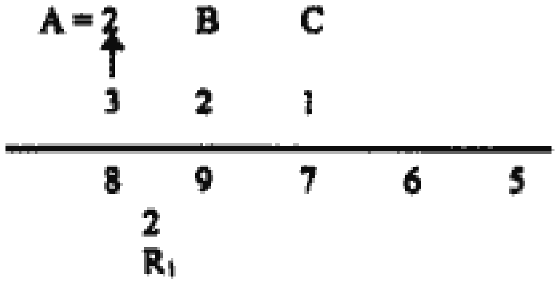
Example 4: 89765 + 321



Step 1:

$3A = 8$

$A = 2$ with remainder R_1 as 2



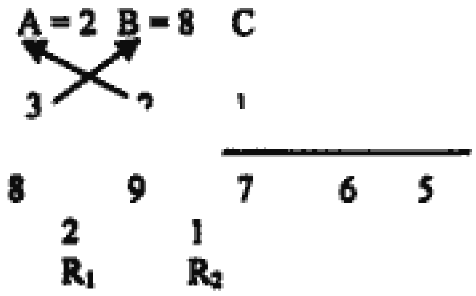
Step 2:

$2A + 3B = 29$

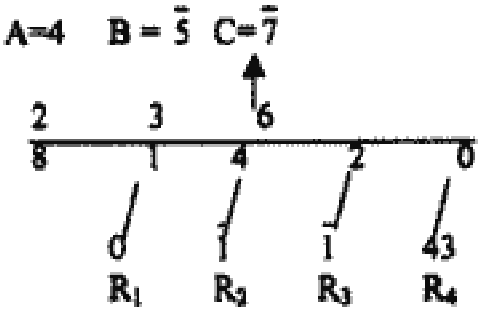
$4 + 3B = 29$

$3B = 25$

B is 8 with remainder R_2 as 1



Step 5



$6C = 430$

But $6C = 6(\bar{7}) = \bar{4}\ \bar{2}$

$\therefore R_5 = 430 - \bar{4}\ \bar{2} = 472$

Remainder = 472 > 236 divisor

$\therefore 472 - 2 \times 236 = 0(n=2)$

Final Quotient $= ABC = 4\bar{5}\ \bar{7} + 2 = 4\bar{5}\ \bar{5} = 345$

Final Remainder $= 0$

Step 3:

A + 3C + 2B = 17

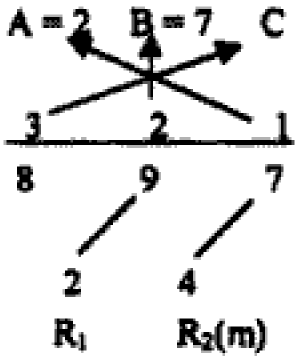
2 + 3C + 16 = 17

C = –1/3

Or

To avoid negative value, we reduce B value

∴ B is 7 with remainder R_2 as 4

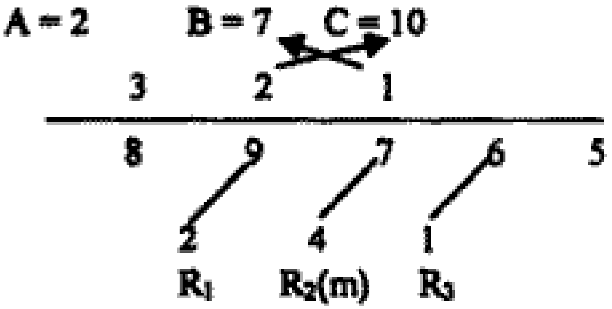


A + 3C + 2B = 47

2 + 3C +14 = 47

C is 10 with remainder R_3 as 1

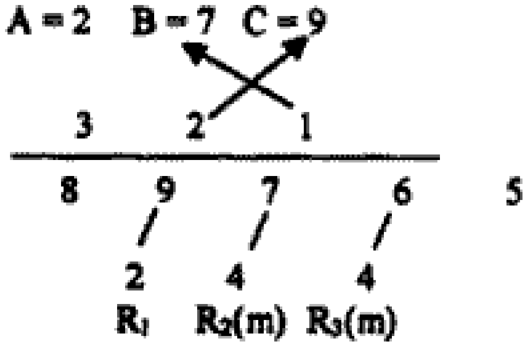
Step 4:



B + 2C = 16

B + 2C = 7 + 20 = 27 which is greater than 16.Hence reduction in the value of c

∴ C is 9 with remainder R_3 as 4



B + 2C = 46

But B + 2C = 7 + 18 = 25

∴ Remainder , R_4 = 46 – 25 = 21

Using Vinculum

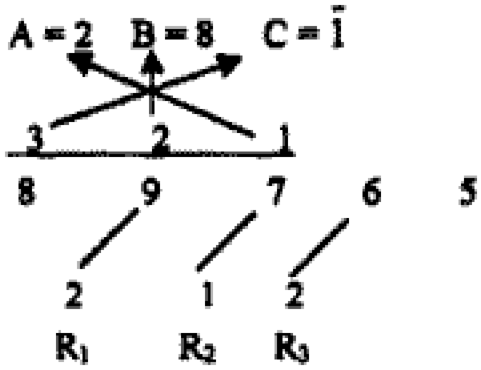
Step 3

A + 3C + 2B = 17

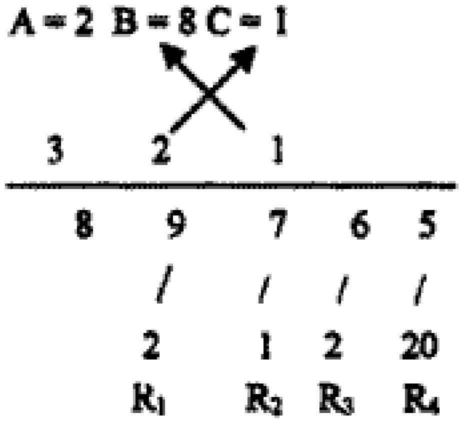
2 + 3C + 16 = 17

$3C = \bar{1}$

$C = \bar{1}$ With $R_3 = 2$



Step 4:

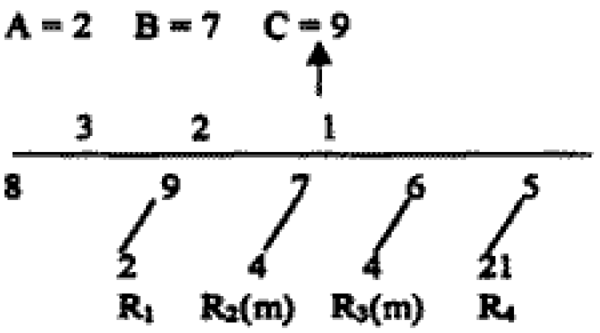


B + 2C = 26

But B + 2C $= 8 + \bar{2} = 6$

∴ Remainder R_4 = 26 – 6 = 20

Step 5:



or

∴ C = 215

But C = 9

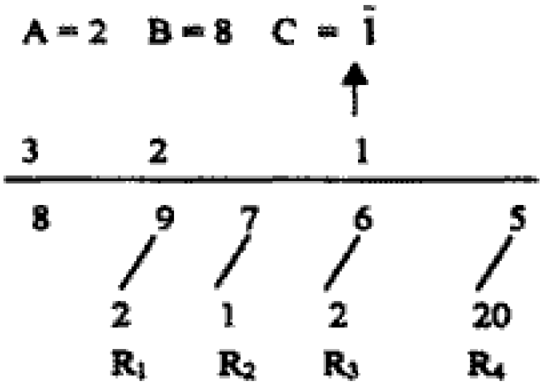
∴ Remainder = 215 − 9 = 206

∴ Final Quotient = A B C

= 279.

Remainder = 206

Step 5:



C = 205 But C = $\bar{1}$

∴ Remainder R_5 = 205 - $\bar{1}$ = 206

∴ Final Quotient = A B C

= $28\bar{1}$ = 279

Quotient = $28\bar{1}$ = 279

Final Remainder = 206

The ease within which the Vinculum method is worked out Can be understood also from Ex. 5

Example 5:

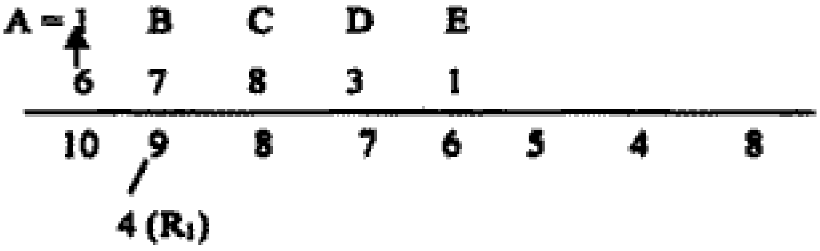
109876548 + 6783

A	B	C	D	E			
6	7	8	3				
10	9	8	7	6	5	4	8
4	6	12	15	19	64	553	

Step 1:

6A = 10

∴ A is 1 with remainder R_1 as 4



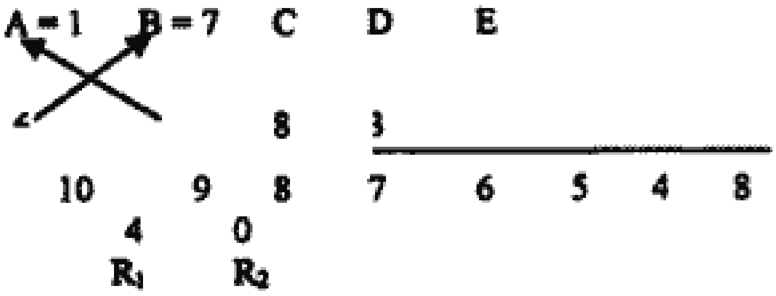
Step 2:

$7A + 6B = 49$

$7 + 6B = 49$

$6B = 42$

B is 7 with remainder R_2 as 0



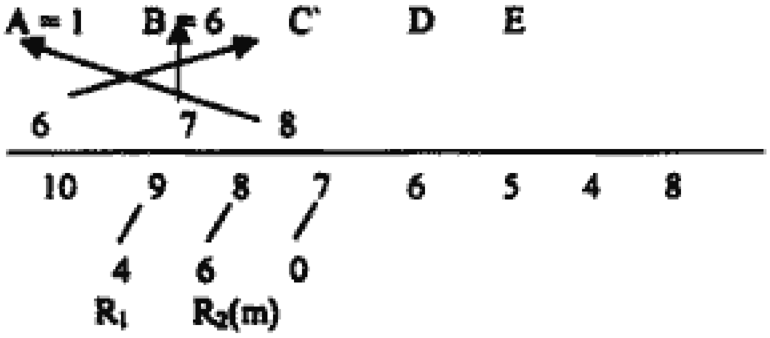
Step 3:

$8A + 7B + 6C = 8$

$8 + 49 + 6C = 8 \Rightarrow 6C = -49$ OR

We reduce B value by 1

∴ B is 6 with remainder R_2 as 6



$8A + 7B + 6C = 68$

$8 + 42 + 6C = 68$

$6C = 18$

C is 3 with remainder R_3 as 0

Step 3: Using Vinculum

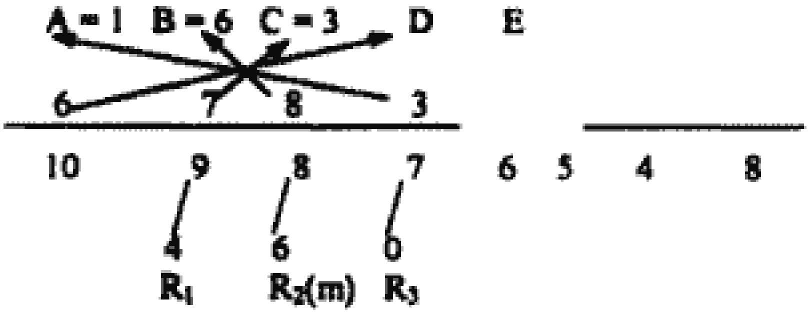
$8A + 7B + 6C = 8$

$8 + 49 + 6C = 8$

$6C = -49 = \bar{4}\,\bar{9}$

$C = \bar{8},\ R_3 = \bar{1}$

Step 4 :



$3A + 8B + 6D + 7C = 7$

$3 + 48 + 6D + 21 = 7$

$6D = -65$

∴ We reduce C value by 1

C is 2 with remainder R_3 as 6

A = 1 B = 6 C = 2 D E

6 7 8 3

10 9 8 7 6 5 4 8

4 6 6

R_1 $R_2(m)$ $R_3(m)$

$3A + 8B + 6D + 7C = 67$

$3 + 48 + 6D + 14 = 67$

$6D = 2$

We reduce C value further by 1

C is 1 with remainder R_3 as 12

Step 4:

A = 1 B = 7 C = $\bar{8}$ D E

6 7 8 3

---

10 9 8 7 6 5 4 8

4 0 1

R_1 R_2 R_3

$3A + 6D + 8B + 7C = 17$

$6D + 3 - \bar{1}7$

$6D = \bar{1}4 = \bar{6}$

$D = \bar{1}$ $R_4 = 0$

Step 5:

A = 1 B = 7 C = $\bar{8}$ D = $\bar{1}$ E

6 7 8 3

10 7 6 5 4 8

4 0 1 0

R_1 R_2 R_3 R_4

$3B + 6E + 8C + 7D = 06$

$21 + 6E + \bar{6}\bar{4} + \bar{7} = 06$

$6E + = 56$

$E = 9$, $R_5 = 2$


A = 1 B = 6 C = 1 D E

6 7 8 3

10 9 8 7 6 5 4 8

4 6 12

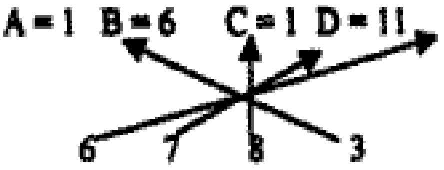
R_1 R_2(m) R_3(m)

3A + 8B + 6D + 7C = 127

3 + 48 + 6D + 7 = 127

6D = 69

D is 11 with Remainder R_4 as 3


A = 1 B = 6 C = 1 D = 11 E

6 7 8 3

10 9 8 7 6 5 4 8

4 6 12 3

R_1 R_2(m) R_3(m) R_4

Step 5:

3B + 6E + 8C + 7D = 36

18 + 6E + 8 + 77 = 36 E = - ve

∴ We reduce D value by 1

D is 10 with remainder R_4 as 9


A = 1 B = 6 C = 1 D = 10 E

6 7 8 3

10 9 8 7 6 5 4 8

4 6 12 9

R_1 R_2(m) R_3(m) R_4(m)

3B + 6E + 8C + 7D = 96

18 + 6E + 8 + 70 = 96

6E = 0

E = 0 R_4 = 0

If E = 0

3B + 6E + 8C + 7D ≠ 06

18 + 8 + 70 > 06

∴ We further reduce D value by 1

∴ D is 9 with Remainder R_4 as 15

Step 6:


A = 1 B = 7 C = $\bar{8}$ D = $\bar{1}$ E = 9

6 7 8 3

10 9 8 7 6 5 4 8

4 0 $\bar{1}$ 0 2

R_1 R_2 R_3 R_4 R_5

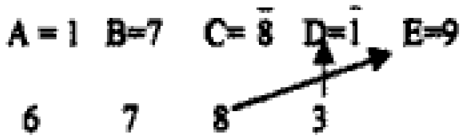
3C + 7E + 8D = 25

But 3C + 7E + 8D = $\bar{2}\bar{4}$ + 63 + $\bar{8}$

= 31

∴ R_6 = 25- 31 = $\bar{6}$1

Step 7:


A = 1 B = 7 C = $\bar{8}$ D = $\bar{1}$ E = 9

6 7 8 3

10 9 8 7 6 5 4 8

4 0 1 0 2 6

R_1 R_2 R_3 R_4 R_5 R_6

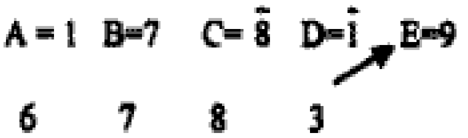
3D + 8E = $\bar{6}$4

But 3D + 8E = $\bar{3}$ + 72 = 69

∴ R_7 = $\bar{6}$4 – 69

= $\bar{1}\bar{2}\bar{5}$

Step 8:


A = 1 B = 7 C = $\bar{8}$ D = $\bar{1}$ E = 9

6 7 8 3

10 9 8 7 6 5 4 8

4 0 $\bar{1}$ 0 2 6 $1\bar{2}\bar{5}$

R_1 R_2 R_3 R_4 R_5 R_6 R_7

10	9	8	7	6	5	4	8
	4	6	12	15			
	R_1	$R_2(m)$	$R_3(m)$	$R_4(m)$			

3B + 6E + 8C + 7D = 156
18 + 6E + 8 + 63 = 156
6E = 67
E is 11 with Remainder R_5 as 1

10	9	8	7	6	5	4	8
	4	6	12	15	1		
	R_1	$R_2(m)$	$R_3(m)$	$R_4(m)$	R_5		

Step 6:

3C + 7E + 8D = 3 + 77 + 72 = 152, which is greater than 15
∴ We reduce E value by 1
E is 10 with Remainder R_5 as 7

10	9	8	7	6	5	4	8
	4	6	12	15	7		
	R_1	$R_2(m)$	$R_3(m)$	$R_4(m)$	$R_5(m)$		

3C + 7E + 8D = 145, which is greater than 75
∴ We reduce E value further by 1
E is 9 with Remainder R_5 as 13

$3E = \bar{1}\bar{2}\bar{5}8$

But $3E = 27$

$$\text{Remainder} = \bar{1}\bar{2}\bar{5}8 - 27$$
$$= \bar{1}\bar{2}\bar{7}1$$
$$= \bar{1}\bar{2}\bar{6}\bar{9}$$

Since the remainder is negative add quotient n times the divisor until it becomes positive.

$$= \bar{1}\bar{2}\bar{6}\bar{9} + 6783 = 552\bar{6}$$
$$= 5514$$

Final Quotient $= 178\bar{8}\bar{1}9 - 1$
$= 17\bar{8}\bar{1}8$
$= 16198$
Final Remainder = 5514

Note:
m - n + 1 = Number of constants
where m = Number of digits in the dividend and
n = Number of digits in the divisor

A = 1 B = 6 C = 1 D = 9 E = 9

6 7 8 3

10 9 8 7 6 $\overline{5 \quad 4 \quad 8}$

4 6 12 15 13
R_1 $R_2(m)$ $R_3(m)$ $R_4(m)$ $R_5(m)$

3C + 7E + 8D = 3 + 63 + 72 = 138 which is greater than 135
∴ We reduce E value further by 1
∴ E is 8 with Remainder R_5 as 19

A = 1 B = 6 C = 1 D = 9 E = 8

6 7 8

10 9 8 7 6 5 4 8

4 6 12 15 19
R_1 $R_2(m)$ $R_3(m)$ $R_4(m)$ $R_5(m)$

3C + 7E + 8D = 131
R_6 = 195 – 131 = 64

A = 1 B = 6 C = 1 D = 9 E = 8

$\underline{6}$ 7 8 3

10 9 8 7 6 5 4 8

4 6 12 15 19 64
R_1 $R_2(m)$ $R_3(m)$ $R_4(m)$ $R_5(m)$ R_6

Step 7:

3D + 8E = 27 + 64 = 91
R_7 = 644 – 91 = 553

A = 1 B = 6 C = 1 D = 9 E = 8
↗

6 7 8

10 9 8 7 6 5 4 8
4 6 12 15 19 64 553
R_1 R_2(m) R_3(m) R_4(m) R_5(m) R_6 R_7

Step 8:

3E = 24
R_8 = 5538 – 24 = 5514

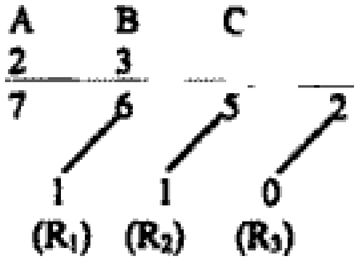
∴ Quotient = 16198, Remainder = 5514

Extension of Argumental Division For Finding Decimals

Example 6: 7652 ÷ 23

m = Number of digits in dividend = 4
n = Number of digits in divisor = 2
∴ Number of constants to be assumed = m – n + 1
= 4 – 2 + 1 = 3

m = 4 n = 2 ∴ 3 constants ABC are to be determined



Step 1 : $2A = 7$

$\boxed{A = 3}$, $R_1 = 1$

Step 2 : $3A + 2B = 16$
$9 + 2B = 16$
$2B = 7$

$\boxed{B = 3}$ $R_2 = 1$

Step 3 : $3B + 2C = 15$
$9 + 2C = 15$
$2C = 6$

$\boxed{C = 3}$ $R_3 = 0$

Step 4 $3C = 2$
But $3C = 9$
∴ $R_4 = 2 - 9 = -7$

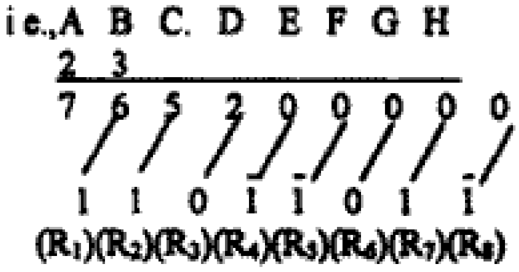
Since R_4 is negative, add divisor 23, once to get the final remainder and subtract '1' from quotient.

∴ Final Reminder = - 7 + 23 = 16

Quotient = ABC – 1
= 333 – 1
= 332
R = 16

For the decimal continuation we can workout with Vinculum easily.

Step5: If decimal points are required, assume as many constants as the number of decimals and proceed in the same way.



The first three steps are same. We start with Step 4.

$$3C + 2D = 2$$

$$9 + 2D = 2$$

$$2D = \bar{7}$$

$D = \bar{3}$, $R_4 = \bar{1}$

Step 6

$3D + 2E = 10$

$\bar{9} + 2E = \bar{1}0$

$2E = \bar{1}9 = \bar{1}$

$E = 0, \; R_5 = \bar{1}$

Step 7:

$3E + 2F = \bar{1}0$

$0 + 2F = \bar{1}0$

$2F = \bar{1}0$

$F = \bar{5}, \; R_6 = 0$

Step 8 :

$3F + 2G = 0$

$\bar{1}\bar{5} + 2G = 0$

$2G = 15$

$G = 7, \; R_7 = 1$

Quotient = A B C D E F G H

$= 3\,3\,3.\,\bar{3}\,0\,\bar{5}\,7\,\bar{5}$

$= 3\,3\,2.\,6\,9\,5\,6\,5$

Step 9 :

$3G + 2H = 10$

$21 + 2H = 10$

$2H = -11$

$H = \bar{5}, \; R_8 = \bar{1}$

Example 7: 91267 + 231

Method I **Argumental Division**

M = 5, N = 3

No. of decimal requred = 5

A B C. D E F G H

2 3 1

9 1 2 6 7 0 0 0 0

/ / / / / / / /

1 1 0 0 1 1 0 0

$(R_1)(R_2)(R_3)(R_4)(R_5)(R_6)(R_7)(R_8)$

Step 1 :

$2A = 9$

$\boxed{A = 4}, \; R_1 = 1$

Step 2 :

$3A + 2B = 11$

$12 + 2B = 11$

$2B = \bar{1}$

$\boxed{B = 0} \; R_2 = \bar{1}$

Step 3 :

$A + 3B + 2C = \bar{1}2$

$4 + 0 + 2C = \bar{1}2$

$2C = \bar{1}2 - 4 = \overline{12}$

$\boxed{C = \bar{6}} \quad R_3 = 0$

Step 4 :

$B + 3C + 2D = 06$

$0 + \bar{1}\bar{8} + 2D = 6$

$2D = 24$

$\boxed{D = 12} \quad R_4$

Step 5 :

$2E + 3D + C = 7$

$2E + 36 + \bar{6} = 7$

$2E = \bar{2}\bar{3}$

$\boxed{E = \bar{1}\bar{1}} \quad R_5 = \bar{1}$

Step 6 :

$2F + 3E + D = \bar{1}0$

$2F + \bar{3}\bar{3} + 12 = \bar{1}0$

$2F = 11$

$\boxed{F = 5}$ $R_6 = 1$

Step 7 :

$2G + 3F + E = 10$

$2G + 15 + \bar{1}\bar{1} = 10$

$2G = 6$

$\boxed{G = 3}$ $R_7 = 0$

Step 8 :

$2H + 3G + F = 0$

$2H + 9 + 5 = 0$

$2H = \bar{1}\bar{4}$

$\boxed{H = \bar{7}}$ $R_8 = 0$

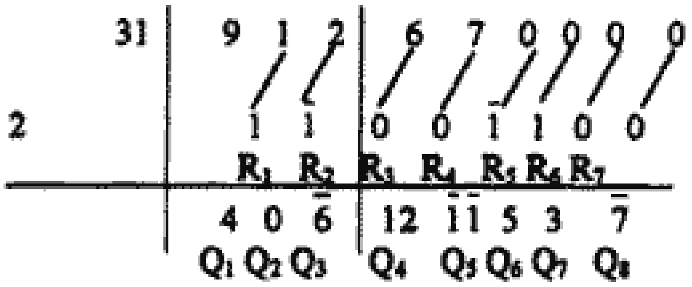
Quotient = A B C, D E F G H

$= 4\ 0\ \bar{6}.12\ \bar{1}\bar{1}\ 5\ 3\ \bar{7}$

$= 4\ 0\ \bar{5}.\ 1\ \bar{1}\ 5\ 2\ 3$

$= 3\ 9\ 5.\ 0\ 9\ 5\ 2\ 3$

Method II Straight Division using Vinculum



(1) $2\,)\,9\,(\,4\,(Q_1)$

$\underline{8}$

$1\ (R_1)$

(2) $11 - \begin{pmatrix} 3 \\ \uparrow \\ 4 \end{pmatrix} = \bar{1}$

$2\,)\,\bar{1}\,(\,0\,(Q_2)$

$\underline{0}$

$\underline{\bar{1}}\ (R_2)$

(3) $\bar{1}2 - \begin{pmatrix} 3 & & 1 \\ & \times & \\ 4 & & 0 \end{pmatrix} = \bar{1}\bar{2}$

$2\,)\,\bar{1}\bar{2}\,(\,\bar{6}\,(Q_3)$

$\underline{\bar{1}\bar{2}}$

$\underline{0}\ (R_3)$

(4) $06 - \begin{pmatrix} 3 & & 1 \\ & \times & \\ 0 & & 6 \end{pmatrix} = 24$

$2\,)\,24\,(12(Q_4)$

$\underline{24}$

$\underline{0}\ (R_4)$

(5) $07 - \begin{pmatrix} 3 & & 1 \\ & \times & \\ \bar{6} & & 12 \end{pmatrix} = \bar{2}\,\bar{3}$

$2\,)\,\bar{2}\,\bar{3}\,(\bar{1}\,\bar{1}\ (Q_5)$

$\underline{\bar{2}\,\bar{2}}$

$\underline{\bar{1}}\ (R_5)$

(6) $\bar{1}0 - \begin{pmatrix}3 & & 1\\ & \times & \\ 12 & & 11\end{pmatrix} = 11$

$2\ )\ 11\ (5(Q_6)$
$\underline{10}$
$1\ (R_6)$

(7) $10 - \begin{pmatrix}3 & & 1\\ & \times & \\ 11 & & 5\end{pmatrix} = 6$

$2\ )\ 6\ (3\ (Q_7)$
$\underline{6}$
$0\ (R_7)$

(8) $00 - \begin{pmatrix}3 & & 1\\ & \times & \\ 5 & & 3\end{pmatrix} = \bar{1}\bar{4}$

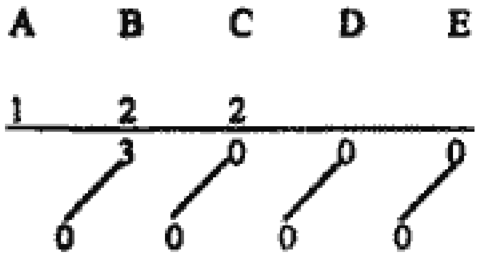
$2\ )\ 14\ (\bar{7}(Q_8)$
$\underline{\bar{1}\bar{4}}$
$0(R_8)$

Quotient $= 4\ 0\ \bar{6}\ \ 12\ \bar{1}\bar{1}\ 5\ 3\ \bar{7}$
$= 3\ 9\ 5\ .\ \ 0\ \ 9\ 5\ 2\ 3$

Current Method

```
231) 91267( 395 09523
     693
     2196
     2079
       1177
       1155
          2200
          2079
            1210
            1155
               550
               462
                 880
                 693
                 197
```

Example 8: 23 ÷ 122
m = 2, n = 3
∴ Number of constants = 2 – 3 + 1 = 0 (This denotes that the quotient part starts with decimal point.)



Current Method

```
122) 230( 0 18852
          122
          1080
           976
           1040
            976
              640
              610
                300
                244
                 56
```

Step 1 : $\boxed{A = 2}$, $R_1 = 0$

Step 2 : $2A + B = 3$

$4 + B = 3$

$\boxed{B = \bar{1}}$ $R_2 = 0$

Step 3 : $2A + 2B + C = 0$

$4 + \bar{2} + C = 0$

$\boxed{C = \bar{2}}$ $R_3 = 0$

Step 4 : $D + 2B + 2C = 0$

$D + \bar{2} + \bar{4} = 0$

$\boxed{D = 6}$ $R_4 = 0$

Step 5 : $E + 2D + 2C = 0$

$E + 12 + \bar{4} = 0$

$\boxed{E = \bar{8}}$ $R_5 = 0$

Quotient $= 0.2\,\bar{1}\,\bar{2}\,6\,\bar{8}$

$0.1\,8\,8\,5\,2$

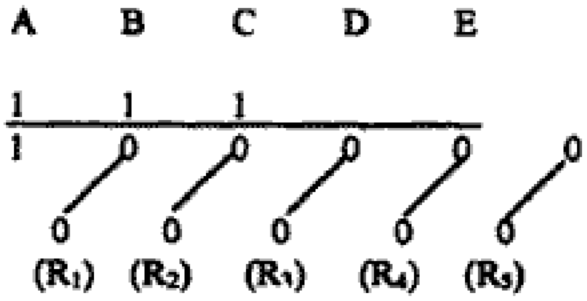
Example 9: 1 ÷ 111

$m = 1,\ n = 3$

Number of constants $= 1 - 3 + 1 = -1$

This denotes that in the quotient the decimal point is followed by one zero.

∴ Quotient 0 0 A B C D E



Step 1 $\boxed{A = 1}$, $R_1 = 0$

Step 2 : $A + B = 0$

$1 + B = 0$

$\boxed{B = \bar{1}}$ $R_2 = 0$

Step 3 : $A + B + C = 0$

$1 + \bar{1} + C = 0$

$\boxed{C = 0}$ $R_3 = 0$

Step 4 : $B + C + D = 0$

$\bar{1} + 0 + D = 0$

$\bar{1} + D = 0$

$\boxed{D = 1}$ $R_4 = 0$

Step 5 : $C + D + E = 0$

$0 + 1 + E = 0$

$1 + E = 0$

$\boxed{E = \bar{1}}$ $R_5 = 0$

Quotient $= 0.\,0\,A\,B\,C\,D\,E$

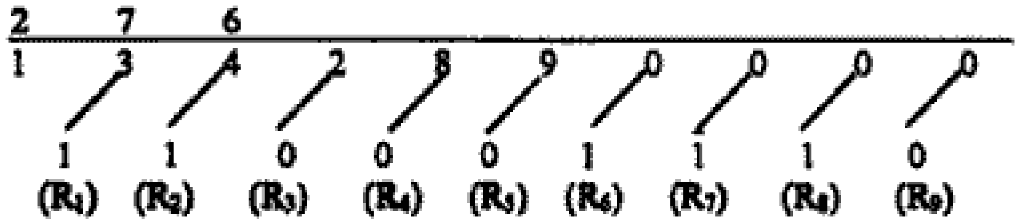
$= 0.\,0\,1\,\bar{1}\,0\,1\,\bar{1}$

$= 0\ 0\,0\,9\,0\,0\,9$

Example 10 : 134.289 + 2.76

m = 3, n = 1
Number of constants = 3 – 1 + 1 = 3

If one wants 5 decimals digits then the quotient is
A B C. D E F G H I



Step 1 : $2A = 1$
$\boxed{A = 0}$, $R_1 = 1$

Step 2 : $7A + 2B = 13$
$0 \quad + 2B = 13$

$\boxed{B = 6}$ $R_2 = 1$

Step 3 : $6A + 7B + 2C = 14$
$0 \quad + 42 \quad + 2C = 14$
$2C = \bar{2}\,\bar{8}$
$\boxed{C = \bar{1}\bar{4}}$ $R_3 = 0$

Step 4 : $2D + 6B + 7C = 2$
$2D + 36 \quad + \bar{9}\,\bar{8} = 2$
$2D = 64$

$\boxed{D = 32}$ $R_4 = 0$

Step 5 : $2E + 7D + 6C = 8$
$2E + 224 + \bar{8}\,\bar{4} = 8$
$2E = \bar{1}\bar{3}\,\bar{2}$

$\boxed{E = \bar{6}\,\bar{6}}$ $R_5 = 0$

Step 6 : $2F + 7E \quad + 6D = 9$
$2F + \bar{4}\,\bar{6}\,\bar{2} + 192 = 9$
$2F = 279$

$\boxed{F = 139}$ $R_6 = 1$

Step 7 : $2G + 7F + 6E \quad = 10$
$2G + 973 + \bar{3}\,\bar{9}\,\bar{6} = 10$
$2G = \bar{5}\,\bar{6}\,\bar{7}$

$\boxed{G = \bar{2}\,\bar{8}\,\bar{3}}$ $R_7 = \bar{1}$

Step 8 : $2H + 7G \quad + 6F \quad = \bar{1}0$
$2H + \bar{1}\;\bar{9}\,\bar{8}\,\bar{1} + 834 = \bar{1}0$
$2H = 1137$

$\boxed{H = 568}$ $R_8 = 1$

Step 9 : $2I + 7H \quad + 6G \quad = 10$
$2I + 3976 + \bar{1}\;\bar{6}\,\bar{9}\,\bar{8} = 10$
$2I = \bar{2}\,\bar{2}\,\bar{6}\,\bar{8}$

$\boxed{I = \bar{1}\;\bar{1}\;\bar{3}\,\bar{4}}$ $R_9 = 1$

Quotient = A B C . D E F G H I
$= 0\;6\;\bar{1}\bar{4}.32\;\bar{6}\;\bar{6}\;139\;\bar{2}\,\bar{8}\,\bar{3}\;568\;\bar{1}\bar{1}\bar{3}\,\bar{4}$
$= \;5\;\bar{1}.\;\bar{3}\;\bar{5}\;5\;2\;5\;\bar{4}$
$= \;4\;8.\;6\;5\;5\;2\;4\;6$

(c) Problems from Swamiji's Text and Hall and Knight Algebra Argumental Division(For Polynomials) - simplified method

The procedure in brief can be explained as follows :

1 One should write down the dividend and divisor in descending order of power of x .

2. To divide the highest power of x in the dividend with the highest power of x in the divisor, which gives the first quotient(Q_1) .

3. Leaving the first term in the divisor, the rest of the terms are used for successive multiplications in Urdhva Tiryak manner. i e , quotients are to be multiplied with the divisor terms.

4 While doing so, a comparison is made between the result of multiplication with the corresponding terms of the dividend, to establish the difference.*

5 The difference is now divided by the highest power of x in the divisor to get the quotients and finally the absolute term

The method explained by swamiji in his book is exemplified through a number of problems in the book
A different method of division is also explained at the end of the notes.

6 For a few problems the current method is demonstrated and for the rest of the problems the reader is expected to complete

Polynomial Division using Urdhva- Tiryak Sutram.
(ARGUMENTAL DIVISION)

Given the dividend and the divisor, the quotient can be worked out (or) given thedividend and quotient the divisor can be found out, both by the division method. Both these methods make use of the Urdhva Tiryagbhyam Sutram used for multiplication. This method is very simple and division can be worked out with ease

The following are a few examples and the various steps are explained.
Example 1 : $x^3 - x^2 - 9x - 12 \ + \ x^2 + 3x + 3$

The dividend is $x^3 - x^2 - 9x - 12$

The divisor is $x^2 + 3x + 3$

Step 1 · Divide x^3 by x^2
(x^3)
i.e., $x^3/x^2 = x$ Q_1(quotient) $\boxed{Q_1 = x}$

Carry out Urdhva Tiryak multiplication of the part divisor 3x + 3 with the successive quotients

*'Original co – efficient' refers to that in the Dividend. 'Difference' means subtracting of the multiplication result from that of the corresponding dividend term . i.e., original.

Step 2 : Now concentrating on the x^2-term of the dividend which is $-x^2$. This can be
(x^2) compared with the x^2 term obtained by the multiplication of quotients so obtained with the suitable terms in the divisor

Divisor = $x^2 + \underset{\overset{\uparrow}{x(Q_1)}}{3x} + 3$ (Urdhva)

$= 3x^2$ (Urdhva)

But the co-efficient of x^2 in the dividend is -1 In order to get $-x^2$ of the dividend, we have to now subtract $3x^2$ from $-x^2 = -x^2 - 3x^2 = -4x^2$
This is to be divided by x^2 to get Q_2 (the highest term in the divisor)

Hence $-4x^2/x^2 = -4(Q_2)$ $\boxed{Q_2 = -4}$

Step 3 : Now the Tiryak multiplication is to be carried out between Quotient Q_1 Q_2 and the part divisor.

$$\begin{array}{ll} \text{Divisor} = & x^2 + 3x + 3 \\ & \quad\;\; \times \qquad = -12x + 3x = -9x(\text{Tiryak}) \\ \text{Quotient} = & \quad\;\; x \; - 4 \\ & \quad\;\; Q_1 \;\; Q_2 \end{array}$$

On Tiryak multiplication we get the value as $-9x$. The original co-efficient of x is also -9. So, the difference between these two is zero.

Step 4 : By Urdhva multiplication of last term of divisor and quotient.

Q_2 we get $\left(\begin{array}{c} 3 \\ \uparrow \\ -4 \end{array}\right)$ = 12 (Urdhva)

But the original absolute term is 12
$\therefore$ The difference zero is the remainder

i.e., Quotient = $Q_1 + Q_2 = x - 4$

Remainder = 0

__Example 2 :__ $28y^4 - 71y^3 - 35y^2 + 30y + 9 \div 4y^2 - 13y + 6$

Step 1: $28y^4 / 4y^2 = 7y^2$ $\boxed{Q_1 = 7y^2}$

Carry out Urdhva Tiryak Multiplication of the part divisor $- 13y + 6$ with the successive quotients as follows :

Step 2 . $\begin{array}{c} 4y^2-13y+6 \\ \uparrow \\ 7y^2 \\ (Q_1) \end{array} = -91y^3$ (Urdhva)

(y3)

But the original co-efficient of $y^3 - 71$

$\therefore$ the difference is $= -71y^3 + 91y^3 = 20y^3$

$Q_2 = 20y^3/4y^2 = 5y$ $\boxed{Q_2 = 5y}$

Step 3:

(y^2) $\begin{array}{c} 4y^2 + 13y + 6 \\ \times \\ 7y^2 + 5y \\ Q_1 \quad Q_2 \end{array}$ $= 42y^2 - 65y^2 = -23y^2$(Tiryak)

But the original co-efficient of $y^2 = -35$

$\therefore$ Difference $= -35y^2 + 23y^2 = -12y^2$

$-12y^2 / 4y^2 = -3$ $\boxed{Q_3 = -3}$

Original coefficient refers to that in the Dividend. Difference means subtracting of the multiplication result from that of the corresponding dividend term.

Step 4 : $\begin{array}{c} 4y^2 - 13y + 6 \\ \times \\ 7y^2 + 5y \quad -3 \\ Q_1 \quad Q_2 \quad Q_3 \end{array}$ $39y + 30y = 69y$ (Tiryak)

(y)

But the original co-efficient of $y = 30$

$\therefore$ The difference $30y - 69y = -39y$ This is Remainder R_1 $\boxed{R_1 = -39y}$

Step 5 :

The last term by Urdhva multiplication -18 of the last term with Q3

$\begin{array}{c} 4y^2 - 13y + 6 \\ \uparrow \\ 7y^2 + 5y - 3 \end{array} = -18$ (Urdhva)

But the original absolute term is 9

$\therefore$ The difference is

$9 + 18 = 27$ This gives the remainder R_2 $\boxed{R_2 = 27}$

$\therefore$ Quotient $= Q_1 + Q_2 + Q_3 = 7y^2 + 5y - 3$

Remainder $= R_1 + R_2 = -39y + 27$

Example 3: $3(a^3/27 - a^2/12 + a/16 - 1/64) + (a/3 - 1/4)$

Step 1 : $(a^3/27) + (a/3) = a^2/9$ (Q_1)
(a^3)

Carry out Urdhva Tiryak Multiplication of the part divisor $-\frac{1}{4}$

$$\boxed{Q_1 = \frac{a^2}{9}}$$

Step 2 :
(a^2)

$$\begin{array}{ll} a/3 & -\ 1/4 \\ & \ \ \uparrow \\ & a^2/9 \\ & Q_1 \end{array} \quad = -a^2/36 \text{ (Urdhva)}$$

But the original co-efficient of $a^2 = -1/12$

$\therefore$ The differences is $-a^2/12 + a^2/36 = (-3+1)a^2/36 = -2a^2/36 = -a^2/18$

$(-a^2/18) / (a/3) = -a/6$ (Q_2)

$$\boxed{Q_2 = \frac{-a}{6}}$$

Step 3 :
(a)

$$\begin{array}{ll} a/3 & -\ 1/4 \\ & \ \ \uparrow \\ a^2/9 & -\ a/6 \\ Q_1 & Q_2 \end{array} \qquad a/24 \quad \text{(Urdhva)}$$

But the original co-efficient of $a = 1/16$

$\therefore$ The difference $= a/16 - a/24 = a/48$

$\frac{(a/48)}{(a/3)} = 1/16$ (Q_3)

$$\boxed{Q_3 = \frac{1}{16}}$$

Step 4 :
(Absolute term)

$$\begin{array}{ccc} & \frac{a}{3} & -\ \frac{1}{4} \\ & & \uparrow \\ -\frac{a^2}{9} & -\ \frac{a}{6} & +\ \frac{1}{16} \\ Q_1 & Q_2 & Q_3 \end{array} \qquad = -\,1/64 \text{ (Urdhva)}$$

The original constant $= -1/64$

The difference is $= -\,1/64 + 1/64 = 0$

Remainder $= 0$

Quotient $Q_1 + Q_2 + Q_3 = a^2/9 - a/6 + 1/16$

Current Method :

$$\frac{a}{3} - \frac{1}{4} \Big) \; \frac{a^3}{27} - \frac{a^2}{12} + \frac{a}{16} - \frac{1}{64} \; \Big(\frac{a^2}{9} - \frac{a}{6} + \frac{1}{16}$$

$$\cancel{\frac{a^3}{27}} - \frac{a^2}{36}$$

$$\cancel{-\frac{a^2}{18}} + \frac{a}{16}$$

$$\cancel{-\frac{a^2}{18}} + \frac{a}{24}$$

$$\cancel{\frac{a}{48}} \quad \cancel{-\frac{1}{64}}$$

$$\cancel{\frac{a}{48}} \quad \cancel{-\frac{1}{64}}$$

$$- \qquad +$$

<u>Example 4</u> : $\dfrac{x^5 - 4x^4 + 3x^3 + 3x^2 - 3x + 2}{x^2 - x - 2}$

Step1 (x^5) $\dfrac{x^5}{x^2} = x^3 \quad (Q_1)$ $\therefore \boxed{Q_1 = x^3}$

Carry out Urdhva Tiryak Multiplication of part of the divisor (the Dhwajanka) – x – 2 with successive quotients as follows :

Step2 (x^4)

$$\begin{array}{c} x^2 - x - 2 \\ \uparrow \\ x^3 \\ Q_1 \end{array} \quad = - x^4 \qquad \boxed{\therefore Q_2 = - 3x^2} \quad \text{(Urdhva)}$$

But the original co-efficient is $- 4x^4$

$\therefore$ The difference is $- 4x^4 + x^4 = - 3x^4$

$- 3x^4 / x^2 = - 3x^2$

Step3 : (x^3)

$$\begin{array}{ccc} x^2 - x & & - 2 \\ & \times & \\ x^3 & & - 3x^2 \end{array} \quad = 3x^2 - 2x^3 = x^3$$

But the original coefficient is $3x^3$ $\therefore$ The difference is $3x^3 - x^3 = 2x^3$, $\dfrac{2x^3}{x^2} = 2x \; (Q_3)$ $\boxed{\therefore Q_3 =}$

Step 4 : (x^2)

$$\begin{array}{ccc} x^2 - x & - & 2 \\ & \times & \\ x^3 - 3x^2 & & + 2x \\ Q_1 \;\; Q_2 & & Q_3 \end{array} \quad = - 2x^2 + 6x^2 = 4x^2 \text{ (Tiryak)}$$

But the original co-efficient of x^2 is 3

$\therefore$ The difference is $3x^2 - 4x^2 = -x^2, -\frac{x^2}{x^2} = -1\ (Q_4)$ $\boxed{\therefore\ Q_4 = -1}$

Step 5 :
(x)

$$\begin{array}{cccc} & x^2 & -x & -2 \\ & & \nwarrow\!\!\!\!\!\nearrow & \\ x^3 & -3x^2 & +2x & -1 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{array} = x - 4x = -3x \qquad \text{(by Tiryak)}$$

But the original co-efficient of x = - 3
$\therefore$ The difference is 0 which is R_1

Step 6 : (To get the absolute term)

$$\begin{array}{cccc} x^2 & - & x & -2 \\ & & & \uparrow \\ x^3 - 3x^2 & + & 2x & -1 \\ Q_1\ \ Q_2 & & Q_3 & Q_4 \end{array} = 2 \quad \text{(Urdhva)}$$

But the original absolute term is 2
$\therefore$ The difference is $R_2 = 0$

Quotient is $\begin{array}{cccc} x^3 & -3x^2 & +2x & -1 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{array}$ $R = R_1 + R_2 = 0$

Example 6 : $\dfrac{2y^3 - 3y^2 - 6y - 1}{2y^2 - 5y - 1}$

Step1: $\frac{2y^3}{2y^2} = y\ (Q_1)$ $\boxed{Q_1 = y}$
(y^1)

carrying out Urdhva Tiryak Multiplication of the part divisor (the part of the divisor means, the divisor excluding the highest term which is used for division.)- 5y - 1 with successive quotients as follows

Step 2:
(y^2)

$$\begin{array}{ccc} 2y^2 & -5y & -1 \\ & \uparrow & \\ & y & \\ & Q_1 & \end{array} = -5y^2 \qquad \text{(Urdhva)}$$

But the original coefficient of y^2 is - 3
$\therefore$ The difference is
$-3y^2 - (-5y^2) = 2y^2$

$\frac{2y^2}{2y^2} = 1\ (Q_2)$ $\boxed{Q_2 = 1}$

Step 3: (y)

$$\begin{array}{ccc} 2y^2-5y & & -1 \\ & \times & \\ y & & +1 \\ Q_1 & & Q_2 \end{array} \quad = -5y - y = -6y \text{ (Tiryak)}$$

The original co-efficient of y is − 6

∴ The difference is

$$-6y-(-6y) = 0 \qquad R_1 = 0$$

Step 4: (to get the absolute term)

$$\begin{array}{ccc} 2y^2-5y & & -1 \\ & & \uparrow \\ y & & +1 \\ Q_1 & & Q_2 \end{array} \quad = -1 \text{ (Urdhva)}$$

The original absolute term is −1

∴ The difference is

$$-1-(-1) = 0 \qquad \boxed{R_2 = 0}$$

Hence Quotient = y + 1 and remainder = 0

Example 6 :

$$\frac{6m^3 - m^2 - 14m + 3}{3m^2 + 4m - 1}$$

Step1 : (m^3) $\frac{6m^3}{3m^2} = 2m \ (Q_1)$

Carrying out Urdhva Tiryak Multiplication of the part of the divisor 4m − 1 with successive quotients as follows .

Step2 : (m^2)

$$\begin{array}{c} 3m^2+4m-1 \\ \uparrow \\ 2m \\ Q_1 \end{array} \quad = 8m^2 \text{ (Urdhva)}$$

The original co-efficient of m^2 is − 1

∴ Difference = $-m^2 - 8m^2 = -9m^2$

$$\frac{-9m^2}{3m^2} = -3 \quad (Q_2) \qquad \boxed{Q_2 = -3}$$

Step3: (m)

$$\begin{array}{ccc} 3m^2+4m & & -1 \\ & \times & \\ 2m & & -3 \\ Q_1 & & Q_2 \end{array} \quad = -12m - 2m = -14m \quad \text{(Tiryak)}$$

But the original co-efficient of m is − 14

∴ The difference is $-14m - (-14m) = 0 \ (R_1)$. $\boxed{R_1 = 0}$

Step4: (The absolute term)

$$\begin{array}{c} 3m^2 + 4m - 1 \\ \uparrow \\ 2m - 3 \\ Q_1 \quad Q_2 \end{array} = 3 \qquad \text{(Urdhva)}$$

The original absolute term is also 3

$\therefore$ The difference is $3 - 3 = 0 \ (R_2)$

Quotient $= Q_1 + Q_2 = 2m - 3$, Remainder $= R_1 + R_2 = 0$

Example 7 : $\dfrac{6a^5 - 13a^4 + 4a^3 + 3a^2}{3a^3 - 2a^2 - a}$

Step1: (a^5)

$$\frac{6a^5}{3a^3} = 2a^2 \ (Q_1) \qquad \boxed{Q_1 = 2a^2}$$

Carrying out Urdhva Tiryak Multiplication of the part divisor $-2a^2 - a$ with successive quotients as follows

Step2 : (a^4)

$$\begin{array}{c} 3a^3 \quad -2a^2 - a \\ \uparrow \\ 2a^2 \\ Q_1 \end{array} = -4a^4 \qquad \text{(Urdhva)}$$

But the co-efficient of original a^4 is -13

$\therefore$ The difference is

$$-13a^4 - (-4a^4) = -9a^4$$

$$\frac{-9a^4}{3a^3} = -3a \ (Q_2) \qquad \boxed{Q_2 = -3a}$$

Step3 : (a^3)

$$\begin{array}{c} 3a^3 - 2a^2 - a \\ \times \\ 2a^2 \ -3a \\ Q_1 \quad Q_2 \end{array} = 6a^3 - 2a^3 = 4a^3 \qquad \text{(Tiryak)}$$

But the co-efficient of original a^3 is 4

$\therefore$ The difference is

$$4a^3 - 4a^3 = 0$$

Step4: (a^2)

$$\begin{array}{c} - - \quad - - \quad - \\ \uparrow \\ 2a^2 - 3a \\ Q_1 \quad Q_2 \end{array} = 3a^2 \quad \text{(Urdhva)} \qquad \boxed{R_2 = 0}$$

But the coefficient of original a^2 is 3

$\therefore$ The difference is $3a^2 - 3a^2 = 0$ $\boxed{R_2 = 0}$

$\therefore$ $Q = Q_1 + Q_2 = 2a^2 - 3a$ $\qquad R = R_1 + R_2 = 0$

Example 8 $\dfrac{x^4 + x^3 + 7x^2 - 6x + 8}{x^2 + 2x + 8}$

Step1: (x^4) $\dfrac{x^4}{x^2} = x^2 \quad (Q_1)$ $\boxed{Q_1 = x^2}$

Carry out the Urdhva and Tiryak multiplication of 2x + 8 of the dividend with successive quotient as follows :

Step2: (x^3)

$$\begin{array}{ccc} x^2 + & 2x & + 8 \\ & x^2 & \\ & Q_1 & \end{array} = 2x^3 \text{ (Urdhva)}$$

But the coefficient of original x^3 is 1
∴ the difference is

$$x^3 - 2x^3 = -x^3 \;\&\; \frac{-x^3}{x^2} = -x \quad (Q_2) \qquad \boxed{Q_2 = -x}$$

Step3: (x^2)

$$\begin{array}{ccc} x^2 + & 2x & +8 \\ & x^2 & -x \\ & Q_1 & Q_2 \end{array} = 8x^2 - 2x^2 = 6x^2 \qquad \text{(Tiryak)}$$

But the coefficient of original x^2 is 7.
∴ the difference is

$$7x^2 - 6x^2 = x^2 ; \; \frac{x^2}{x^2} = 1 \; (Q_3) \qquad Q_3 = 1$$

Step4: (x)

$$\begin{array}{ccc} x^2 + & 2x & + 8 \\ x^2 & -x & +1 \\ Q_1 & Q_2 & Q_3 \end{array} \qquad 2x - 8x = -6x \qquad \text{(Tiryak)}$$

But the coefficient of original x term is – 6
∴ the difference is

$$-6x + 6x = 0 \qquad R_1 = 0$$

Step5:
(To get the Absolute term)

$$\begin{array}{ccc} x^2 + & 2x + & 8 \\ & & \uparrow \\ x^2 - & x & + 1 \\ Q_1 & Q_2 & Q_3 \end{array} \qquad \text{(Urdhva)}$$

But the original absolute term is 8
∴ the difference is 8 – 8 = 0 $\boxed{R_2 = 0}$

Quotient, $Q = Q_1 + Q_2 + Q_3 = x^2 - x + 1 ; R = R_1 + R_2 = 0$
Remainder, $R = R_1 + R_2 = 0$

Example 9 : $\dfrac{a^4 - a^3 - 8a^2 + 12a - 9}{a^2 + 2a - 3}$

Step1:
(a^4) $\dfrac{a^4}{a^2} = a^2 \quad (Q_1)$ $\boxed{Q_1 = a^2}$

Carrying out Urdhva Tiryak multiplication of the remaing part of the divisor 2a – 3 by the successive quotients

Step2:
(a^3)
$$\begin{array}{c} a^2 + \overset{\uparrow}{2}a - 3 \\ a^2 \\ Q_1 \end{array} = 2a^3$$

But the coefficient of original a^3 is - 1

∴ the difference is

$$-a^3 - 2a^3 = -3a^3 ; \quad \frac{-3a^3}{a^2} = -3a \quad (Q_2) \qquad \boxed{Q_2 = -3a}$$

Step3:
(a^2)
$$\begin{array}{ccc} a^2 + & 2a & -3 \\ & \times & \\ & a^2 & -3a \\ & Q_1 & Q_2 \end{array} = -3a^2 - 6a^2 = -9a^2$$

But the coefficient of original a^2 is – 8

∴ the difference is

$$-8a^2 - (-9a^2) = a^2$$

$$\frac{a^2}{a^2} = 1 \quad (Q_3) \qquad \boxed{Q_3 = 1}$$

Step4:
(a)
$$\begin{array}{ccc} a^2 + & 2a & -\ 3 \\ & \times & \\ a^2 - & 3a & +\ 1 \\ Q_1 & Q_2 & Q_3 \end{array} = 9a + 2a = 11a \text{ (Tiryak)}$$

But the coefficient of original is 12

∴ the difference is

$$12a - 11a = a \quad (R_1) \qquad \boxed{R_1 = a}$$

Step5:
(To get the absolute term)
$$\begin{array}{c} a^2 + 2a - \overset{\uparrow}{3} \\ a^2 - 3a + 1 \\ Q_1 \ \ Q_2 \ \ Q_3 \end{array} = -3$$

But the original absolute term is - 9

∴ the difference is

$$-9 - (-3) = -6 \quad (R_2) \qquad \boxed{R_2 = -6}$$

$$Q = Q_1 + Q_2 + Q_3 = a^2 - 3a + 1 \ \ \& \ \ R = R_1 + R_2 = a - 6$$

Example 10 $\dfrac{a^4 + 6a^3 + 13a^2 + 12a + 4}{a^2 + 3a + 2}$ $Q = a^2 + 3a + 2$

$R = 0$

Step1: (a^4) $\dfrac{a^4}{a^2} = a^2 \quad (Q_1)$ $\boxed{Q_1 = a^2}$

Carrying out Urdhva Tiryak multiplication of the remaining part 3a + 2 of the divisor with the successive quotients

Step2: (a^3)

$$\begin{array}{c} a^2 + 3a + 2 \\ \uparrow \\ a^2 \\ Q_1 \end{array} = 3a^3 \text{(Urdhva)}$$

But the coefficient of the given a^3 is 3

∴ the difference is

$$6a^3 - 3a^3 = 3a^3\ ; \quad \frac{3a^3}{a^2} = 3a \quad (Q_2) \quad \boxed{Q_2 = 3a}$$

Step3: (a^2)

$$\begin{array}{c} a^2 + 3a + 2 \\ \times \\ a^2 + 3a \\ Q_1 \quad Q_2 \end{array} = 2a^2 + 9a^2 = 11a^2 \quad \text{(Tiryak)}$$

But the coefficient of given a^2 is 13

∴ the difference is

$$13a^2 - 11a^2 = 2a^2, \quad \frac{2a^2}{a^2} = 2\ (Q_3) \quad \boxed{Q_3 = 2}$$

Step4: (a)

$$\begin{array}{c} a^2 + 3a + 2 \\ \times \\ a^2 + 3a + 2 \\ Q_1 \quad Q_2 \quad Q_3 \end{array} = 6a + 6a = 12a \quad \text{(Tiryak)}$$

But the coefficient of given a is 12

∴ the difference is

$$12a - 12a = 0 \ (R_1) \quad \boxed{R_1 = 0}$$

Step5: (To get the absolute term)

$$\begin{array}{c} a^2 + 3a \quad + 2 \\ \uparrow \\ a^2 + 3a \quad + 2 \\ Q_1 \quad Q_2 \quad Q_3 \end{array} = 4 \ \text{(Urdhva)} \quad \boxed{R_2 = 0}$$

But the given absolute term is 4

∴ the difference is 4 – 4 = 0

$$Q = Q_1 + Q_2 + Q_3 = a^2 + 3a + 2 \quad \& \quad R = R_1 + R_2 = 0 + 0 = 0$$

Example 11 $\dfrac{2x^4 - x^3 + 4x^2 + 7x + 1}{x^2 - x + 3}$ $Q = 2x^2 + x - 1$

$R = 3x + 4$

Step1: (x^4) $\dfrac{2x^4}{x^2} = 2x^2 \quad (Q_1)$ $\boxed{Q_1 = 2x^2}$

Carrying out the multiplication of the remaining part $-x+3$ of the divisor with the successive quotients as follow

Step2: (x^3)

$$\begin{array}{c} x^2 - x + 3 \\ \uparrow \\ 2x^2 \\ Q_1 \end{array} \quad = -2x^3 \quad \text{(Urdhva)}$$

But the coefficient of original x^3 is -1
$\therefore$ the difference is

$$-x^3 - (-2x^3) = x^3 \quad \& \quad \frac{x^3}{x^2} = x\,(Q_2) \qquad \boxed{Q_2 = x}$$

Step3: (x^2)

$$\begin{array}{c} x^2 - x + 3 \\ \times \\ 2x^2 + x \\ Q_1 \quad Q_2 \end{array} \quad = 6x^2 - x^2 = 5x^2 \text{ (Tiryak)}$$

the coefficient of the original x^2 is 4

$\therefore$ the difference is $\quad 4x^2 - 5x^2 = -x^2 \;,\; \dfrac{-x^2}{x^2} = -1 \;(Q_3) \qquad \boxed{Q_3 = -1}$

Step4: (x)

$$\begin{array}{c} x^2 - x + 3 \\ \times \\ 2x^2 + x - 1 \\ Q_1 \quad Q_2\,Q_3 \end{array} \quad = 3x + x = 4x \qquad \text{(Tiryak)}$$

the coefficient of the original x is 7
$\therefore$ the difference is

$$7x - 4x = 3x \;(R_1) \qquad \boxed{R_1 = 3x}$$

Step5: (To get the Absolute term)

$$\begin{array}{c} x^2 \quad x + 3 \\ \uparrow \\ 2x^2 + x \quad 1 \\ Q_1 \quad Q_2\,Q_3 \end{array} \quad - 3 \qquad \text{(Urdhva)}$$

The absolute term of the original is 1
$\therefore$ the difference is

$$1 - (-3) = 4 \;(R_2) \qquad \boxed{R_2 = 4}$$

$$Q = Q_1 + Q_2 + Q_3 = 2x^2 + x - 1 \quad \& \quad R = R_1 + R_2 = 3x + 4$$

<u>**Example 12**</u> $\quad \dfrac{x^5 - 5x^4 + 9x^3 - 6x^2 - x + 2}{x^2 - 3x + 2} \qquad Q = x^3 - 2x^2 + x + 1$

$R = 0$

Step1: (x^5) $\quad \dfrac{x^5}{x^2} = x^3 \;(Q_1) \qquad \boxed{Q_1 = x^3}$

Carrying out Urdhva Tiryak multiplication of the part divisor $-3x + 2$ with successive quotients as follows

Step2:
(x^4)

$$\begin{array}{c} x^2 - 3x + 2 \\ \uparrow \\ x^3 \\ Q_1 \end{array} \quad = -3x^4 \text{ (Urdhva)}$$

The coefficient of the original x^4 is -5

∴ the difference is

$$-5x^4 - (-3x^4) = -2x^4 \quad \& \quad \frac{-2x^4}{x^2} = -2x^2 \qquad \boxed{Q_2 = -2x^2}$$

Step3:
(x^3)

$$\begin{array}{cc} x^2 - 3x & +2 \\ \times & \\ x^3 & -2x^2 \\ Q_1 & Q_2 \end{array} \quad 2x^3 + 6x^3 = 8x^3 \text{ (Tiryak)}$$

The coefficient of the original x^3 is 9

∴ the difference is

$$9x^3 - 8x^3 = x^3 \quad \& \quad \frac{x^3}{x^2} = x \qquad \boxed{Q_3 = x}$$

Step4:
(x^2)

$$\begin{array}{ccc} x^2 - 3x & + & 2 \\ & \times & \\ x^3 - 2x^2 & + & x \\ Q_1 \quad Q_2 & & Q_3 \end{array} \quad = -4x^2 - 3x^2 = -7x^2 \quad \text{(Tiryak)}$$

The coefficient of the original x^2 is -6

∴ the difference is

$$-6x^2 - (-7x^2) = x^2 \quad \& \quad \frac{x^2}{x^2} = 1 \qquad \boxed{Q_4 = 1}$$

Step5:
(x)

$$\begin{array}{ccc} x^2 - 3x & + & 2 \\ & \times & \\ x^3 - 2x^2 + x & & +1 \\ Q_1 \; Q_2 \quad Q_3 & & Q_4 \end{array} \quad = 2x - 3x = -x \text{ (Tiryak)}$$

The coefficient of the original x is also -1

∴ the difference is

$$-x - (-x) = 0 \quad (R_1)$$

Step6:
(To get the Absolute Term)

$$\begin{array}{c} x^2 - 3x + 2 \\ \uparrow \\ x^3 - 2x^2 + x + 1 \\ Q_1 \; Q_2 \quad Q_3 \quad Q_4 \end{array} \quad = 2 + 1 = 2 \qquad \boxed{R_2 = 0}$$

The original absolute term is 2

∴ the difference is

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = x^3 - 2x^2 + x + 1 \quad \& \quad R = R_1 + R_2 = 0$$

Example 13

$$\frac{x^5 - 4x^4 + 3x^3 + 3x^2 - 3x + 2}{x^2 - x - 2} \qquad \begin{array}{l} Q = x^3 - 3x^2 + 2x - 1 \\ R = 0 \end{array}$$

Step1:
(x^5)

$$\frac{x^5}{x^2} = x^3 \quad (Q_1) \qquad \boxed{Q_1 = x^3}$$

Carrying out Urdhva-Tiryak multiplication of the remaining part of the divisor $- x - 2$ by the successive quotients

Step2:
(x^4)

$$\begin{array}{ccc} x^2 & -x & -2 \\ & \uparrow & \\ & x^3 & \\ & Q_1 & \end{array} \qquad = -x\,(x^3) = -x^4 \quad \text{(Urdhva)}$$

But the coefficient of the original is $-4x^4$
∴ the difference is

$$-4x^4 - (-x^4) = -3x^4$$

$$\frac{-3x^4}{x^2} = -3x^2 \quad (Q_2) \qquad\qquad \boxed{Q_2 = -3x^2}$$

Step3:
(x^3)

$$\begin{array}{ccc} x^2 & -x & -2 \\ & \times & \\ & x^3 & -3x^2 \\ & Q_1 & Q_2 \end{array} \qquad = -2x^3 + 3x^3 = x^3 \quad \text{(Tiryak)}$$

But the coefficient of original is x^3 is 3
∴ the difference is

$$3x^3 - x^3 = 2x^3$$

$$\frac{2x^3}{x^2} = 2x \quad (Q_3) \qquad\qquad \boxed{Q_3 = 2x}$$

Step4:
(x^2)

$$\begin{array}{cccc} & x^2 & -x & -2 \\ & & \times & \\ x^3 & -3x^2 & +2x & \\ Q_1 & Q_2 & Q_1 & \end{array} \qquad \begin{array}{l} = (-2)(-3x^2) + (2x)(-x) \\ = 6x^2 - 2x^2 = 4x^2 \end{array} \quad \text{(Tiryak)}$$

But the coefficient of original is $3x^2$
∴ the difference is

$$3x^2 - 4x^2 = -x^2 \;\;\&\;\; \frac{-x^2}{x^2} = -1 \;(Q_4) \qquad\qquad \boxed{Q_4 = -1}$$

Step5:
(x)

$$\begin{array}{ccccc} & & x^2 & -x & -2 \\ & & & \times & \\ x^3 & -3x^2 & +2x & -1 & \\ Q_1 & Q_2 & Q_3 & Q_4 & \end{array} \qquad - \;4x + x = -3x \quad \text{(Tiryak)}$$

But the coefficient of original is also $-3x$
∴ the difference is

$$-3x - (-3x) = 0 \quad (R_1) \qquad\qquad \boxed{R_1 = 0}$$

Step6:
Absolute

$$\begin{array}{ccccc} & & x^2 & -x & -2 \\ & & & \times & \\ x^3 & -3x^2 & +2x & -1 & \\ Q_1 & Q_2 & Q_3 & Q_4 & \end{array} \qquad = 2$$

But the original absolute is also 2

$$\therefore \quad 2 - 2 = 0 \quad (R_2) \qquad\qquad \boxed{R_2 = 0}$$

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = x^3 - 3x^2 + 2x - 1 \;\;\&\;\; R = R_1 + R_2 = 0 + 0 = 0$$

Example 14

$$\frac{30x^4+11x^3-82x^3 \;-\;-\; 5x+3}{3x^3+2x-4}$$

Step1.
(x^4)

$$\frac{30x^4}{3x^3} = 10x \quad (Q_1) \qquad \boxed{Q_1 = 10x}$$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor $0.x^2 + 2x - 4$ by the successive quotients.

Step2.
(x^3)

$$\underset{\underset{Q_1}{+10x}}{3x^3 + \overset{}{\underset{\uparrow}{0x^2}} + 2x - 4} = 0\ x^3 \qquad \text{(Urdhva)}$$

But the coefficient of the original x^3 is -71
the difference is

$$-71x^3 - 0\ x^3 = -71x^3 \quad \& \quad \frac{-71x^3}{3x^3} = \frac{-71}{3} \qquad \boxed{Q_2 = \frac{-71}{3}}$$

Step 3 :
(x^4)

$$3x^2 + 0x^2 + 2x - 4 \qquad \times \qquad = 20\ x^2 \qquad \text{(Tiryak)}$$

$$\underset{Q_1}{10x} \ \underset{Q_2}{-\frac{71}{3}}$$

But the coefficient of the original x^2 is 0
$\therefore$ the difference is $0\ x^2 - 20x^2 = -20x^2$ $\qquad \boxed{R_1 = -20x^2}$

Step 4 : $3x^3 + 0x^2 + 2x - 4$
(x)

$$\times \qquad = -\frac{142}{3}x - 40x = \frac{-262}{3}x \qquad \text{(Tiryak)}$$

$$\underset{Q_1}{10x} \quad \underset{Q_2}{\frac{-71}{3}}$$

But the coefficient of original x is -5
$\therefore$ the difference $-5x + \frac{262}{3}x = \frac{247}{3}x$ $\qquad \boxed{R_2 = \frac{247}{3}x}$

Step 5 : $3x^3+0x^2+ 2x - \underset{\uparrow}{4}$
(Absolute)

$$= \frac{284}{3} \quad \text{(Urdhva)}$$

$$\underset{Q_1}{10x} \ - \underset{Q_2}{\frac{71}{3}}$$

But the original absolute value is 3
$\therefore$ the difference is $3 - \frac{284}{3} = \frac{-275}{3}$ $\qquad \boxed{R_3 = \frac{-275}{3}}$

Quotient $= 10x - \frac{71}{3}$ $\qquad$ Remainder $= -20x^2 + \frac{247}{3}x - \frac{275}{3}$

Example 15 $\frac{6k^5 - 15k^4 + 4k^3 + 7k^2 - 7k + 2}{3k^3 - k + 1}$ $Q = 2k^2 - 5k + 2$

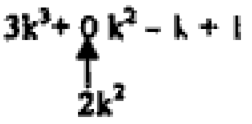
Step1: $\frac{6k^5}{3k^3} = 2k^2$ (Q_1) $\boxed{Q_1 = 2k^2}$

(k^5)

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor $0k^2 - k + 1$ by the successive quotients

Step2:

(k^4)



$= 0\ k^4$ (Urdhva)

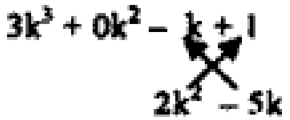
But the coefficient of original k^4 is $-15x^4$

the difference is

$-15k^4 - 0 = -15k^4$ & $\frac{-15k^4}{3k^3} = -5k$ (Q_2) $\boxed{Q_2 = -5k}$

Step 3:

(k^3)



$= -2k^3$ (Tiryak)

The coefficient of original k^3 is 4

the difference is

$4k^3 - (-2k^3) = 6k^3$ & $\frac{6k^3}{3k^3} = 2$ $\boxed{Q_3 = 2}$

Step 4

(k^2)

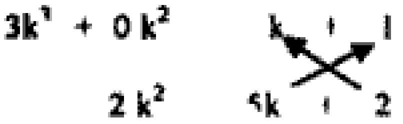
$2k^2 + 5k^2 = 7k^2$ (Tiryak)

The coefficient of original k^2 is also 7

∴ The difference is 0

Step 5 :

(k)



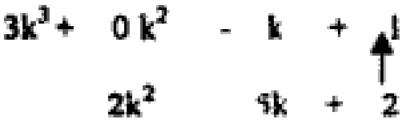
$2k \quad 5k = -7k$ (Tiryak)

The coefficient of original k is also -7 $\boxed{R_2 = 0}$

∴ the difference is 0

Step 6 :

(Absolute)



2 (Urdhva)

The original absolute value is also 2

∵ the difference = 0

∴ Quotient $= 2k^2 - 5k + 2$, Remainder 0

Example 16. $$\frac{15+m^5+2m^4+4m^3+9m^2-31m}{3-2m-m^2} \qquad Q = 5-7m-m^3$$

given problem can be written as $$\frac{m^5+2m^4+4m^3+9m^2-31m+15}{-m^2-2m+3}$$

Step 1:
(m^5) $$\frac{m^5}{-m^2} = -m^3 \; (Q_1) \qquad \boxed{Q_1 = -m^3}$$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor $-2m+3$ with the successive quotients

Step 2:
(m^4) $$\begin{array}{c} -m^2 \; -2m \; + \; 3 \\ \uparrow \\ -m^3 \\ Q_1 \end{array} \qquad = 2m^4$$

But the original coefficient of m^4 is also 2

∴ the difference =

$$2m^4 - 2m^4 = 0 \; (Q_2) \qquad \boxed{Q_2 = 0m^2}$$

Step 3:
(m^3) $$\begin{array}{ccc} -m^2 - & 2m & + \; 3 \\ & \times & \\ -m^3 & + & 0m^2 \\ Q_1 & & Q_2 \end{array} \qquad = -3m^3 \quad \text{(Tiryak)}$$

But the original coefficient of m^3 is 4

∴ difference is

$$4m^3 - (-3m^3) = 7m^3$$

$$\frac{7m^3}{-m^2} = -7m \; (Q_3) \qquad \boxed{Q_3 = -7m}$$

Step 4:
(m^2) $$\begin{array}{cccc} & -m^2 - & 2m & + \; 3 \\ & & \times & \\ -m^3 + & 0m^2 & & -7m \\ Q_1 & Q_2 & & Q_3 \end{array} \qquad = 14m^2 \qquad \text{(Tiryak)}$$

But the original coefficient of m^2 is 14

∴ difference is = $9m^2 - 14m^2 = -5m^2$

$$\frac{-5m^2}{-m^2} = 5 \; (Q_4) \qquad \boxed{Q_4 = 5}$$

Step 5:
(m) $$\begin{array}{ccccc} & & -m^2 - & 2m & + \; 3 \\ & & & \times & \\ -m^3 + & 0m^2 & - & 7m & + \; 5 \\ Q_1 & Q_2 & & Q_3 & Q_4 \end{array} \qquad = -10m - 21m = -31m \qquad \text{(Tiryak)}$$

But the original coefficient of m is -31

the difference is

$$-31m - (-31m) = 0 \; (R_1) \qquad \boxed{R_1 = 0}$$

Step 6:
Absolute

$$\begin{array}{cccc} & -m^2 & -\ 2m & +\ 3 \\ & & & \uparrow \\ -m^3 & +\ 0m^2 & -\ 7m & +\ 5 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{array} = 15$$

The original absolute value is also 15

$\therefore$ The difference = $15 - 15 = 0$ (R_2) $\boxed{R_2 = 0}$

$$Q = Q_1 + Q_2 + Q_3 \quad \& \quad R = R_1 + R_2$$
$$Q = -m^3 - 7m + 5 \;\; \& \;\; R = 0$$

Example 17 : $\dfrac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{x^2 - 2x - 9}$

Step 1:
(x^4)

$\dfrac{6x^4}{x^2} = 6x^2$ (Q_1) $\boxed{Q_1 = 6x^2}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the divisor $-2x - 9$ with the successive quotients

Step 2:
(x^3)

$$\begin{array}{ccc} x^2 & -\ 2x & -\ 9 \\ & \uparrow & \\ & 6x^2 & \\ & Q_1 & \end{array} = -12x^3 \qquad \text{(Urdhva)}$$

But the original coefficient of x^3 is 13

$\therefore$ the difference = $13x^3 + 12x^3 = 25x^3$

$\dfrac{25x^3}{x^2} = 25x$ (Q_2) $\boxed{Q_2 = 25x}$

Step 3:
(x^2)

$$\begin{array}{ccc} x^2 & -\ 2x & -\ 9 \\ & \nwarrow \!\!\!\! \nearrow & \\ & 6x^2 \quad + \quad 25x & \\ & Q_1 \qquad Q_2 & \end{array} = -50x^2 \;\; 54x^2 = -104x^2 \qquad \text{(Tiryak)}$$

The original coefficient of x^2 is + 39

$\therefore$ the difference = $(39 + 104)x^2 = 143x^2$

$\dfrac{143x^2}{x^2} = 143$ (Q_3) $\boxed{Q_3 = 143}$

Step 4:
(x)

$$\begin{array}{ccc} x^2 & -\ 2x & -\ 9 \\ & & \nwarrow \!\!\!\! \nearrow \\ 6x^2 & +\ 25x & +\ 143 \\ Q_1 & Q_2 & Q_3 \end{array} = (-286 - 225)x = -511x \qquad \text{(Tiryak)}$$

The original coefficient of x is 37

$\therefore$ the difference = $(37+511)x = 548x$ (R_1) $\boxed{R_1 = 548x}$

Step 5:
(Absolute)

$$\begin{array}{ccc} x^2 & -\ 2x & -\ 9 \\ & & \uparrow \\ 6x^2 & +\ 25x & +\ 143 \\ Q_1 & Q_2 & Q_3 \end{array} = -1287 \quad \text{(Urdhva)}$$

The original absolute value is 45

$\therefore$ the difference = $45 + 1287 = 1332$ (R_2) $\boxed{R_2 = 1332}$

$$Q = Q_1 + Q_2 + Q_3 = 6x^2 + 25x + 143$$

$$R = R_1 + R_2 = 548x + 1332$$

Comparison of all the methods

Example 18 : $$\frac{7x^{10} + 26x^9 + 53x^8 + 56x^7 + 43x^6 + 40x^5 + 41x^4 + 38x^3 + 19x^2 + 8x + 5}{x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1}$$

Method 1 **Argumental Division – (Urdhva Tiryak)**

Step 1: (x^{10}) $\frac{7x^{10}}{x^5} = 7x^5$ (Q_1) $\boxed{Q_1 = 7x^5}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend $3x^4 + 5x^3 + 3x^2 + x + 1$ with the successive quotients

Step 2: (x^9) $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

$7x^5$

Q_1

$= 21x^9$ (Urdhva)

But the original coefficient of x^9 is 26

$\therefore$ The difference = $26x^9 - 21x^9 = 5x^9$, $\frac{5x^9}{x^5}$ (Q_2) $\boxed{Q_2 = 5x^4}$

Step 3 (x^8) $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

$7x^5 + 5x^4$

Q_1 Q_2

$= 15x^8 + 35x^8 = 50x^8$ (Tiryak)

But the original coefficient of x^8 is 50

$\therefore$ The difference = $53x^8 - 50x^8 = \frac{3x^8}{x^5} = 3x^3$ (Q_3) $\boxed{Q_3 = 3x^3}$

Step 4: (x^7) $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

$7x^5 + 5x^4 + 3x^3$

Q_1 Q_2 Q_3

$= 9x^7 + 21x^7 + 25x^7 = 55x^7$ (Tiryak)

The original coefficient of x^7 is 56

The difference = $56x^7 - 55x^7 = x^7$

$$\frac{x^7}{x^5} = x^2 \quad (Q_4)$$ $\boxed{Q_4 = x^2}$

Step 5: (x^6) $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

$7x^5 + 5x^4 + 3x^3 + x^2$

Q_1 Q_2 Q_3 Q_4

$= 3x^6 + 7x^6 + 15x^6 + 15x^6 = 40x^6$ (Tiryak)

The original coefficient of x^6 is 43

The difference $43x^6 - 40x^6 = 3x^6$, $\frac{3x^6}{x^5} = 3x$ (Q_5) $\boxed{Q_5 = 3x}$

Step 6: $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

(x^5) $= 7x^5 + 9x^5 + 5x^5 + 5x^5 + 9x^5 = 35x^5$ (Tiryak)

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x$

Q_1 Q_2 Q_3 Q_4 Q_5

The original coefficient of x^5 is 40

∴ The difference = $40x^5 - 35x^5 = 5x^5$, $\frac{5x^5}{x^5} = 5$ (Q_6) $\boxed{Q_6 = 5}$

Step 7: $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

(x^4) $= 15x^4 + 5x^4 + 15x^4 + 3x^4 + 3x^4 = 41x^4$ (Tiryak)

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

The original coefficient of x^4 is also 41

∴ Difference = 0 $\boxed{R_1 = 0}$

Step 8: $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

(x^3) $= 25x^3 + 3x^3 + 9x^3 + x^3 = 38x^3$

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

The original coefficient of x^3 is also 38

∴ Difference = 0 $\boxed{R_2 = 0}$

Step 9: $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

(x^2) $= 15x^2 + x^2 + 3x^2 = 19x^2$ (Tiryak)

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

The original coefficient of x^2 is also 19

∴ Difference = 0 $\boxed{R_3 = 0}$

Step 10 : $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

$= 5x + 3x = 8x$ (Tiryak)

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$ $\boxed{R_4 = 0}$

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

The original coefficient of x is also 8

∴ The difference = 0

Step 11 : $x^5 + 3x^4 + 5x^3 + 3x^2 + x + 1$

(Absolute) $= 5$ (Urdhva)

$7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$ $\boxed{R_1 = 0}$

Q_1 Q_2 Q_3 Q_4 Q_5 Q_6

The original absolute value is also 5

∴ Difference = 0

Quotient = $Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 = 7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$

Remainder = $R_1 + R_2 + R_3 + R_4 + R_5 = 0$

Method 2 **Straight Division**

$$\begin{array}{l|l|l}
\begin{array}{l} D_1\ D_2\ D_3\ D_4 D_5 \\ 3x^4+5x^3+3x^2+x+1 \end{array} & 7x^{10}+26x^9+53x^8+56x^7+43x^6+40x^5 & +\ 41x^4+38x^3+19x^2+8x+5 \\
x^5 & \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 & 0 \\ \hline
 & 7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5 & \quad 0
\end{array}$$

Step 1· $\frac{7x^{10}}{x^5} = 7x^5\ (Q_1),$ $R_1 = 0$ $\boxed{Q_1 = 7x^5}$

(x^{10})

Step 2: $ID_1 = 26x^9 - \begin{pmatrix} D_1 \\ 3x^4 \\ \uparrow \\ 7x^5 \end{pmatrix} = 26x^9 - 21x^9 = \frac{5x^9}{x^5} = 5x^4\ Q_2,\ R_2 = 0$ $\boxed{Q_2 = 5x^4}$

(x^9) $\qquad Q_1$

Step 3: $ID_2 = 53x^8 - \begin{pmatrix} D_1 & D_2 \\ 3x^4 & 5x^3 \\ 7x^5 & 5x^4 \\ Q_1 & Q_2 \end{pmatrix} = 53x^8 - 50x^8 = \frac{3x^8}{x^5} = 3x^3\ Q_3, R_3 = 0$ $\boxed{Q_3 = 3x^3}$

(x^8)

Step 4: $ID_3 = 56x^7 - \begin{pmatrix} D_1 & D_2 & D_3 \\ 3x^4 & 5x^3 & 3x^2 \\ 7x^5 & 5x^4 & 3x^3 \\ Q_1 & Q_2 & Q_3 \end{pmatrix} = 56x^7 - (9x^7 + 21x^7 + 25x^7)$

(x^7)

$= 56x^7 - 55x^7 = \frac{x^7}{x^5} = x^2\ \ Q_4,$ $R_4 = 0$ $\boxed{Q_4 = x^2}$

Step 5: $ID_4 = 43x^6 - \begin{pmatrix} D_1 & D_2 & D_3 & D_4 \\ 3x^4 & 5x^3 & 3x^2 & x \\ 7x^5 & 5x^4 & 3x^3 & x^2 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{pmatrix} = 43x^6 - 40x^6 = 3x^6$

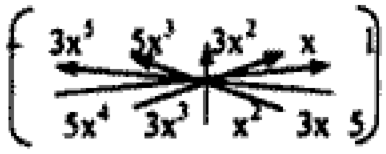
$= \frac{3x^6}{x^5} = 3x\ \ Q_5,$ $R_5 = 0$ $\boxed{Q_5 = 3x}$

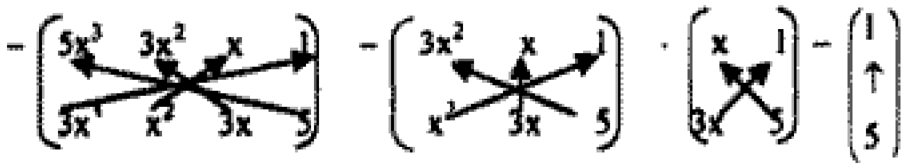
Step 6: $ID_5 = 40x^5 - \begin{pmatrix} D_1 & D_2 & D_3 & D_4 & D_5 \\ 3x^4 & 5x^3 & 3x^2 & x & 1 \\ 7x^5 & 5x^4 & 3x^3 & x^2 & 3x \end{pmatrix} = 40x^5 - 35x^5 = 5x^5$

$\qquad Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \ Q_5$

$\frac{5x^5}{x^5} = 5\ \ Q_6,$ $R_6 = 0$ $\boxed{Q_6 = 5}$

Step 7. Remainder = $41x^4 + 38x^3 + 19x^2 + 8x + 5 -$





$$= 41x^4 + 38x^3 + 19x^2 + 8x + 5 - 41x^4 - 38x^3 - 19x^2 - 8x - 5 = 0$$

Quotient = $7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$, Remainder = 0

Method 3 Paravartya Yojavat :

$x^5+3x^4+5x^3+3x^2+x+1$	x^{10}	x^9	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	absolute
–3 –5 –3 –1 –1	7	26	53	56	43	40	41	38	19	8	5
		–21	35	–21	–7	7					
			15	–25	–15	5	–5				
				–9	–15	9	–3	–3			
					–3	5	–3	–1	–1		
						9	-15	–9	–3	–3	
							-15	–25	–15	–5	–5
	7	5	3	1	3	5	0	0	0	0	0

Quotient = $7x^5 + 5x^4 + 3x^3 + x^2 + 3x + 5$ Remainder = 0

Example 19 :

$$\frac{8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6}{7x^2 + 2x + 1}$$

Step 1 : $\frac{8x^5}{7x^2} = \frac{8x^3}{7}$ (Q_1) $\boxed{Q_1 = \frac{8x^3}{7}}$

(x^5)

Carrying out the Urdhva Tiryak multiplication with the remaining part of the divisor 2x + 1 with the successive quotients

Step 2 : $7x^2 + 2x + 1$

(x^4) $\begin{array}{c} \uparrow \\ \frac{8x^3}{7} \\ Q_1 \end{array} = \frac{16x^4}{7}$ (Urdhva)

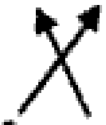
$$\frac{\frac{47x^4}{7}}{7x^2} = \frac{47x^4}{49x^2} = \frac{47x^2}{49} \qquad \boxed{Q_2 = \frac{47x^2}{49}}$$

The original coefficient of x^4 is 9

$$\therefore \quad \text{Difference} = 9x^4 - \frac{16x^4}{7} = \frac{47x^4}{7}$$

Step 3 : $7x^2 + 2x \;+\; 1$

(x^3)



$$= \frac{8x^3}{7} + \frac{94x^3}{49} \quad \text{(Tiryak)}$$

$$\underset{Q_1}{\frac{8x^3}{7}} \qquad \underset{Q_2}{\frac{47x^2}{49}}$$

$$\frac{8x^3}{7} + \frac{94x^3}{49} = \frac{(56+94)x^3}{49} = \frac{150x^3}{49}$$

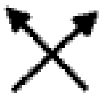
But the original coefficient of x^1 is 7

$$\therefore \quad \text{The difference} = 7x^1 - \frac{150x^3}{49} = \frac{343x^3 - 150x^3}{49} = \frac{193x^3}{49}$$

$$\frac{\frac{193x^3}{49}}{7x^2} = \frac{193x}{343}\ (Q_3) \qquad \boxed{Q_3 = \frac{193x}{343}}$$

Step4 : $7x^2 \;+\; 2x \;+\; 1$

(x^2)



$$= \left(\frac{386}{343} + \frac{47}{49}\right) x^2 = \frac{715}{343}x^2$$

$$\underset{Q_1}{\frac{8}{7}x^3} + \underset{Q_2}{\frac{47}{49}x} + \underset{Q_3}{\frac{193}{343}x}$$

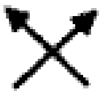
But the original coefficient of x^2 is 3

$$\therefore \quad \text{the difference} = 3x^2 - \frac{715}{343}x^2 = \frac{314}{343}x^2$$

$$\frac{\frac{314}{343}x^2}{7x^2} \cdot \frac{314}{2401} \qquad \boxed{Q_4 = \frac{314}{2401}}$$

Step5 : $7x^2 \;+\; 2x \;+\; 1$

(x)



$$= \left(\frac{628}{2401} + \frac{193}{343}\right) x = \frac{1979}{2401}x$$

$$\underset{Q_1}{\frac{8}{3}x^3} + \underset{Q_2}{\frac{47}{49}x^2} + \underset{Q_3}{\frac{193}{343}x} + \underset{Q_4}{\frac{314}{2401}}$$

But the original coefficient of x is 5

$\therefore$ Difference $= 5x - \frac{1979}{2401}x = \frac{10026}{2401}x$ (R_1)

$$\boxed{R_1 = \frac{10026}{2401}x}$$

Step6 : $7x^2 + 2x + 1$

(Absolute term) $\uparrow = \frac{314}{2401}$

$$\underset{Q_1}{\frac{8}{3}x^3} + \underset{Q_2}{\frac{47}{49}x^2} + \underset{Q_3}{\frac{193}{343}x} + \underset{Q_4}{\frac{314}{2401}}$$

But the original absolute term is 6

$\therefore$ Difference $= 6 - \frac{314}{2401} = \frac{14092}{2401}$

$$_2 = \frac{14092}{2401}$$

$\therefore$ Quotient, $Q = Q_1 + Q_2 + Q_3 + Q_4 = \frac{8}{7}x^3 + \frac{47}{49}x^2 + \frac{193}{343}x + \frac{314}{2401}$

Remainder, $R = R_1 + R_2 = \frac{10026}{2401}x + \frac{14092}{2401}$

Division of Polynomial using urdhva Tiryak :
(Problems from Swamiji's text) Page . 79

Problem 1: $\frac{x^4 - 3x^3 + 7x^2 + 5x + 7}{x - 4}$

Step 1: $x^4 + x = x^3$ (Q_1) $\boxed{Q_1 = x^3}$
(x^4)

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend – 4 with the successive quotients.

Step 2 : $\begin{matrix} x & -4 \\ & \uparrow \\ & x^3 \\ & Q_1 \end{matrix} = -4x^3$. The original coefficient of x^3 is -3
(x^3)

$\therefore$ Difference $= -3x^3 + 4x^3 = x^3$; $x^3 + x = x^2$ $\boxed{Q_2 = x^2}$

Step 3 $\begin{matrix} x - 4 \\ \uparrow \\ x^3 + x^2 \\ Q_1 \quad Q_2 \end{matrix} = -4x^2$ The original coefficient of x^2 is 7
(x^2)

$\therefore$ Difference $= 7x^2 + 4x^2 = 11x^2$, $11x^2 + x = 11x$ $\boxed{Q_3 = 11x}$

Step 4 :
(x)

$$\begin{array}{l} \qquad\quad x - 4 \\ \qquad\qquad \uparrow \\ x^3 + x^2 + 11x \\ Q_1 \quad Q_2 \quad Q_3 \end{array} = -44x$$

The original coefficient of x is 5

$\therefore$ Difference = 5x + 44x = 49x ; 49x ÷ x = 49 $\boxed{Q_4 = 49}$

Step 5 :
(Absolute)

$$\begin{array}{l} \qquad\qquad x - 4 \\ \qquad\qquad\quad \uparrow \\ x^3 + x^2 + 11x + 49 \\ Q_1\, Q_2 \quad Q_3 \quad Q_4 \end{array} = -196 \quad \text{(Urdhva)}$$

But the absolute value is 7

$\therefore$ Difference = 7 + 196 = 203

$\therefore$ Quotient = $x^3 + x^2 + 11x + 49$; Remainder = 203

Problem 2 :

$$\frac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{x^2 - 2x - 9}$$

Step 1: $6x^4 \div x^2 = 6x^2$ (Q_1) $\boxed{Q_1 = 6x^2}$
(x^4)

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend – 2x – 9 with the successive quotients.

Step 2 :
(x^3)

$$\begin{array}{l} x^2 - 2x \quad - 9 \\ \qquad\ \uparrow \\ \qquad 6x^2 \\ \qquad Q_1 \end{array} = -12x^3 \quad \text{(Urdhva)}$$

The original coefficient of x^3 is 13

$\therefore$ Difference = $13x^2 + 12x^3 = 25x^3$, $25x^3 \div x^2 = 25x$ (Q_2) $\boxed{Q_2 = 25x}$

Step 3 :
(x^2)

$$\begin{array}{l} x^2 - 2x - 9 \\ \qquad \uparrow \nearrow \\ 6x^2 + 25x \\ Q_1 \qquad Q_2 \end{array} = -50x^2 - 54x^2 = -104x^2 \quad \text{(Tiryak)}$$

The original coefficient of x is 39

$\therefore$ Difference = $39x^2 + 104x^2 = 143x^2$, $143x^2 \div x^2 = 143$ (Q_3) $\boxed{Q_3 = 143}$

Step 4 :
(x)

$$\begin{array}{l} x^2 - 2x \quad - 9 \\ \qquad\quad \nwarrow \nearrow \\ 6x^2 + 25x + 143 \\ Q_1 \quad Q_2 \quad Q_3 \end{array} = -286x - 225x = -511x \quad \text{(Tiryak)}$$

The original coefficient of x is 37

$\therefore$ Difference = 37x + 511x = 548x (R_1) $\boxed{R_1 = 548x}$

Step 5 :
(Absolute)

$$\begin{array}{l} x^2 - 2x \quad - 9 \\ \qquad\qquad\quad \uparrow \\ 6x^2 + 25x + 143 \\ Q_1 \quad Q_2 \quad Q_3 \end{array} = -1287 \quad \text{(Urdhva)}$$

The original absolute value is 45

$\therefore$ Difference = 45 + 1287 = 1332 $\boxed{R_2 = 1332}$

Quotient = $6x^2 + 25x + 143$, Remainder = 548x + 1332

Problem 3 $\frac{x^4 - 4x^2 + 12x - 9}{x^2 - 2x + 3}$

Step 1: (x⁴) $\frac{x^4}{x^2} = x^2\ (Q_1)$ $\boxed{Q_1 = x^2}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend – 2x + 3 with the successive quotients

Step 2: (x^3)

$$\begin{array}{l} x^2 - 2x + 3 \\ \quad\ \ \uparrow \\ \quad\ \ x^2 \\ \ Q_1 \end{array} = -2x^3$$

The original coefficient of x^3 is 0

∴ Difference = $0x^3 + 2x^3 = 2x^3$

$\frac{2x^3}{x^2} = 2x\ (Q_2)$ $\boxed{Q_2 = 2x}$

Step 3: (x^2)

$$\begin{array}{ccc} x^2 & -2x & +\ 3 \\ & \times & \\ x^2 & + & 2x \\ Q_1 & & Q_2 \end{array} = -4x^2 + 3x^2 = -x^2 \quad \text{(Tiryak)}$$

The original coefficient of x^2 is –4

∴ Difference = $-4x^2 + x^2 = -3x^2$

$\frac{-3x^2}{x^2} = -3\ (Q_3)$ $\boxed{Q_3 = -3}$

Step 4: (x)

$$\begin{array}{ccc} x^2 & -2x & +\ 3 \\ & \times & \\ x^2 & +\ 2x & -\ 3 \\ Q_1 & Q_2 & Q_3 \end{array} = 6x + 6x = 12x \quad \text{(Tiryak)}$$

The original coefficient of x is also 12

∴ Difference = 0 (R_1)

Step 5: (Absolute)

$$\begin{array}{ccc} x^2 & -\ 2x & +\ 3 \\ & & \uparrow \\ x^2 & +\ 2x & -\ 3 \\ Q_1 & Q_2 & Q_3 \end{array} = -9 \quad \text{(Urdhva)}$$

The original absolute value is –9

∴ Difference = 0 (R_2)

Quotient = $x^2 + 2x - 3$; Remainder = 0

Problem 4. $\frac{6x^4 + 13x^3 + 39x^2 + 37x + 45}{3x^2 + 2x + 9}$

Step 1: (x⁴) $\frac{6x^4}{3x^2} = 2x^2\ (Q_1)$ $\boxed{Q_1 = 2x^2}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend 2x + 9 with the successive quotients

Step 2: (x^3)

$$\begin{array}{l} 3x^2 + 2x + 9 \\ \qquad\ \uparrow \\ \qquad 2x^2 \\ \qquad Q_1 \end{array} = 4x\text{`} \quad \text{(Urdhva)}$$

The original coefficient of x^3 is 13

∴ Difference = $13x^3 - 4x^3 = 9x^3$

$\frac{9x^3}{3x^2} = 3x\ (Q_2)$ $\boxed{Q_2 = 3x}$

Step 3 : (x^2)

$3x^2 + 2x + 9$

$2x^2 + 3x$ $\quad = 6x^2 + 18x^2 = 24x^2$

$Q_1 \quad Q_2$

The original coefficient of x^2 is 39

$\therefore$ Difference $= 39x^2 - 24x^2 = 15x^2$

$\frac{15x^2}{3x^2} = 5 \quad (Q_3)$ $\quad \boxed{Q_3 = 5}$

Step 4 : (x)

$3x^2 + 2x + 9$

$2x^2 + 3x + 5$ $\quad = 10x + 27x = 37x$

$Q_1 \quad Q_2 \quad Q_3$

The original coefficient of x is 37

$\therefore$ Difference $= 0$ (R_1)

Step 5 : (Absolute)

$3x^2 + 2x + 9$

$2x^2 + 3x + 5$ $\quad = 45$

$Q_1 \quad Q_2 \quad Q_3$

The original absolute value is 45

$\therefore$ Difference $= 0$ (R_2)

Quotient $= 2x^2 + 3x + 5$, Remainder $= 0$

Problem 5: $\frac{16x^4 + 36x^2 + 81}{4x^2 + 6x + 9}$

Step 1 : (x^4) $\quad \frac{16x^4}{4x^2} = 4x^2 \quad (Q_1)$ $\quad \boxed{Q_1 = 4x^2}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend 6x + 9 with the successive quotients

Step 2 : (x^3)

$4x^2 + 6x + 9$

$4x^2$ $\quad = 24x^3$ (Urdhva)

Q_1

The original coefficient of x^3 is 0

$\therefore$ Difference $= -24x^3$

$\frac{-24x^3}{4x^2} = -6x \quad (Q_2)$ $\quad \boxed{Q_2 = -6x}$

Step 3 : (x^2)

$4x^2 + 6x + 9$

$4x^2 - 6x$ $\quad = -36x^2 + 36x^2 = 0.x^2$ (Tiryak)

$Q_1 \quad Q_2$

The original coefficient of x^2 is 36

$\therefore$ Difference $= 36x^2 - 0x^2 = 36x^2$

$\frac{36x^2}{4x^2} = 9 \quad (Q_3)$ $\quad \boxed{Q_3 = 9}$

Step 4 : (x)

$4x^2 + 6x + 9$

$4x^2 - 6x + 9$ $\quad = 54x - 54x = 0x$ (Tiryak)

$Q_1 \quad Q_2 \quad Q_3$

The original coefficient of x is 0

$\therefore$ Difference $= 0x$ (R_1)

Step 5 : (Absolute)

$4x^2 + 6x + 9$

$4x^2 - 6x + 9$ $\quad = 81$, The original absolute value is also 81

$Q_1 \quad Q_2 \quad Q_3$

$\therefore$ Difference $= 81 - 81 = 0$ (R_2)

Quotient $= 4x^2 - 6x + 9$, Remainder $= 0$

Problem 6 $\dfrac{-2x^5 - 7x^4 + 2x^3 + 18x^2 - 3x - 8}{x^3 - 2x^2 + 0.x + 1}$

Step 1: (x^5) $\dfrac{-2x^5}{x^3} = -2x^2 \quad (Q_1) \quad \boxed{Q_1 = -2x^2}$

Carrying out the Urdhva Tiryak multiplication of the remaining part of the dividend

$2x^2 + 0x + 1$ with the successive quotients

Step 2 : (x^4) $x^3 - 2x^2 + 0x + 1 = 4x^4$, (Urdhva) The original coefficient of x^4 is -7

$-2x^2$
Q_1

∴ Difference $= -7x^4 - 4x^4 = -11x^4$

$-11x^4 = -11x \quad (Q_2) \quad \boxed{Q_2 = -11x}$

Step 3 : (x^3) $x^3 - 2x^2 + 0x + 1 = 22x^3$, (Tiryak) The original coefficient of x^3 is 2

$-2x^2 - 11x$
$Q_1 \quad Q_2$

∴ Difference $= 2x^3 - 22x^3 = -20x^3$

$\dfrac{-20x^3}{x^3} = -20 \quad (Q_3) \quad \boxed{Q_3 = -20}$

Step 4 : (x^2) $x^3 - 2x^2 + 0x + 1 = 40x^2 - 2x^2 = 38x^2$ (Tiryak)

$-2x^2 \quad 11x \quad -20$
$Q_1 \quad Q_2 \quad Q_3$

The original coefficient of x^2 is 18

Difference $= 18x^2 - 38x^2 = -20x^2 \quad (R_1)$

Step 5 : (x) $x^3 - 2x^2 + 0x + 1 = 0 - 11x$, (Tiryak) The original coefficient of x is -3

$-2x^2 \quad 11x \quad 20$
$Q_1 \quad Q_2 \quad Q_3$

∴ Difference $= -3x + 11x = 8x \quad (R_2)$

Step 6 : (Absolute) $x^3 - 2x^2 + 0x + 1 = -20$, (Tiryak) The original absolute value is -8

$-2x^2 - 11x - 20$
$Q_1 \quad Q_2 \quad Q_3$

∴ Difference $= -8 + 20 = 12 \quad (R_3)$

Quotient $= -2x^2 - 11x - 20$

Remainder $= -20x^2 + 8x + 12$

CHAPTER – VI

POLYNOMIAL DIVISION USING STRAIGHT DIVISION METHOD:

(a) for single variable

Using the same procedure as used for numbers, the straight division process can be applied for polynomials also

Problem 1 Consider an example $5x^2 + 2x + 1 \quad + \quad 3x + 2$

The divisor $3x + 2$ is partitioned to form the part divisor (PD) 3x and dwajanka 'D' as 2 The Dhwajanka has one term 2, so the dividend is also partitioned by taking one term from the right which is designated as remainder region, following the usual rules of the partition as in the number division

$$\begin{array}{cc|cccccc}
 & (D) & & & & & & \\
 & 2 & 5x^2 & + & 2x & : & + & 1 \\
(PD) & 3x & & 0 & & : & 0 & \\
\hline
 & & \frac{5x}{3} & & \frac{-4}{9} & : & & \frac{17}{9} \\
 & & Q_1 & & Q_2 & & &
\end{array}$$

The division can be carried out in two ways

(1) For zero intermediate remainder

(2) For non-zero intermediate remainder

(a) For zero intermediate remainder .

Step1: The first term of the dividend is divided by 3x to get Q_1 (quotient) i.e., $\frac{5x^2}{3x} = \frac{5x}{3}$ (Q_1) and the remainder is zero. The remainder is placed between the first and the second terms, below the dividend

Step2 : The next intermediate dividend (ID) is 0+2x The Urdhva multiplication of Dhwajanka with Q_1 is first subtracted from this ID and the result is then divided by the P D , 3x to obtain the next term in the quotient(Q_2)

$$\text{i e., } 2x - \underset{Q_1}{\left(\begin{array}{c} 2 \\ \uparrow \\ \frac{5x}{3} \end{array}\right)} = 2x - \frac{10x}{3} = \frac{-4x}{3}$$

$$\left(\frac{-4x}{3}\right) \div 3x = \left(\frac{-4x}{3}\right)\left(\frac{1}{3x}\right) = \frac{-4}{9} \quad (Q_2)$$

The remainder R_2 is zero

Step3 : Now one enters the remainder region This has the new ID 01 from which the Urdhva multiplication of Dhwajanka with Q_2 is subtracted to obtain the final remainder

$$01 - \underset{Q_2}{\overset{D}{\left(\begin{array}{c} 2 \\ \uparrow \\ \frac{-4}{9} \end{array}\right)}} = 1 - \left(\frac{-8}{9}\right) = \frac{17}{9}$$

$$\text{Quotient} = \frac{5x}{3} - \frac{4}{9}, \quad \text{Remainder} = \frac{17}{9}$$

Vedic Method 1 :

$$\begin{array}{r|cc|c} 2 & 5x^2 + & 2x & + \quad 1 \\ 3x & & 0 & 0 \\ & & (R_1) & (R_2) \\ \hline & \frac{5x}{3} & \frac{-4}{9} & \frac{17}{9} \\ & Q_1 & Q_2 & \end{array}$$

Current Method :

$$\begin{array}{l} 3x + 2 \;)\; \cancel{5x^2} + 2x + 1 \;(\;\frac{5x}{3} - \frac{4}{9} \\ \qquad\qquad \cancel{5x^2} + \frac{10x}{3} \\ \qquad\qquad (-) \quad (-) \\ \qquad\qquad\qquad -\cancel{\frac{4x}{3}} + 1 \\ \qquad\qquad\qquad -\cancel{\frac{4x}{3}} - \frac{8}{9} \\ \qquad\qquad\qquad (+) \quad (+) \\ \hline \qquad\qquad\qquad\quad \frac{17}{9} \\ \hline \end{array}$$

It can be proved that quotient multiplied by the total divisor when added to the final remainder gives the dividend.

$$\begin{array}{ll} \text{Quotient is} & \frac{5x}{3} - \frac{4}{9} \\ \text{Divisor is} & \underline{\quad 3x + 2 \quad} \\ & 5x^2 + 2x - \frac{8}{.} \\ \text{Remainder is} & \qquad + \underline{\frac{17}{9}} \\ & 5x^2 + 2x + 1 = \text{Dividend} \end{array}$$

It is shown that the dividend = (quotient) (divisor) + remainder. = q × d + r = Div
Vedic Method 1 is valid for any value of x
There is another method in which one need not aim at zero remainder as the intermediate stage (Thus fractions can be avoided)

(b) For non-zero intermediate remainder

$$\begin{array}{c|c|c} \begin{matrix}(D)\\ 2\end{matrix} & 5x^2 + 2x & +1 \\ 3x & 2x^2 & 2x \\ (PD) & (R_1) & (R_2) \\ \hline & x \quad + \quad 6 & 2x-11 \\ & Q_1 \qquad Q_2 & \end{array}$$

In this method also the partition rules are followed in the same manner

Step1: The first term of the dividend is divided by 3x i e , $(5x^2) \div (3x)$ to obtain the first term of the quotient as x (Q_1) and the remainder $2x^2$ (R_1).

Step2 : Now the intermediate dividend is ID $2x^2 + 2x$
The Urdhva multiplication of D and Q_1 is subtracted from ID and the result is divided by PD . i.e.,

$$2x^2 + 2x - \overset{D}{\underset{Q_1}{\begin{bmatrix} 2 \\ \uparrow \\ x \end{bmatrix}}} \quad = 2x^2 + 2x - 2x = 2x^2$$

The second degree term is reduced to first degree term as follows
The result $2x^2$ is written as (2x) (x) = (2x) (10) = 20x (as x = 10)

$$20x \div 3x = \underset{Q_2}{6}, \ \underset{R_2}{2x}$$

Step 3 : One enters into the remainder region by taking the new ID as 2x + 1. From this the Urdhva multiplication of D and Q_2 is subtracted to obtain the remainder.

D

$2x + 1 -$ $\qquad 2x + 1 - 12 = 2x - 11$

Quotient = $x + 6$
Remainder = $2x - 11$
This method is valid only for $x = 10$

Verifying the division using
Dividend = (Divisor) (Quotient) + Remainder

(Divisor) = $3x + 2$
(Quotient)= $x + 6$

$3x^2 + 20x + 12$
Remainder = $2x - 11$

$x^2 + 22x + 1$ $\quad (20x = 2x^2 \text{ for } x = 10)$

$5x^2 + 2x + 1$

i e, $\dfrac{5x^2 + 2x + 1}{3x + 2} = \dfrac{3x^2 + 22x + 1}{3x + 2}$ (When $x = 10$)

In this method, the quotient when multiplied with divisor along with the addition of remainder does not directly give the dividend. But in case of x value being considered as 10, it can be shown that the value so obtained can be equated to the dividend with following reading of the terms.

The result of multiplication after adding the remainder = $3x^2 + 22x + 1$
When we consider the second digit to be carried over to the next term, the result is $5x^2 + 2x + 1$. This is justified because 22x can be treated as 2x + 20x, where 20x can be considered as $2x^2$ ($x = 10$), Further we can say that the dividend $5x^2 + 2x + 1 = 521$ ($x = 10$).and $3x^2 + 22x + 1 = 521$

The result obtained in non-zero intermediate remainder method is

$$\begin{aligned} 3x^2 + 22x + 1 &= 300 + 220 + 1 \ (x=10) \\ &= (300 + 200) + 20 + 1 \\ &= 500 + 20 + 1 \\ &= 5x^2 + 2x + 1 \end{aligned}$$

This method is also valid for any value of x, provided we re-write the expressions suitably to give the original dividend.
For example one can work-out for $x = 1, 2, 3 \ldots$ etc
(C) $x = 1$ given $5x^2 + 2x + 1 \div 3x + 2$

$$\begin{array}{r|cc|c} 2 & 5x^2 & + \; 2x & + \; 1 \\ 3x & & 2x^2 & 2x \\ \hline & x & + \; 0 & 2x + 1 \\ & & & \end{array}$$

(1) $\dfrac{5x^2}{3x} = x \; (Q_1) \, , \quad 2x^2 \; (R_1)$

(2) $2x^2 + 2x - \begin{pmatrix} 2 \\ \uparrow \\ x \end{pmatrix} = 2x^2 + 2x - 2x = 2x^2 = (2x)(x) = 2x \; (x=1)$

$\dfrac{2x}{3x} \quad 0 \; (Q_2), \; 2x \; (R_2)$

(3) $2x + 1 - \begin{pmatrix} 2 \\ \uparrow \\ 0 \end{pmatrix} = 2x + 1$ Remainder

Quotient = x, Remainder = 2x + 1
Quotient * Divisor = x(3x+2) = $3x^2 + 2x$

Remainder = 2x + 1

Q x divisor + R = $3x^2 + 4x + 1$

On comparison with the dividend 4x can be written as 2x + 2x, and 2x can be written as $2x^2$ as x = 1.
$3x^2 + 4x + 1 = 3x^2 + 2x + 2x + 1$
$= (3x^2 + 2x^2) + 2x + 1$ (since $x = 1$; $2x^2 = 2x$)
$= 5x^2 + 2x + 1$

Also : $5x^2 + 2x + 1 = 5 + 2 + 1 = 8$ and $3x^2 + 4x + 1 = 3 + 4 + 1 = 8$
Divisor = 3x + 2 = 5
8 / 5 = 1 Q, 3 R
Q = x = 1
R = 2x + 1 = 3

(d) Base x = 2 :

$$\begin{array}{r|ccc|cc}
+2 & 5x^2 & + \diagup & 2x & + \diagup & 1 \\
3x & & 2x^2 & & x & \\
\hline
 & x & + & 1 & x - 1 & \\
 & & & & &
\end{array}$$

1) $5x^2 + 3x = x \quad (Q_1), \quad 2x^2 \quad (R_1)$

(2) $2x^2 + 2x - \begin{pmatrix} 2 \\ \uparrow \\ x \end{pmatrix} = 2x^2 + 2x - 2x - 2x^2 - (2x)(x) = (2x)(2)(x = 2) = 4x \quad (\because x = 2)$

$4x + 3x = 1 \ (Q_2), \quad x \quad (R_2)$

(3) $x + 1 - \begin{pmatrix} 2 \\ \uparrow \\ 1 \end{pmatrix} = x + 1 - 2 = x - 1$ Remainder

Quotient = $x + 1$, Remainder = $x - 1$

$$\begin{array}{lcr}
\text{(Quotient) (Divisor)} & = & x + 1 \\
 & & 3x + 2 \\
\cline{3-3}
 & & 3x^2 + 5x + 2 \\
\text{Remainder} & = & x - 1 \\
\cline{3-3}
 & & 3x^2 + 6x + 1
\end{array}$$

On comparison with the dividend 6x can be written as 2x + 4x and 4x as $2x^2$ (x = 2)

$$\begin{aligned}
3x^2 + 6x + 1 &= 3x^2 + (4x + 2x) + 1 \\
&= (3x^2 + 2x^2) + 2x + 1 \\
&= 5x^2 + 2x + 1
\end{aligned}$$

Also $5x^2 + 2x + 1 = 5*4 + 2*2 + 1 = 25$

$3x + 2 = 3*2 + 2 = 8$

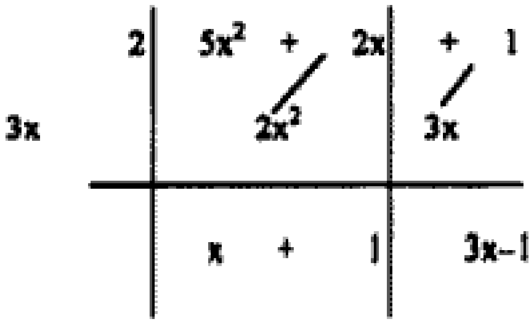
25 / 8 = 3 Q, 1 R

Quotient = x + 1 = 2 + 1 = 3 (Q)

Remainder = x − 1 = 2 − 1 = 1 (R)

(e) Base x = 3

When x = 3 the division has to be carried out in the same way and one has to take care that in the reduction x = 3 has to be considered as followed :



(1) $5x^2 + 3x = \underset{(Q_1)}{(x)}\ ,\ \underset{(R_1)}{2x^2\,(R_1)}$

(2) $2x^2 + 2x = 2x(x+1) = 2x(3+1) = 8x - \begin{pmatrix}2\\ \uparrow\\ x\end{pmatrix} = 8x - 2x = 6x$,

$6x + 3x = \underset{(Q_2)}{1}\ ,\ \underset{(R_2)}{3x}$

(3) $3x + 1 - \begin{pmatrix}2\\ \uparrow\\ 1\end{pmatrix} = 3x + 1 - 2 = 3x - 1$ Remainder.

Verifying the Division

$$\begin{array}{lr}
\text{(Divisor) (quotient)} = & 3x + 2 \\
 & x + 1 \\
\hline
 & 3x^2 + 5x + 2 \\
\text{Remainder} = & 3x - 1 \\
\hline
 & 3x^2 + 8x + 1
\end{array}$$

i.e., $\frac{5x^2 + 2x + 1}{3x + 2} = \frac{3x^2 + 8x + 1}{3x + 2}$ (for x = 3)

On comparison of $3x^2 + 8x + 1$ with the dividend $5x^2 + 2x + 1$, it can be seen that 8x can be written as 2x + 6x and again 6x can be written as $2x^2$ (For x = 3)

$$\begin{aligned}
3x^2 + 8x + 1 &= 3x^2 + (6x + 2x) + 1 \\
&= (3x^2 + 2x^2) + 2x + 1 \\
&= 5x^2 + 2x + 1
\end{aligned}$$

Also $5x^2 + 2x + 1 = (5)(3^2) + (2)(3) + 1$

$= 45 + 6 + 1 = 52$

$3x + 2 = (3)(3) + 2 = 9 + 2 = 11$

$$\begin{array}{r l}
11\,)\,52\,(& 4\ (Q) \\
\underline{44} & \\
\underline{8} & (R)
\end{array}$$

Quotient $= x + 1 = 3 + 1 = 4$

Remainder $= 3x - 1 = (3)(3) - 1 = 9 - 1 = 8$

Problem 2 : Consider $3x^2 + 7x + 1 + 2x + 5$

CURRENT METHOD

$$\begin{array}{l}
2x + 5\,)\,3x^2 + 7x + 1\,(\,\frac{3x}{2} \quad -\frac{1}{4} \\
\qquad 3x^2 + \frac{15x}{2} \\
\qquad (-) \qquad (-) \\
\hline
\qquad\qquad \frac{-x}{2} + 1 \\
\qquad\qquad \frac{-x}{2} - \frac{5}{4} \\
\qquad\qquad + \quad + \\
\hline
\qquad\qquad\qquad \frac{9}{4} \\
\hline
\end{array}$$

$Q = \frac{3x}{2}\ \frac{-1}{4} \qquad R = \frac{9}{4}$

This is valid for all values of x ,
x = 1, 2,3, 10

VEDIC METHOD

(a) DIVISION WITH ZERO REMAINDER :

$$\begin{array}{c|ccc|cc}
D_1 & & & & & \\
5 & 3x^2 & + & 7x & + & 1 \\
2x & & & \overset{}{0} & 0 & \\
 & & & R_1 & R_2 & \\
\hline
 & \frac{3x}{2} & & -\frac{1}{4} & \frac{9}{4} & \\
 & Q_1 & & Q_2 & & \\
\end{array}$$

Remainder

Step 1:

$$\begin{array}{l}
2x)\,3x^2\,(\frac{3x}{2} \quad (Q_1) \\
\ \ \underline{3x^2} \\
\ \ \underline{0} \quad R_1
\end{array}$$

Step 2:

$$0 + 7x - \underset{Q_1}{\begin{pmatrix} 5 \\ \uparrow \\ \frac{3x}{2} \end{pmatrix}} = 7x - \frac{15x}{2} = -\frac{x}{2}$$

$$\begin{array}{l}
2x\,)\,\frac{-x}{2}\,(-\frac{1}{4} \quad (Q_2) \\
\quad \frac{-x}{2} \\
\quad \underline{0} \qquad (R_2)
\end{array}$$

Step 3:

$$0 + 1 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix} 5 \\ \uparrow \\ -\frac{1}{4} \end{pmatrix}}} = 1 - \left(-\frac{5}{4}\right) = 1 + \frac{5}{4}$$

$$Q = Q_1 + Q_2 = \left(\frac{3x}{2} - \frac{1}{4}\right) \text{ and } R = \frac{9}{4}$$

(b) **Base x = 10** (non – zero remainder)

$$\begin{array}{cc|c|c}
 & D_1 & & \\
 & 5 & 3x^2 + 7x & + \ 1 \\
2x & & x^2 & 0 \\
 & & R_1 & R_2 \\
\hline
 & & x \quad + \quad 6 & -2x-9 \\
 & & Q_1 \qquad Q_2 & \\
\end{array}$$

Step1:

$$2x\,)\,3x^2\,(\,x \quad Q_1$$
$$2x^2$$
$$\underline{(-)\quad}$$
$$\underline{x^2} \quad R_1$$

$$(x^2 + 7x) - \overset{D_1}{\underset{Q_1}{\begin{pmatrix} 5 \\ \uparrow \\ x \end{pmatrix}}} = x^2 + 7x - 5x = x^2 + 2x = x(x+2)$$

$$= x(10 + 2) = 12x \qquad [\,x = 10\,(\text{Base})\,]$$

Step 2:

$$2x\,)\,12x\,(\,6 \quad (Q_2)$$
$$\underline{12x}$$
$$\underline{0} \quad (R_2)$$

$$0 + 1 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix} 5 \\ \uparrow \\ 6 \end{pmatrix}}} = 1 - 30 = -29 \text{ Remainder}$$

$$= -20 - 9 = -(20 + 9) = -2x - 9$$

$$Q = Q_1 + Q_2 = x + 6 \quad \& \quad R = -2x - 9$$

From non – zero remainder procedure we get quotient as x+ 6 and remainder –2x –9
Quotient (Q) × Divisor (D) + Remainder (R) = Dividend (Div)
Here $(x+6) \times (2x+5) + (-2x-9) = 2x^2 + 15x + 21 \ \because\ 15x = 10x + 5x$
and $10x = x^2$ also $21 = 20 + 1 = 2x + 1$

When the base x = 10 , this expression can be written as $(2+1)\,x^2 + (5+2) \times x + 1$
$= 3x^2 + 7x + 1$
Carrying out the last digit to the next immediate left with its status gives the dividend used

(c) Base x = 1

$$\begin{array}{cc|c|c} & D_1 & & \\ & 5 & 3x^2 \; + \; 7x & + \quad 1 \\ 2x & & x^2 & x \\ & & R_1 & R_2 \\ \hline & & x \; + \; 1 & \\ & & Q_1 \quad Q_2 & \end{array}$$

Step 1

$$\begin{array}{l} 2x\,)\,3x^2\,(\,x \quad (Q_1) \\ \quad\;\; 2x^2 \\ \hline \quad\;\; x^2 \quad (R_1) \\ \hline \end{array}$$

$$(x^2 + 7x) - \overset{D_1}{\underset{Q_1}{\begin{pmatrix} 5 \\ \uparrow \\ x \end{pmatrix}}} = x^2 + 7x - 5x = x^2 + 2x = x(x + 2) = 3x$$

[x = 1 (Base)]

Step2:

$$\begin{array}{l} 2x\,)\,3x\,(\,1 \quad (Q_2) \\ \quad\;\; 2x \\ \hline \quad\;\; x \quad (R_2) \\ \hline \end{array}$$

$$x + 1 - \overset{D_1}{\underset{Q_2}{\begin{pmatrix} 5 \\ \uparrow \\ 1 \end{pmatrix}}} = x + 1 - 5 = x - 4 \text{ Remainder.}$$

$$Q = Q_1 + Q_2 = x + 1 \text{ and } R = (x - 4)$$

When x= 1 , the quotient is x+1 and the remainder is x-4

Verifying : $Q \times D + R = \text{Div}$

$Q = x + 1$

$D = 2x + 5$

$Q \times D = 2x^2 + 7x + 5$

$R \quad = \quad x - 4$

Div = $2x^2 + 8x + 1$ This can be written as $(2 + 1)x^2 + 7x + 1 = 3x^2 + 7x + 1$ (Since $8x = 7x + 1$) and ($1x = 1x^2$ when) $x = 1$

(d) Base x = 2

$$\begin{array}{cc|ccc|cc}
 & D_1 & & & & & \\
 & 5 & 3x^2 & + & 7x & + & 1 \\
2x & & & & x^2 & 0 & \\
 & & & & R_1 & R_2 & \\
\hline
 & & x & + & 2 & & -9 \\
 & & Q_1 & & Q_2 & & R
\end{array}$$

Step 1

$$2x\ )\ 3x^2\ (\ x \quad (Q_1)$$
$$\underline{2x^2}$$
$$\underline{x^2} \quad (R_1)$$

$$(x^2 + 7x) - \underset{Q_1}{\overset{D_1}{\begin{pmatrix}5\\ \uparrow\\ x\end{pmatrix}}} = x^2 + 7x - 5x = x^2 + 2x = x(x + 2) = x(2 + 2) = 4x$$

$[x = 2 \quad (\text{Base})]$

Step2:

$$2x\ )\ 4x\ (\ 2 \quad (Q_2)$$
$$\underline{4x}$$
$$\underline{0} \quad (R_2)$$

$$(0 + 1) - \underset{Q_2}{\overset{D_1}{\begin{pmatrix}5\\ \uparrow\\ 2\end{pmatrix}}} = 1 - 10 = -9 \text{ Remainder}$$

$$Q = Q_1 + Q_2 = (x + 2) \quad \text{and} \quad R = -9$$

The quotient is x + 2 remainder is –9

Verifying : Q × D + R = Div

Q = x +2

D = 2x+5

Q x D= $2x^2 + 9x + 10$

R = -9 $\quad 9x = 7x + 2x$

$2x^2 + 9x + 1 = 3x^2 + 7x + 1 \quad (\because 2x = x^2 \text{ When } x=2)$

(e) Base x = 3

$$\begin{array}{c|c|c}
 & D_1 & & \\
 & 5 & 3x^2 \; + \; 7x & + \; 1 \\
2x & & x^2 & x \\
 & & R_1 & R_2 \\
\hline
 & & x \; + \; 2 & (x\text{-}9) \\
 & & Q_1 \quad Q_2 &
\end{array}$$

Step 1: 2x) $3x^2$ (x (Q₁)

$2x^2$

x^2 (R_1)

$$(x^2+7x)-\overset{D_1}{\underset{Q_1}{\begin{pmatrix}5\\ \uparrow \\ x\end{pmatrix}}} = x^2+7x-5x = x^2+2x = x(x+2)$$

$= x(3+2),\ x=3$ (Base)

$= 5x$

Step 2: 2x) 5x (2 (Q_2)

4x

x (R_2)

$$(x+1)-\overset{D_1}{\underset{Q_2}{\begin{pmatrix}5\\ \uparrow \\ 2\end{pmatrix}}} = x+1-10 = (x-9)$$

$Q = Q_1 + Q_2 = x + 2$ and $R = x - 9$

This needs to be worked out for each value of x separately.

The quotient is x + 2 remainder is x–9

Verifying : Q × D + R = Div

$Q \;=\; x+2$

$D \;=\; 2x+5$

$Q \times D = 2x^2 + 9x + 10$

$R = \qquad x \; -9$

$2x^2 + 10x + 1 \;=\; (\because 3x = x^2 \text{ When } x = 3)$

$= 3x^2 + 7x + 1$

Problem 3 : $8x^5 + 9x^4 + 7x^3 + + 3x^2 + 5x + 6 \div 7x^2 + 2x + 1.$

(A) Zero Remainder method

Let us consider the Dividend as $8x^5 + 9x^4 + 7x^3 + + 3x^2 + 5x + 6$

The divisor as $7x^2 + 2x + 1$ and the division is carried out digit by digit in this method with zero remainder

Step 1: The Divisor is split into two parts, the Dhwajanka (D) $(2x + 1)$ part divisor (PD) $7x^2$.

Step 2: As there are two terms in Dhwajanka a line of partition is drawn after counting two terms from the last in the Dividend, indicating the remained region

The divided is $8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6$

Step 3: To write down the data as follow

$$
\begin{array}{l|ccccccc|ccc}
 & & & \text{Quotient Region} & & & & & & \text{Remainder Region} & \\
2x + 1 & 8x^5 & + & 9x^4 & + & 7x^3 & + & 3x^2 & 5x & + & 6 \\
\text{D} & & & & & & & & & & \\
7x^2 & & & 0 & & 0 & & 0 & 0 & & 0 \\
\hline
\text{(PD)} & \dfrac{8x^3}{7} & + & \dfrac{47x^2}{49} & + & \dfrac{193x}{343} & + & \dfrac{314}{2401} & \dfrac{10026x}{2401} & + & \dfrac{14092}{2401} \\
 & Q_1 & & Q_2 & & Q_3 & & Q_4 & & &
\end{array}
$$

Step 4: Divide $8x^5$ by $7x^2$ to get zero remainder when the quotient Q_1 is $\dfrac{8x^3}{7}$

(Q_1)

Step 5: The in terminate dividend is $0 + 9x^4$ (digit by digit division)

(Q_2)

subtract from this the Urdhva product $\begin{pmatrix} D_1 \\ \uparrow \\ Q_1 \end{pmatrix} = \begin{pmatrix} 2x \\ \uparrow \\ \dfrac{8x^3}{7} \end{pmatrix} = \dfrac{16x^4}{7}$

i.e., $9x^4 - \dfrac{16x^4}{7} = \dfrac{47x^4}{7}$ and dividing by $7x^2$

5 one gets $\dfrac{47}{49}x^2$ as Q_2

The remainder is zero

Step 6: The next divisor is $(Q_3)\ 0 + 7x^3 = 7x^3$

Subtract from $7x^3$ the product as the

$$\begin{array}{c} \begin{array}{ccc} D_1 & & D_2 \end{array} \\ \left(\begin{array}{ccc} 2x & & 1 \\ & \times & \\ \frac{8x}{7} & & \frac{47x^2}{49} \end{array}\right) \\ \begin{array}{ccc} Q_1 & & Q_2 \end{array} \end{array} = \frac{150x^3}{49} \quad \text{Tiryak product}$$

$$\text{To get } 7x^3 - \frac{150x^3}{49} = \frac{193x^3}{49} = \frac{193x^3}{49} = Q_3$$

This is to be divided by $7x^2$ to get Q_3

to get $Q_3 = \frac{193x}{343}$, the remainder is zero

Step 7: The next Tiryak product part dividend is $0 + 3x^2 = 3x^2$
From this subtract

$$\begin{array}{c} \begin{array}{ccc} D_1 & & D_2 \end{array} \\ \left(\begin{array}{ccc} 2x & & 1 \\ & \times & \\ \frac{47x^2}{49} & & \frac{193x}{343} \\ Q_2 & & Q_3 \end{array}\right) \end{array} = \frac{386x^2}{343} + \frac{47x^2}{49} = \frac{715x^2}{343}$$

$$\text{To get} \quad 3x^2 - \frac{715x^2}{343} = \frac{314x^2}{343}$$

This is to be divided by $7x^2$ to get $Q_4 = \frac{314}{343} \times \frac{x^2}{7x^2} = \frac{314}{2401} = Q_4$. The remainder is zero.

$5x + 6 = 5x + 6$

Step 8: The working enters into the remainder region. The terms in the remainder region are $0 + 5x + 6 = 5x + 6$. To obtain the remainder, Subtract from this the Tiryak product

$$\left(\begin{array}{ccc} D_1 & & D_2 \\ & \nwarrow\!\!\!\!\nearrow & \\ Q_3 & & Q_4 \end{array}\right) \text{ and also the urdhva product } \left(\begin{array}{c} D_2 \\ \uparrow \\ Q_4 \end{array}\right)$$

$$\text{i.e. } 5x + 6 - \begin{array}{c} \left(\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \times & \\ \frac{193x}{343} & & \frac{314}{2401} \end{array}\right) \\ \begin{array}{ccc} Q_3 & & Q_4 \end{array} \end{array} - \begin{array}{c} D_2 \\ \left(\begin{array}{c} 1 \\ \uparrow \\ \frac{314}{2401} \end{array}\right) \\ Q_4 \end{array}$$

$$=5x+6-\frac{1979x}{2401}-\frac{314}{2401}=\frac{10026x}{2401}+\frac{14092}{2401}$$

$$\text{Quotient} = \begin{array}{ccccccc} \frac{8x^3}{7} & + & \frac{47x^2}{49} & + & \frac{193x}{343} & + & \frac{314}{2401} \\ Q_1 & + & Q_2 & + & Q_3 & + & Q_4 \end{array}$$

$$\text{Remainder} = \begin{array}{ccc} \frac{10026x}{2401} & + & \frac{14092}{2401} \\ R_1 & & R_2 \end{array}$$

It can be proved that quotient × divisor + remainder = Dividend. This working is based on zero remainder at every stage of finding out the quotients

(A) Vedic Method - I (Working for zero remainder-any base x)

$$\begin{array}{l|cccc|cc} 2x+1 & 8x^5 + & 9x^4 + & 7x^3 + & 3x^2 + & 5x & + 6 \\ & / & / & / & & / & / \\ 7x^2 & 0 & 0 & 0 & & 0 & 0 \\ \hline & \frac{8x^3}{7} + & \frac{47x^2}{49} + & \frac{193x}{343} + & \frac{314}{2401} & \frac{10026x}{2401} + & \frac{14092}{2401} \end{array}$$

II But one need not aim at zero remainders during the working of the quotients. But the quotients can be converted to zero remainder at every stage of working, so that it is valid for any base

This is as follows

(B) Vedic Method – II (x = 10)

$$\begin{array}{l|cccc|cc} 2x+1 & 8x^5 + & 9x^4 + & 7x^3 + & 3x^2 & 5x & + 6 \\ & / & / & / & & / & \\ 7x^2 & x^5 & 3x^4 & 4x^3 & & 5x^2 & \\ & (R_1) & (R_2) & (R_3) & & (R_4) & \\ \hline & x^3 + & 2x^2 + & 4x & + 4 & 43x & + 2 \\ & (Q_1) & (Q_2) & (Q_3) & (Q_4) & & \end{array}$$

After the preliminary partitioning as given earlier

Step I : Dividing $8x^5 + 7x^2 = \underset{(Q_1)}{x^1} , \underset{(R_1)}{x^5}$

Step 2: The next Intermediate dividend is $R_1 + 9x^4 = x^5 + 9x^4$ with $x = 10$ we can simplify this as follow:

$$x^5 + 9x^4 - \begin{pmatrix} 2x \\ \uparrow \\ x^3 \end{pmatrix} = x^5 + 9x^4 - 2x^4 = x^5 + 7x^4 = x^4\,(x+7) = 17x^4$$

$$17x^4 \div 7x^2 = \underset{(Q_2)}{2x^2} \quad , \quad \underset{(R_2)}{3x^4}$$

Step 3: The next intermediate dividend is $3x^4 + 7x^3$ On simplifying this further we get

$$\begin{aligned} 3x^4 + 7x^3 - \begin{pmatrix} 2x & & 1 \\ & \times & \\ x^3 & & 2x^2 \end{pmatrix} &= 3x^4 + 7x^3 - (4x^3 + x^3) \\ &= 3x^4 + 2x^3 = x^3\,(3x+2) \\ &= 32x^3 \end{aligned}$$

$$32x^3 + 7x^2 = \underset{(Q_3)}{4x} \quad , \quad \underset{(R_3)}{4x^3}$$

Step4 : The next intermediate dividend is $4x^3 + 3x^2$ This is further simplified as .

$$\begin{aligned} 4x^3 + 3x^2 - \begin{pmatrix} 2x & & 1 \\ & \times & \\ 2x^2 & & 4x \end{pmatrix} &= 4x^3 + 3x^2 - (8x^2 + 2x^2) \\ &= 4x^3 - 7x^2 \\ &= x^2\,(4x - 7) \\ &= 33x^2 \end{aligned}$$

$$33x^2 - 7x^2 = \underset{(Q_4)}{4} \quad , \quad \underset{(R_4)}{5x^2}$$

Step5 : The next intermediate dividend in the remainder region is $5x^2 + 5x$. This is simplified as

$$5x^2 + 5x - \begin{pmatrix} 2x & & 1 \\ & \times & \\ 4x & & 4 \end{pmatrix} = 5x^2 + 5x - 12x = 5x^2 - 7x = x\,(5x - 7) = 43x$$

No division is required as this is in the remainder region

Step 6: The next term in the remainder region is 6 which is simplified as

$$6 - \begin{pmatrix} 4 \\ \uparrow \\ 1 \end{pmatrix} = 6 - 4 = 2$$

Quotient = $x^3 + 2x^2 + 4x + 4$
Remainder = $43x + 2$

Verification : Quotient x Divisor + remainder = Dividend

$$\begin{array}{ll}
\text{Quotient} = & x^3 + 2x^2 + 4x + 4 \\
\text{Divisor} = & \underline{0 + 7x^2 + 2x + 1} \\
7x^5 + 16x^4 & + 33x^3 + 38x^2 + 12x + 4 \\
 & \underline{\qquad\qquad\qquad 43x + 2} \\
7x^5 + 16x^4 & + 33x^3 + 38x^2 + 55x + 6
\end{array}$$

We can show that the quotient set so obtained will also give the same dividend as the original, when the frames is multiplied by the divisor $7x^2 + 2x + 1$ and the remainder is added to the result . In order to get the final dividend, a reorientation of the terms taking x as 10 is necessary as shown below.

The quotient is $x^3 + 2x^2 + 4x + 4$
Divisor is $7x^2 + 2x + 1$

Multiplication of these two results in $7x^5 + 16x^4 + 33x^3 + 38x^2 + 12x + 4$
To this the remainder $(43x + 2)$ when added gives
$7x^5 + 16x^4 + 33x^3 + 38x^2 + 55x + 6$

When $x = 10$

(1) $55x = (50 + 5)x$
$50x = 5x^2$

$\therefore 55x = 5x^2 + 5x$

(2) $38x^2 + 5x^2 = 43x^2$
$43x^2 = (40 + 3)x^2$
$40x^2 = 4x^3$

$\therefore 43x^2 = 4x^3 + 3x^2$

(3) $4x^3$ When added to $33x^3$ results in $37x^3$

$37x^3 = (30 + 7)x^3$
$30x^3 = 3x^4$
$\therefore 37x^3 = 3x^4 + 7x^3$

(4) This $3x^4$ when added $16x^4$ to becomes $19x^4$

$$19x^4 = (10 + 9)\,x^4$$

$$10x^4 = x^5$$

$$19x^4 = x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6$$

(5) This x^5 when added to $7x^5$ becomes $8x^5$

$\therefore$ The dividend is $8x^5 + 9x^4 + 7x^3 + 3x^2 + 5x + 6$

To deduce the results of (A) from the results of (B).

The division using the straight division method is first carried out to obtain the quotients and the remainder. The results are given in (B) This method is simpler to workout from which the zero remainder quotients can be deduced The procedure is as follows (successive deduction method)

Step 1: Q_1 (B) is x^3

$$Q_1(A) = \frac{8x^5}{7x^2} = \frac{8}{7}x^3$$

$$\therefore\ Q_1(A) = \frac{8}{7}Q_1\ (B)$$

This 8 is Co-efficient of x^5 the first term of the dividend and 7 is Co-efficient of part divisor

Step 2: To obtain Q_2 (A) from Q_2 (B) one has to consider the working details of B and also the results obtained in the (A) prior to Q.

For example (a) $Q_2(B) = 2x^2$

$$\text{(b)}\ \underset{(PD)}{7x^2} \times \underset{(Q_2\,B)}{2x^2} + \underset{(R_2B)}{3x^4} + \underset{(Q_1\,B\,\times\,D_1)}{(x^3 \times 2x)} = 2x^4 - \underset{(R_1B)}{x^5}$$

$$19x^4 - x^5 = x^4(19\quad x)$$

$$9x^4 \text{ if } x \quad 10$$

(c) $\dfrac{9x^4}{7x^2}$ $\Big($This is also writen as $\dfrac{63x^2}{49}\Big)$ $\dfrac{9x^2}{7}$

(In comparison with method A)

(d) Consider $\begin{matrix}(2x)\\ D_1\end{matrix}\left\{\begin{matrix}\dfrac{8x^3}{7}\\ Q_1A\end{matrix}\right.\quad \dfrac{16x^4}{7}$

(e) Divide this by $7x^2$, the part divisor

$$\frac{16x^4}{7} = \frac{16x^4}{49x^2} = \frac{16}{49}x^2$$

(f) Subtracting this value from the value in (c)

$\frac{63x^2}{49}$ -to get $(Q_2\,A)$

i e, $\frac{63x^2}{49} - \frac{16x^2}{49} = \frac{47}{49}x^2$

similar is the procedure for the other co-efficient in (A).

Step 3 : From $Q_3\,B = 4x$, to deduce $Q_3\,A$

$$(P.D)\ (Q_3B) + R_3\,B + \begin{pmatrix} D_1 & & D_2 \\ & \nwarrow\!\!\!\nearrow & \\ Q_1B & & Q_2B \end{pmatrix} - R_2\,B - \begin{pmatrix} 2x^2 & & 1 \\ & \nwarrow\!\!\!\nearrow & \\ \frac{8}{7}x^3 & & \frac{47}{49}x^2 \end{pmatrix}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad Q_1(A)\quad Q_2(A)$$

This belongs to (A)

The substitution of values

$$(7x^2)\,(4x) + 4x^3 + \begin{pmatrix} 2x & & 1 \\ & \nwarrow\!\!\!\nearrow & \\ & & 2x^2 \end{pmatrix} - 3x^4 - \left[\frac{94}{49}x^3 + \frac{8}{=}x^3\right.$$

$$28x^3 + 4x^3 + 5x^3 - 3x^4 - \left(\frac{94+56}{49}\right)x^3$$

Since $x^3\,(37 - 3x) = 7x^3$

$$= \left(37x^3 - 3x^4\right) - \frac{150}{49}x^3 = 7x^3 - \frac{150}{49}x^3$$

$$= \left(\frac{343-150}{49}\right)x^3 = \frac{193}{49}x^3$$

This is to be divided by $7x^2$ (PD) which gives $\frac{193}{343}x$ for $Q_3\,A$

Step 4: From $Q_4 B = 4$, to obtain $Q_4 A$

$$(P.D)\,(Q_4 B) + R_4 B + \begin{pmatrix} D_1 & & D_2 \\ & \times & \\ Q_2B & & Q_3B \end{pmatrix} - R_3 B - \underset{Q_2A \qquad Q_3A}{\begin{pmatrix} 2x & & 1 \\ & \times & \\ \frac{47}{49}x^2 & & \frac{193}{343}x^2 \end{pmatrix}}$$

$$= (7x^2)\,4 + 5x^2 + \begin{pmatrix} 2x & & 1 \\ & \times & \\ 2x^2 & & 4x \end{pmatrix} - 4x^3 - \left(\frac{386}{343}x^2 + \frac{47}{49}x^2\right)$$

$$= (33x^2 + 10x^2 - 4x^3) - \left(\frac{386+329}{343}x^2\right)$$

$$= x^2(43 - 4x) - \left(\frac{715}{343}x^2\right)$$

$$= 3x^2 - \frac{715}{343}x^2 \quad (\text{Since } 4x = = 40)$$

$$= \frac{1029 - 715}{343}x^2$$

$$= \frac{314}{343}x^2 \text{ This divided by } 7x^2 \text{ gives } \frac{314}{2401} \quad Q_4(A)$$

Step 5: To deduce the corresponding remainder of (A) from (B)

$$\text{I part of remainder} \quad = 43x + \begin{pmatrix} 2x & & 1 \\ & \times & \\ 4x & & 4 \end{pmatrix} - 5x^2 - \underset{Q_3(A) \qquad Q_4(A)}{\begin{pmatrix} 2x & & 1 \\ & \times & \\ \frac{193}{343}x & & \frac{314}{2401} \end{pmatrix}}$$

$$= (43x + 12x - 5x^2) - \left(\frac{628}{2401} + \frac{193}{343}\right)x$$

$$= (55x - 5x^2) - \left(\frac{628+1351}{2401}\right)x$$

$$= 5x(11-x) - \frac{1979}{2401}x \quad \text{(since } x = 10\text{)}$$

$$= 5x - \frac{1979}{2401}x = \left(\frac{12005-1979}{2401}\right)x$$

$$= \frac{10026}{2401}x \qquad R_1(A)$$

$$\text{II Part of remainder} = \underset{(R_2B)}{2} + \underset{Q_4(B)}{\begin{bmatrix}1\\ \uparrow\\ 4\end{bmatrix}} - \underset{Q_4(A)}{\begin{bmatrix}1\\ \uparrow\\ \frac{314}{2401}\end{bmatrix}} = 6 - \frac{314}{2401}$$

$$= \frac{14406-314}{2401}$$

$$= \frac{14092}{2401} \qquad R_2(A)$$

CURRENT METHOD

$$7x^2+2x+1\,\big)\ \cancel{8x^5}+9x^4+7x^3+3x^2+5x+6\ \Big(\frac{8}{7}x^3+\frac{47}{49}x^2+\frac{193}{343}x+\frac{314}{2401}$$ Quotient

$$\cancel{8x^5}+\frac{16}{7}x^4+\frac{8}{7}x^3$$

$$(-)\quad(-)\quad(-)$$

$$\cancel{\frac{47}{7}x^4}+\frac{41}{7}x^3+3x^2$$

$$\cancel{\frac{47}{7}x^4}+\frac{94}{49}x^3+\frac{47}{49}x^2$$

$$(-)\quad(-)\quad(-)$$

$$\cancel{\frac{193}{49}x^3}+\frac{100}{49}x^2+5x$$

$$\cancel{\frac{193}{349}x^3}+\frac{386}{343}x^2+\frac{193}{343}x$$

$$(-)\quad(-)\quad(-)$$

$$\cancel{\frac{314}{343}x^2}+\frac{772}{343}x+6$$

$$\cancel{\frac{314}{343}x^2}+\frac{628}{2401}x+\frac{314}{2401}$$

$$(-)\quad(-)\quad(-)$$

$$\frac{10026}{2401}x+\frac{14092}{2401}$$ Remainder

Problem 4:

Consider $(8x^5-9x^4+^4 7x^3-3x^2+5x+2)+(7x^2+2x+1)$

(a) **Zero Remainder method:**

$$\begin{array}{l|cccccccc|ccc}
D_1\ D_2 & & & & & & & & & & & \\
2x+1 & 8x^5 & - & 9x^4 & + & 7x^3 & - & 3x^2 & & + & 5x & +\ 2 \\
7x^2 & & & 0 & & 0 & & 0 & & & 0 & \\
\hline
 & \frac{8}{7}x^3 & - & \frac{79}{49}x^2 & + & \frac{445}{343}x & & -\frac{1366}{2401} & & + & \frac{1162}{2401}x & +\frac{6168}{2401} \\
 & Q_1 & & Q_2 & & Q_3 & & Q_4 & & & R_1 & R_2
\end{array}$$

(1) $8X^3 + 7X^2 = \frac{8}{7}x^3$, (Q_1)

(2) $$-9X^4 - \left(\begin{array}{c} D_1 \\ 2x \\ \uparrow \\ \frac{8}{7}x^3 \\ Q_1 \end{array}\right) = -9x^4 - \frac{16}{7}x^4 = \frac{-63-16}{7}x^4 = \frac{-79}{7}x^4$$

$$\frac{-79}{7}x^4 + 7x^2 = \frac{-79}{49}x^2 \quad (Q_2)$$

(3) $$7x^3 - \begin{array}{ccc} D_1 & & D_2 \\ \left(\begin{array}{c} 2x \\ \frac{8}{7}x \end{array}\right. & \overset{\nwarrow\ \nearrow}{\times} & \left.\begin{array}{c} 1 \\ -\frac{79}{49}x^2 \end{array}\right) \\ Q_1 & & Q_2 \end{array} = 7x^3 - \left(-\frac{102}{49}x^3\right) = \frac{445}{49}x^3$$

$$\frac{445}{49}x^3 + 7x^2 = \frac{445}{343}x \quad (Q_3)$$

(4) $$-3x^2 - \left(\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \overset{\nwarrow\ \nearrow}{\times} & \\ \frac{-79}{49}x^2 & & \frac{445}{343}x \\ Q_2 & & Q_3 \end{array}\right) = -3x^2 - \left(\frac{890}{343}x - \frac{79}{49}x^2\right)$$

$$= \left(-3 - \frac{890}{343} + \frac{79}{49}\right)x^2 = \frac{-1366}{343}x^2$$

$$\frac{1366}{343}x^2 + 7x^2 = \frac{-1366}{2401}(Q_4)$$

(5) $$5x - \left(\begin{array}{ccc} D_1 & & D_2 \\ 2x & & 1 \\ & \overset{\nwarrow\ \nearrow}{\times} & \\ \frac{445}{343}x & & \frac{-1366}{2401} \\ Q_3 & & Q_4 \end{array}\right) = 5x - \left(\frac{-2732}{2401}x + \frac{445}{343}x\right)$$

$$= \left(5 + \frac{2732}{2401} - \frac{445}{343}\right)x$$

$$= \frac{11622}{2401}x \quad (R_1)$$

(6) $$2 - \begin{pmatrix} 1 \\ \uparrow \\ -\frac{1366}{2401} \end{pmatrix} = 2 + \frac{1366}{2401} = \frac{6168}{2401}(R_2)$$

$$\text{Quotient} = \frac{8}{7}x^3 - \frac{79}{49}x^2 + \frac{445}{343}x - \frac{1366}{2401} \quad , \quad \text{Remainder} = \frac{11622}{2401}x + \frac{6168}{2401}$$

(b) **Current Method:**

$$7x^2 + 2x + 1 \overline{\big)\, 8x^5 - 9x^4 + 7x^3 - 3x^2 + 5x + 2 \,\big(} \frac{8}{7}x^3 - \frac{79}{49}x^2 + \frac{445}{343}x - \frac{1366}{2401}$$

$$8x^5 + \frac{16}{7}x^4 + \frac{8}{7}x^3$$

$$(-) \quad (-) \qquad (-)$$

$$-\frac{79}{7}x^4 + \frac{41}{7}x^3 - 3x^2$$

$$-\frac{79}{7}x^4 - \frac{158}{49}x^3 - \frac{79}{49}x^2$$

$$(+) \qquad (+) \qquad (+)$$

$$\frac{445}{49}x^3 - \frac{68}{49}x^2 + 5x$$

$$\frac{445}{49}x^3 + \frac{890}{343}x^2 + \frac{445}{343}x$$

$$(-) \quad (-) \qquad (-)$$

$$-\frac{1366}{343}x^2 + \frac{1270}{343}x + 2$$

$$-\frac{1366}{343}x^2 - \frac{2732}{2401}x - \frac{1366}{2401}$$

$$(+) \qquad (+) \qquad (+)$$

$$\frac{11622}{2401}x + \frac{6168}{2401}$$

(C) With remainder and 10 base

$$\begin{array}{l|cccccccc|ccc} 2x+1 & 8x^5 & - & 9x^4 & + & 7x^3 & - & 3x^2 & & + & 5x & +\ 2 \\ 7x^2 & & & x^5(R_1) & & 6x^4(R_2) & & 5x^3(R_3) & & 2x^2(R_4) & & \\ \hline & 1x^3 & + & \bar{1}x^2 & + & 9x & + & 4 & & & 9x & -\ 2 \\ & (Q_1) & & (Q_2) & & (Q_3) & & (Q_4) & & & (R_1) & (R_2) \end{array}$$

(1) $8x^5 + 7x^2 = 1.x^3,\ 1.x^5$

$\qquad Q_1 \quad R_1$

(2) $x^5 + \bar{9}\ x^4 \begin{pmatrix} 2x \\ \uparrow \\ \bar{1}.x^3 \end{pmatrix} = x^5 + \bar{9}\ x^4 - 2x^4 = x^5 + \bar{1}\ \bar{1}\ x^4 = x^4\,(x + \bar{1}\ \bar{1}) = \bar{1}x^4$

$\bar{1}\ x^4 + 7x^2 = \bar{1}.x^2,\ 6x^4$

$\qquad Q_2 \quad R_2$

(3) $6x^4 + 7x^3 - \begin{pmatrix} 2x & & 1 \\ & \times & \\ x^3 & & \bar{1}x^2 \end{pmatrix} = 6x^4 + 7x^3 - \bar{1}x^3 = 6x^4 + 8x^3 = x^3\,(6x + 8) = \bar{6}8x^3$

$68x^3 + 7x^2 = 9x,\ 5x^3$

$\qquad Q_3 \quad R_3$

(4) $5x^3 + \bar{3}x^2 - \begin{pmatrix} 2x & & 1 \\ & \times & \\ \bar{1}x^2 & & 9x \end{pmatrix} = 5x^3 + \bar{3}x^2 - 17x^2$

$= 5x^3 - 20x^2$

$= x^2\,(5x - 20) = 30x^2$

$30x^2 + 7x^2 = 4,\ \ 2x^2$

$\qquad Q_4 \quad R_4$

(5) $2x^2 + 5x + 2 - \begin{pmatrix} 2x & & 1 \\ & \times & \\ 9x & & 4 \end{pmatrix} - \begin{pmatrix} 1 \\ \uparrow \\ 4 \end{pmatrix}$

$= 2x^2 + 5x + 2 - 17x - 4$

$- 2x^2 - 12x - 2 - 2x\,(x \quad 6) - 2$

$\cdot\ 8x \quad 2$

Quotient $= 1.x^3 + \bar{1}\,x^2 + 9x + 4$

$= 0.x^3 + 9x^2 + 9x + 4$

Remainder $= 8x - 2$

Problem 5 :

Consider $(5x^4 + 3x^3 + 2x^2 + x + 2) + (3x^2 + x + 4)$

(a) Working for zero Remainder.

$$\begin{array}{c|c|c} x \; + \; 4 & 5x^4 + \; 3x^3 + \; 2x^2 + & x \; + \; 2 \\ 3x^2 & \quad 0 \quad\quad 0 & 0 \\ \hline & \frac{5}{3}x^2 + \frac{4}{9}x - \frac{46}{27} & \frac{25}{27}x + \frac{238}{27} \end{array}$$

(1) $5x^4 + 3x^2 = \frac{5}{3}x^2$

(2) $3x^3 - \begin{pmatrix} x \\ \uparrow \\ \frac{5}{3}x^2 \end{pmatrix} = 3x^2 - \frac{5}{3}x^3 = \frac{4}{3}x^3$

$\frac{4}{3}x^3 + 3x^2 = \frac{4}{9}x$

(3) $2x^2 - \begin{pmatrix} x & & +4 \\ & \times & \\ \frac{5}{3}x^2 & & +\frac{4}{9}x \end{pmatrix} = 2x^2 - \frac{4}{9}x^2 - \frac{20}{3}x^2 = \frac{-46}{9}x^2$

$\frac{-46}{9}x^2 + 3x^2 = -\frac{46}{27}$

(4) Remainder $= x + 2 - \begin{pmatrix} & & +4 \\ & \times & \\ \frac{4}{9} & & \frac{46}{27} \end{pmatrix} \begin{pmatrix} 4 \\ \\ -\frac{46}{27} \end{pmatrix}$

$$= x + 2 + \frac{46}{27}x - \frac{16}{9}x + \frac{184}{27}$$

$$= \frac{25}{27}x + \frac{238}{27}$$

Quotient $= \frac{5}{3}x^2 + \frac{4}{9}x - \frac{46}{27}$

(b) **Current Method:**

$$\begin{array}{l|l|l}
3x^2 + x + 4 & 5x^4 + 3x^3 + 2x^2 + x + 2 & \frac{5}{3}x^2 + \frac{4}{9}x - \frac{46}{27} \\
 & 5x^4 + \frac{5}{3}x^3 + \frac{20}{3}x^2 & \\
 & (-) \quad (-) \qquad (-) & \\
\hline
 & \frac{4}{3}x^3 - \frac{14}{3}x^2 + x & \\
 & \frac{4}{3}x^3 + \frac{4}{9}x^2 + \frac{16}{9}x & \\
 & (-) \quad (-) \quad (-) & \\
 & \qquad -\frac{46}{9}x^2 - \frac{7}{9}x + 2 & \\
 & \qquad -\frac{46}{9}x^2 - \frac{46}{27}x - \frac{184}{27} & \\
 & \qquad (+) \quad (+) \quad (+) & \\
 & \qquad\qquad \frac{25}{27}x + \frac{238}{27} &
\end{array}$$

(C) Working for base x=10

$$\begin{array}{r|l|l}
x+4 & 5x^4 + 3x^3 + 2x^2 + & x + 2 \\
 & \quad / \qquad / & \; / \\
3x^2 & \quad 2x^4 \quad x^3 & x^2 \\
\hline
 & x^2 + 7x + 0 & 14x+6 \\
 & Q_1 \quad Q_2 \quad Q_3 &
\end{array}$$

1) $5x^4 + 3x^2 = x^2, 2x^4$
$\qquad Q_1 \; R_1$

2) $2x^4 + 3x^{-3}$ _ $= 2x^4 + 3x^3 - x^3 = 2x^4 + 2x^3 = 2x^3(x+1) = 22x^3 (\because x = 10)$

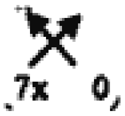
$22x^3 + 3x^2 = \bar{7}\bar{x}, x^3$
$\qquad Q_2 \; R_2$

3) $x^3 + 2x^2$ _

$= x^3 + 2x^3 - 7x^2 - 4x^2$
$= x^3 - 9x^2 = x^2(x - 9) = x^2$

$3x^2 = 0, \; x^2$
$\qquad Q_3 \; R_3$

(4) $x^2 + x + 2$ _



$= x^2 + x + 2 - 28x - 0 = x^2 - 27x + 2$
$= x(x-27) + 2 = 17x + 2$

Quotient $= x^2 + 7x$,
Reminder $= - 17x + 2$

Verification
$Q = x^2 + 7x$
$D = 3x^2 + x + 4$

$$\begin{array}{rr} & 3x^4 + 22x^3 + 11x^2 + 28x \\ \text{Remainder} = & \underline{\qquad\qquad\qquad\qquad 17x + 2} \\ & 3x^4 + 22x^3 + 11x^2 + 11x + 2 \end{array}$$

$= 3x^4 + (20+2)x^3 + (10+1)x^2 + (10+1)x +$

$= 3x^4 + (2x+2)x^3 + (x+1)\; x^2 + (x+1)\; x +$

$= 3x^4 + 2x^4 + 2x^3 + x^3 + x^2 + x + 2$

$= 5x^4 + 3x^3 + 2x^2 + x + 2$
= Dividend

(d) **Working for base x =2**

$$\begin{array}{r|lll|l} x+4 & 5x^4 + & 3x^3 & +2x^2 & +x+2 \\ & / & / & & / \\ 3x^2 & \underset{R_1}{\enclose{circle}{2x^4}} & \underset{R_2}{\enclose{circle}{0}} & & \underset{R_3}{\enclose{circle}{2x^2}} \\ \hline & \underset{Q_1}{1x^2} + & \underset{Q_2}{2x} & \underset{Q_3}{-2} & 2x^2 - 5x + 10 \end{array}$$

(1) $5x^4 + 3x^2 = \underset{Q_1}{1\,x^2}, \underset{R_1}{2x^4}$

(2) $2x^4 + 3x^3 - \begin{pmatrix} x \\ \uparrow \\ x^2 \end{pmatrix} = 2x^4 + 2x^1 = 2x^3\,(x+1) = 6x^1$

$6x^3 + 3x^2 = \underset{Q_2}{2x}, \underset{R_2}{0}$

(3) $2x^2 - \begin{pmatrix} x & & 4 \\ & \times & \\ x^2 & & 2x \end{pmatrix} = 2x^4 - 6\,x^2 = -\ 4x^2$

$-4x^2 + 3x^2 = \underset{Q_3}{-2}, \underset{R_3}{2x^2}$

(4) $2x^2 + x + 2 - \begin{pmatrix} x & & 4 \\ & \times & \\ 2x & & -2 \end{pmatrix} - \begin{pmatrix} 4 \\ \uparrow \\ -2 \end{pmatrix}$

$= 2x^2 + x + 2 - 6x + 8$
$= 2x^2 - 5x + 10$

$\therefore$ Quotient $=$ $x^2 + 2x - 2$
Remainder $=$ $2x^2 \quad 5x + 10$

Verifying for base $x = 2$:

$$Q = x^2 + 2x - 2$$

$$\text{Divisor} = \underline{3x^2 + \ \ x + 4}$$

$$3x^4 + 7x^3 + \ 0x^2 + 6x - \ \ 8$$

$$\underline{\qquad\qquad 2x^2 - 5x + \ 10}$$

$$3x^4 + 5x^3 + \ 2x^2 + \ x + 2$$

$$= 3x^4 + (4 + 3)x^3 + 2x^2 + x + 2$$

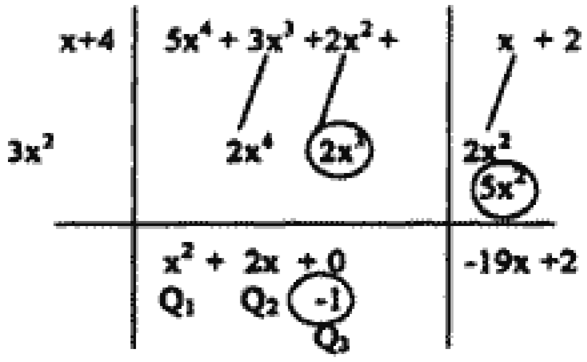
$$= 3x^4 + (2x + 3)x^3 + 2x^2 + x + 2$$

$$= 3x^4 + 2x^4 + 3x^3 + 2x^2 + x + 2$$

$$= 5x^4 + 3x^3 + 2x^2 + x + 2$$

= Dividend

(e) **Working for base $x = 3$**



(1) $5x^3 + 3x^2 = \underset{Q_1}{x^2}, \underset{R_1}{2x^4}$

(2) $2x^4 + 3x^3 = x^3\,(2x + 3) = 9x^3 \_ \begin{pmatrix} x \\ \uparrow \\ x^2 \end{pmatrix} = 9x^3 - x^3$

$8x^3 + 3x^2 = \underset{Q_2}{2x}, \underset{R_2}{2x^3}$

(3) $$2x^3 + 2x^2 = 2x^2(x+1) = 8x^2 - \left(\begin{array}{ccc} x & & 4 \\ & \nwarrow\!\!\!\nearrow & \\ x^2 & & 2x \end{array}\right) = 8x^2 - 6x^2 = 2x^2$$

$$\underset{Q_3}{2x^2 + 3x^2 = 0}\,,\ \underset{R_3}{2x^2}$$

(4) $$2x^2 + x + 2 - \left(\begin{array}{ccc} x & & 4 \\ & \nwarrow\!\!\!\nearrow & \\ 2x & & 0 \end{array}\right) - \begin{pmatrix} 4 \\ \uparrow \\ 0 \end{pmatrix} \begin{array}{l} = 2x^2 + x + 2 - 8x = 2x^2 - 7x + 2 \\ = x(2x-7) + 2 = -19x + 2 \end{array}$$

(5) $$5x^2 + x + 2 - \left(\begin{array}{ccc} x & & +4 \\ & \nwarrow\!\!\!\nearrow & \\ 2x & & -1 \end{array}\right) - \begin{pmatrix} 4 \\ \uparrow \\ 1 \end{pmatrix}$$

$$= 5x^2 + x + 2 + x - 8x + 4$$
$$= 5x^2 - 6x + 6$$
$$= x(5x - 6) + 6 = 9x + 6$$

Verifying for base x = 3

Quotient = $x^2 + 2x - 1$
Division = $3x^2 + x + 4$

$$3x^4 + 7x^3 + 3x^2 + 7x - 4$$

Remainder = $9x + 6$

Calculated $3x^4 + 7x^3 + 3x^2 + 16x + ② = 5x^4 + 3x^3 + 2x^2\, x + 2$

Given is x But calculated is +16x

$x \;+②$

$5x^2 = \underline{15x}$ Calculated difference $15x = 5x^2$ (x=3)

$8x^2$ After the e becomes $3x^2 + 5x^2 = 8x^2$

But given is $\underline{2x^2}$

$\underline{\text{②}x^2}$ $6x^2$

$\frac{2x^3}{9x^3} =$ $6x^2$ Difference $6x^2 = 2x^3$ (x = 3)

Now the Calculated value becomes $7x^3 + 2x^3 = 9x^3$

But the given value is $3x^3$

$\underline{\boxed{3x^3}}$ ∴ Difference is $= 6x^3$

$\underline{\boxed{2x^4}} = 6x^3$ $6x^3 = 2x^4$ (x=3)

$\boxed{5x^4}$ Now the calculated value $3x^4 + 2x^4 = 5x^4$

And the given is $5x^4$

$\therefore$ **Dividend** $= 5x^4 + 3x^3 + 2x^2 + x + 2$

i.e. , $3x^4 + 7x^3 + 3x^2 + 16x + 2 = 3x^4 + 7x^3 + 3x^2 + (15 + 1)x + 2$

$= 3x^4 + 7x^3 + 3x^2 + (5x + 1)x + 2$ $(15 = 5x$ when $x = 3)$

$= 3x^4 + 7x^3 + (3x^2 + 5x^2) + x + 2$

$= 3x^4 + 7x^3 + 8x^2 + x + 2$

$= 3x^4 + 7x^3 + (6+2)x^2 + x + 2$

$= 3x^4 + 7x^3 + (2x + 2)x^2 + x + 2$ $(6 = 2x$ when $x = 2)$

$= 3x^4 + (7x^3 + 2x^3) + 2x^2 + x + 2$

$= 3x^4 + 9x^3 + 2x^2 + x + 2$

$= 3x^4 + (6 + 3)x^3 + 2x^2 + x + 2$

$= 3x^4 + 2x^4 + 3x^3 + 2x^2 + x + 2$ $(6x^3 = 2x^4$ when $x = 3)$

$= 5x^4 + 3x^3 + 2x^2 + x + 2$

(b) Extension of evaluation of Quotients and Remainders

In the present investigation, the authors have used the straight division method as implied by swamiji where in a partition method is applied and the remainder is derived. The same method can be applied to workout the division continuously to write the result in the form of quotients and also to get the remainders at every stage of the division. The results stand the test of division at any stage of division

$(2 + 3x + 5x^2 + 3x^3) + (2 - x + 3x^2)$The details of the work by the authors are given below

$$\begin{array}{l|c|cccccc}
-x + 3x^2 & 2 + 3x + & 5x^2 & + 3x^3 & 0x^4 & 0x^5 & 0x^6 & \\
2 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\hline
 & 1 + 2x & 2x^2 & -\frac{1}{2}x^3 & -\frac{13}{4}x^4 & -\frac{7}{8}x^5 & +\frac{71}{10}x^6 & \\
 & Q_1 \; Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 &
\end{array}$$

Terms in the Remainder region are $5x^2 + 3x^3$
The absolute remainder is calculated as

$$0 + 5x^2 + 3x^3 - \begin{pmatrix} -x & 3x^2 \\ 1 & 2x \end{pmatrix}$$
$= 5x^2 + 3x^3 + 2x^2 - 3x^2 - 6x^3$
$R = 4x^2 - 3x^3$

Verification I with R

Divisor $= 2 - x + 3x^2$
Quotient $= \underline{1 + 2x + 0}$
$2 + 3x + x^2 + 6x^3$
$R_1 = \underline{4x^2 - 3x^3}$
Dividend$= 2 + 3x + 5x^2 + 3x^3$

Verification II

Continuation of the Division in the Remainder region term by term i.e., (a) $5x^1$ (b) $3x^2$

a) $5x^2 - \left(\begin{matrix} -x & 3x^2 \\ 1 & 2x \end{matrix} \right. = 4x^2 + 2 = 2x^2$ Q_3

When the term $4x^2$ is again divided by the P.D then the quotient is $2x^2$

b) $3x^3 - \begin{pmatrix} -x & & 3x^2 \\ & \times & \\ 2x & & 2x^2 \end{pmatrix} = -x^3 + 2 = -\frac{1}{2}x^3$ Q_4

This is the next quotient.

Division is stopped and the absolute remainder is evaluated as

$$\text{Remainder } R_2 = 0 - \begin{matrix} -x & +3x^2 \\ 2x^3 & -\frac{1}{2}x^3 \end{matrix} \Big| - \begin{matrix} 3x^2 \\ \uparrow \\ -\frac{1}{2}x^3 \end{matrix}$$

$$= -\frac{1}{2}x^4 - 6x^4 + \frac{3}{2}x^5 = -\frac{13}{2}x^4 + \frac{3}{2}x^5$$

one can verify at this stage also

Verification with R_2:

Divisor $= 2 - x + 3x^2$

Quotient $= 1 + 2x + 2x^2 - \frac{1}{2}x^3$

Q x Divisor $2 + 3x + 5x^2 + 3x^3 + \frac{13}{2}x^4 - \frac{3}{2}x^5$

$R_2 = -\frac{13}{2}x^4 + \frac{3}{2}x^5$

Thus the dividend (original) is obtained

Verification III: It is further proceeded to obtain three more quotients (Q_5, Q_6 and Q_7) starting with zero dividend terms in the reminder region

$$0x^4 - \begin{pmatrix} -x & +3x^2 \\ 2x^2 & -\frac{1}{2}x^3 \end{pmatrix} = -\frac{1}{2}x^4 - 6x^4 = -\frac{13}{2}x^4$$

$$-\frac{13}{2}x^4 + 2 = -\frac{13}{4}x^4 \qquad Q_5$$

$$0x^5 - \overset{-x}{-\frac{1}{2}x^3} \quad \overset{+3x^2}{-\frac{13}{4}x^4} \qquad -\frac{13}{4}x^5 - \frac{3}{2}x^5$$

$$= -\frac{7}{4}x^5$$

$$-\frac{7}{4}x^5 \div 2 = -\frac{7}{8}x^5$$

$$0x^6 - \overset{-x}{-\frac{13}{4}x^4} \quad \overset{+3x^2}{-\frac{7}{8}x^5} \qquad --\frac{7}{8}x^6 + \frac{39}{4}x^6$$

$$= \frac{71}{8}x^6$$

$$\frac{71}{8}x^6 \div 2 = \frac{71}{16}x^6 \qquad Q_7$$

At this stage again the absolute remainder is calculated

$$\text{Remainder } R_1 = 0 - \left[\; -\frac{7}{8}x^5 \times \overset{+3x^2}{-\frac{71}{16}x^6} \qquad \overset{3x^2}{\frac{71}{16}x^6} \right]$$

$$Q_7$$

$$-\frac{71}{16}x^7 + \frac{21}{8}x^7 \quad \frac{213}{16}x^8$$

$$R_1 = \frac{113}{16}x^7 \quad \frac{213}{16}x^8$$

<u>Verification</u> At the 3rd stage

$$\begin{array}{lllllllll} & & Q_1 & Q_2 & Q_3 & Q_4 & Q_5 & Q_6 & Q_7 \\ & \text{Quotient} & = 1 & +2x & +2x^2 & -\frac{1}{2}x^3 & -\frac{13}{4}x^4 & -\frac{7}{8}x^5 & +\frac{71}{16}x^6 \\ & \text{Divisor} & = 2 & -x & +3x^2 & +0 & +0 & +0 & +0 \\ \hline Q \times & \text{Divisor} & = 2 & +3x & +5x^2 & +3x^3 & +0x^4 & +0x^5 & +0x^6 \quad -\frac{113}{16}x^7 + \frac{213}{16}x^8 \\ & \text{Remainder } (R_1) & & & & & & & \qquad \frac{113}{16}x^7 \qquad \frac{213}{16}x^8 \end{array}$$

$$2 + 3^x + 5x^2 + 3x^3$$

One can proceed still further to get the quotients and absolute remainder as per ones own choice

(c) Division of Bipolynomials Straight Division Method :

The Usual straight division method developed by Swamiji by partitioning the divisor into Dhwajanka and part divisor is also extendable to Bipolynomials. The method described here is exactly on the basis of the method described by Swamiji and the details are given below with one example, where in all the different steps to obtain the final quotient and the remainder are clearly shown.

Problem1 :

The divisor is $5+7x+4y$

The dividend is $5+2x+4x^2+5x^3+3y+7xy+8x^2y+5y^2+8xy^2+6y^3$ (both the powers of x and y are taken in the ascending order)

A partition is shown in the divisor with $7x+4y$ as the dhwajanka and 5 as the part divisor. In accordance with this the dividend is partitioned at $8xy^2$, indicating that from 5 through $5y^2$ depicts the quotient region, whereas the two terms $8xy^2+6y^3$ come under remainder region.

In brief, the following details are worked out.

1) The quotients are obtained from the quotient region by applying straight division method. While doing so, it may be necessary that the terms of the dividend (this includes also the remainder region) may get modified.

2) The quotients are also to be sorted out from the remainder region.

3) The remainders are obtained under two different categories

   a) From the quotient (modified quotient) region.

   b) From the modified quotients in the remainder region as a unit which is clearly indicated in the following working details.

Step1 :

In the first instance, the table below shows the problem and partitions drawn in the divisor and dividend

	Quotient Region	Remainder Region
$7x + 4y$	$5 + 2x + 4x^2 + 5x^3 + 3y + 7xy + 8x^2y + 5y^2$	$+8xy^2 + 6y^3$
	$7x^2 - \frac{77}{5}x^3 \quad - 4y + 4xy - \frac{44}{5}x^2y + \frac{4}{5}y^2$	$\frac{-248}{25}xy^2 - \frac{116}{25}y^3$
	$\frac{7}{5}xy - \frac{434}{25}x^2y$	$- \frac{203}{25}xy^2$
	$11x^2 - \frac{52}{5}x^3 \quad - y \quad + \frac{62}{5}xy \; \frac{454}{25}x^2y \frac{29}{5}y^2$	$\frac{251}{25}xy^2 + \frac{34}{25}y^3$
5	0 0 0 0 0 0 0	
	$1 - x + \frac{11}{5}x^2 - \frac{52}{25}x^1 - \frac{1}{5}y + \frac{62}{25}xy - \frac{454}{125}x^2y + \frac{29}{25}y^2$	$- \frac{251}{125}xy^2 + \frac{34}{125}y^3$
	$Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_6 \quad Q_7 \quad Q_8$	$Q_9 \quad Q_{10}$

Here 5 acts as the part divisor (PD) and 7x + 4y as the Dhwajanka and is used in multiplication. In the straight division the partition in the dividend is shown by counting the same number of terms from the right end of the dividend towards left equivalent to the number of terms in the Dhwajanka

Step 2 :

The division is carried out term by term of the dividend by obtaining the corresponding quotients through the formation of new dividends with the help of Dhwajanka These new dividends are then divided by the part divisor (PD) to obtain the final quotients.

The first term of the dividend 5 is to be divided by PD, 5. The result is 1 shown exactly below 5 in the answer line as Q_1

The Intermediate dividend (ID) is 0+2x = 2x ID is converted into the new dividend through working in the following way

$$2x - \left(\begin{array}{c} 7x \\ \uparrow \\ 1 \end{array}\right) = -5x$$

The new dividend -5x has to be divided by PD, 5 to get the corresponding quotient Q_2

$-5x + 5 = -x\ (Q_2)$.

Step 3 :

The next ID is $0 + 4x^2 = 4x^2$

$$4x^2 - \left(\begin{array}{ccc} 7x & & 4y \\ & \nwarrow\!\!\!\!\nearrow & \\ 1 & & -x \end{array}\right) = 11x^2 - 4y$$

Now $11x^2$ is to be divided by 5 to represent the quotient Q_3 under x^2 – term

$11x^2 + 5 = \frac{11}{5}x^2\ (Q_3)$

-4y actually has to be added to the term 3y of the dividend thus the y-term gets modified to –y (i.e., - 4y + 3y= -y)

This is the modified dividend to be considered for division under the term y

The placement is shown in the table concerned with the problem

Step 4:

Let us consider the x^3-term

$0+5x^3 = 5x^3$ as ID. The new dividend is $5x^3 - \left(\begin{array}{ccc} 7x & & 4y \\ & \nwarrow\!\!\!\!\nearrow & \\ -x & & \frac{11}{5}x^2 \end{array}\right) = -\frac{52}{5}x^3 + 4xy$

Thus the term, x^3 now is $-\frac{52}{5}x^3$

This is divided by 5 to get the corresponding quotient $Q_4 = -\frac{52}{25}x^3$

The 4xy is now to be added to the term 7xy to get the modified dividend.

Step 5 :

The modified y-term is –y (step 3). The corresponding new dividend obtained is

$$-y - \begin{pmatrix} 7x & & 4y \\ & \times & \\ \frac{11}{5}x^2 & & -\frac{52}{25}x^3 \end{pmatrix} = -y + \frac{364}{25}x^4 - \frac{44}{5}x^2y$$

As y is not further simplified, it can be divided by 5 to get the corresponding quotient (-y/5) as (Q_5).

There is no x^4-term in the dividend so it can be taken to represent the remainder (R_1) = $\frac{364x^4}{25}$

The result $-\frac{44}{5}x^2y$ is to be added to the corresponding term $8x^2y$ of the dividend which is finally modified to (step 7).

Step 6:

Consider the modified xy-term $7xy + 4xy = 11xy$

The new dividend of the modified term is

$$11xy - \begin{pmatrix} 7x & & 4y \\ & \times & \\ -\frac{52}{25}x^3 & & -y/5 \end{pmatrix} = -\frac{62}{5}xy + \frac{208}{25}x^3y$$

xy is term divided by 5 to get the quotient (62/25)xy (Q_6).

The term $\frac{208}{25}x^3y$ can be taken to be the remainder R_2 as x^3y -term is not in the dividend.

Step 7:

The x^2y- term is modified as

$$8x^2y - \frac{44}{5}x^2y = -\frac{4}{5}x^2y$$

The new dividend is further simplified as

$$\frac{4}{5}x^2y - \begin{pmatrix} 7x & & 4y \\ & \times & \\ -\frac{y}{5} & & \frac{62}{25}xy \end{pmatrix} = -\frac{454}{25}x^2y + \frac{4}{5}y$$

The simplified term of x^2y with the addition is $(-454/25)\,x^2y$ This is divided by 5 to get the corresponding quotient, $(-454/125)x^2y$ (Q_7)

The term $(4/5)y^2$ can be simplified with the corresponding y^2-terms of the dividend

Step 8 : The modified y^2-term is $5y^2 + \frac{4}{5}y^2 = \frac{29}{5}y^2$

The new dividend is $\frac{29}{5}y^2 - \begin{pmatrix} 7x & & 4y \\ & \times & \\ \frac{62}{25}xy & & -\frac{454}{125}x^2y \end{pmatrix} = \frac{29}{5}y^2 + \frac{3178}{125}x^1y - \frac{248}{25}xy^2$

$\frac{29}{5}y^2$ can be divided by 5 to get the corresponding quotient $\frac{29}{25}y^2$ (Q_8)

The term $(-248/25)\,xy^2$ is added to the corresponding $8xy^2$ term of the dividend and shown in the remainder region.

The term $(3178/125)\,x^3y$ is the remainder R_3 as x^3y term is not present in the dividend

Step 9: Remainder region

Consider the modified xy^2-term $8xy^2 - \frac{248}{25}xy^2 = -\frac{48}{25}xy^2$

The new dividend is

$$\frac{48}{25}xy^2 - \begin{pmatrix} 7x & & 4y \\ & \times & \\ -\frac{454}{125}x^2y & & \frac{29}{25}y^2 \end{pmatrix} = -\frac{251}{25}xy^2 + \frac{1816}{125}x^2y^2$$

This result of xy^2-term is divided by 5, the quotient $(-251/125)xy^2$ (Q_9) is worked out from the terms of the remainder region. The term $(1816/125)\,x^2y^2$ is to be considered as the remainder (R_4) as the dividend has no such term

Step 10:

The new dividend is concerned with y^3-term

$$6y^3 - \begin{pmatrix} 7x & \nwarrow \quad \nearrow & 4y \\ & \times & \\ \frac{29}{25}y^2 & & -\frac{251}{125}xy^2 \end{pmatrix} = 6y^3 - \frac{116}{25}y^3 + \frac{1757}{125}x^2y^2$$

There is an addition of y^3-term by $-\frac{116}{25}y^3$, so that the simplified result is

$$6y^3 - \frac{116}{25}y^3 = \frac{34}{25}y^3$$

This is divided by 5 to get Q_{10} as $\frac{34}{125}y^3$

In addition there is a term $\frac{1757}{125}x^2y^2$ which can be taken to be the remainder, R_5

which when added to the similar term R_4, $\frac{1816}{125}x^2y^2$ results in $\frac{3573}{125}x^2y^2$

At this stage, let us write down the results obtained as quotients and remainders

$$\text{Quotient} = 1 - x + \frac{11}{5}x^2 - \frac{52}{25}x^3 - \frac{y}{5} + \frac{62}{25}xy - \frac{454}{125}x^2y + \frac{29}{25}y^2 - \frac{251}{125}xy^2 + \frac{34}{125}y^3$$

$$\begin{array}{llllll} \text{Remainder} = & \frac{364}{25}x^4 + & \frac{208}{25}x^3y + & \frac{3178}{125}x^3y + & \frac{1816}{125}x^2y^2 + & \frac{1757}{125}x^2y^2 \\ & R_1 & R_2 & R_3 & R_4 & R_5 \end{array}$$

R_1, R_2, R_3 are those obtained from dividend terms in the quotient region where as R_4, R_5 are from remainder region.

Step 11:

To obtain the remaining remainders from the quotients in the modified remainder region, for example, the two modified quotients obtained in the remainder region are

$$-\frac{251}{125}xy^2 + \frac{34}{125}y^3$$

The remainders from the above two are obtained in the following manner

$$) - \begin{pmatrix} 7x & & 4y \\ & \times & \\ \frac{-251}{125}xy^2 & & \frac{34}{125}y^3 \end{pmatrix} - \begin{pmatrix} 4y \\ \uparrow \\ \frac{34}{125}y^3 \end{pmatrix}$$

$$\underset{R_6}{\frac{1004}{125}xy^3} - \underset{R_7}{\frac{238}{125}xy^3} - \underset{R_8}{\frac{136}{125}y^4}$$

The remainders are from R_1 to R_8 put to together
Straight division method as applied to Bipolynomials is tested by the rule
(Quotient)(divisor) + Remainder is the given dividend.

<u>Problem 2</u> :

Another example is worked out as follows

$(2+4x+6x^2+4y+8xy+x^2y) + (2+x+x^2+2y+3xy)$

$$\begin{array}{l|l} x + x^2 + 2y + 3xy & 2 + 4x + 6x^2 + 4y + 8xy + x^2y \\ 2 & \quad\ 0 \quad\ 0 \quad\ 0 \quad\ 0 \quad\ 0 \\ \hline & 1 + \frac{3}{2}x + \frac{7}{4}x^2 + y + \frac{xy}{2} - \frac{17}{4}x^2y \\ & Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6 \end{array}$$

Step 1: $2 + 2 = 1\ (Q_1)$

Step2: $4x - \begin{pmatrix} x \\ \uparrow \\ 1 \end{pmatrix} = 3x$

$3x + 2 = \frac{3}{2}x\ (Q_2)$

Step 3: $6x^2 - \begin{pmatrix} x & & x^2 \\ & \times & \\ 1 & & \frac{3x}{2} \end{pmatrix} = \frac{7}{2}x^2$

$\frac{7}{2}x^2 + 2 \qquad x^2\ (Q_3)$

Step 4: $4y - \begin{pmatrix} x & x^2 & 2y \\ 1 & \frac{3}{2}x & \frac{7}{4}x^2 \end{pmatrix} = 2y - \frac{13}{4}x^2$

$2y \div 2 = y(Q_4)$, $R_1 = -\frac{13}{4}x^3$

Step 5: $8xy - \begin{pmatrix} x & x^2 & 2y & 3xy \\ 1 & \frac{3x}{2} & \frac{7x^2}{4} & y \end{pmatrix}$

$= xy - \frac{7}{4}x^4$

$xy \div 2 = xy/2\ (Q_5)$ $R_2 = (-7/4)\,x^4$

Step 6: $x^2y - \begin{pmatrix} x & x^2 & 2y & 3xy \\ \frac{3}{2}x & \frac{7}{4}x^2 & y & \frac{xy}{2} \end{pmatrix}$

$= -\frac{17}{2}x^2y \div 2 = (-17/4)x^2y(Q_6)$

Step 7: **Remaining Remainders**

$$0 - \begin{pmatrix} x & x^2 & 2y & 3xy \\ \frac{7}{4}x^2 & y & \frac{xy}{2} & -\frac{17}{4}x^2y \end{pmatrix} - \begin{pmatrix} & 2y & 3xy \\ & xy & -\frac{17}{4}x^2y \end{pmatrix}$$

$$- \begin{pmatrix} 2y & 3xy \\ \frac{xy}{2} & -\frac{17}{4}x^2y \end{pmatrix} - \begin{pmatrix} 3xy \\ -\frac{17}{4}x^2y \end{pmatrix}$$

$$= -\frac{3}{2}x^3y - 2y^2 + \frac{17}{4}x^4y - 4xy^2 + 7x^2y^2 + \frac{51}{4}x^3y^2$$

R_3 R_4 R_5 R_6 R_7 R_8

Verification

Quotient $Q = 1 + \frac{3}{2}x + \frac{7}{4}x^2 + y + \frac{xy}{2} - \frac{17}{4}x^2y$

Divisor $D = 2 + x + x^2 + 2y + 3xy + 0$

Q X D $= 2 + 4x + 6x^2 + 4y + 8xy + x^2y + \frac{13}{4}x^3 + \frac{7}{4}x^4 + \frac{3}{2}x^3y + 2y^2 - \frac{17}{4}x^4y$
$+ 4xy^2 - 7x^2y^2 - \frac{51}{4}x^3y^2$

Remainder $= -\frac{13}{4}x^3 - \frac{7}{4}x^4 - \frac{3}{2}x^3y - 2y^2 + \frac{17}{4}x^4y - 4xy^2 + 7x^2y^2 + \frac{51}{4}x^3y$

Q X D+ R $= 2 + 4x + 6x^2 + 4y + 8xy + x^2y$ = Dividend

Problem 3 :

Divide $(3 + 4x + x^2 + 2x^3 + 2x^4 + 4y + 17xy + 12x^2y + 2x^3y + 10x^4y + 4y^2 + 7xy^2 + 20x^2y^2 + 9x^3y^2 + 5x^4y^2 + 4y^3 - 5xy^3 + x^2y^3 + 3x^3y^3 + y^4 - xy^4 - 3x^2y^4 - 4x^3y^4 - 2x^4y^4)$
by $(3 - 2x + 2x^2 + 4y + 2x^2y + y^2 + xy^2 + x^2y^2)$

$-2x+2x^2+4y+2x^2y+y^2+xy^2+x^2y^2$ | $3+4x+x^2+2x^3+2x^4+4y+17xy+12x^2y+2x^3y+10x^4y+4y^2+7xy^2+20x^2y^2$

$-4y-8xy \quad -2x^2y \quad -4x^3y-2x^4y \quad -y^2-3xy^2-4x^2y^2$

$\text{—} \quad \text{—} \quad \underline{-4x^2y} \quad \text{—} \quad \text{—} \quad \text{—} \quad \underline{-12xy^2}-\underline{16x^2y^2}$

$0.y \quad 9xy \quad 6x^2y \quad -2x^3y \quad 8x^4y \quad 3y^2 \quad -8xy^2 \quad 0x^2y^2$

3 | $0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$1+2x+x^2+0.x^3+0.x^4+0y+3xy+4x^2y+0x^3y+0x^4y+y^2-2xy^2-2x^2y^2$

Continued

$9x^3y^2+5x^4y^2+4y^3-5xy^3+x^2y^3+3x^3y^3+y^4-xy^4-3x^2y^4-4x^3y^4-2x^4y^4$

$-3x^3y^2-x^4y^2-4y^3-3xy^3-7x^2y^3-7x^3y^3-y^4-xy^4+3x^2y^4+4x^3y^4+2x^4y^4$

$\underline{-6x^3y^2}-\underline{8x^4y^2} \quad \text{—} \quad +\underline{xy^3}-2x^2y^3+\underline{4x^3y^3} \quad \text{—} \quad \text{—} \quad \text{—} \quad \text{—} \quad \text{—}$

$0x^3y^2 \quad -4x^4y^2 \quad 0.y^3 \quad 0.xy^3 \quad +8x^2y^3 \quad 0.x^3y^3 \quad y^4 \quad 0.xy^4 \quad 0.x^2y^4 \quad 0.x^3y^4 \quad 0.x^4y^4$

$0x^2y^3$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$0x^3y^2+0.x^4y^2+0.y^3+0.xy^3+0.x^2y^3+0.x^3y^3+0y^4+0.xy^4+0.x^2y^4+0.x^3y^4+0.x^4y$

$-4x^4y^2 \quad +4x^4y^3$

$R_1 \quad R_2$

1. $3 + 3 = 1$

2. $4x - \begin{pmatrix} -2x \\ \uparrow \\ 1 \end{pmatrix} = 4x + 2x = 6x + 3 = 2x$

3. $x^2 - \begin{pmatrix} -2x & & 2x^2 \\ & \times & \\ 1 & & 2x \end{pmatrix} = x^2 + 4x^2 - 2x^2 = 3x^2 + 3 = x^2$

4 $2x^3 - \begin{pmatrix} -2x & 2x^2 & 4y \\ 1 & 2x & x^2 \end{pmatrix} = 2x^3 + 2x^3 - 4y - 4x^3$

$= -2x^3 + 2x^3 - 4y = -4y + 0\,x^3$

$0\,x^3 + 3 = 0.x$

5. $2x^4 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y \\ 1 & 2x & x^2 & 0 \end{pmatrix} = 2x^4 - 2x^2y - 2x^4 - 8xy = 0\,x^4 - 2x^2y - 8xy$

$\therefore 0.x^4 + 3 = 0\,x^4$

6. $0.y - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 \\ 1 & 2x & x^2 & 0 & 0 \end{pmatrix} = -y^2 - 4x^3y - 4x^2y + 0.y$

7. $9xy - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 \\ 1 & 2x & x^2 & 0 & 0 & 0 \end{pmatrix} = 9xy - xy^2 - 2xy - 2x^4y$

$= 9xy - 3xy^2 - 2x^4y$

$9xy + 3 = 3xy$

8. $6x^2y - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 1 & 2x & x^2 & 0 & 0 & 0 & 3xy \end{pmatrix}$

$= 6x^2y + 6x^2y - x^2y^2 - 2x^2y^2 - x^2y^2 = 12x^2y - 4x^2y^2 ; \therefore 12x^2y + 3 = 4x^2y$

9 $-2x^3y - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 2x & x^2 & 0 & 0 & 0 & 3xy & 4x^2y \end{pmatrix}$

$= -2x^3y + 8x^3y - 2x^3y^2 - 6x^3y - x^3y^2 = 0x^3y - 3x^3y^2 \quad \therefore 0.x^3y + 3 = 0.x^3y$

10. $8x^4y - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ x^2 & 0 & 0 & 0 & 3xy & 4x^2y & 0 \end{pmatrix}$

$= 8x^4y - x^4y^2 - 8x^4y - 12xy^2 = 0x^4y - x^4y^2 - 12xy^2 \quad \therefore 0.x^4y + 3 = 0.x^4y$

11. $3y^2 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & 0 & 3xy & 4x^2y & 0 & 0 \end{pmatrix}$

$= 3y^2 - 16x^2y^2 - 6x^3y^2 \qquad \therefore 3y^2 + 3 = y^2$

12. $-8xy^2 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & 3xy & 4x^2y & 0 & 0 & y^2 \end{pmatrix}$

$= -8xy^2 + 2xy^2 - 3xy^3 - 8x^4y^2 = -6xy^2 - 3xy^3 - 8x^4y^2$

$-6xy^2 + 3 = -2xy^2$

13. $0x^2y^2 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 3xy & 4x^2y & 0 & 0 & y^2 & -2xy^2 \end{pmatrix}$

$= 0x^2y^2 - 4x^2y^2 - 2x^2y^2 - 3x^2y^1 - 4x^2y^3 = -6x^2y^2 - 7x^2y^3$

$\therefore -6x^2y^2 + 3 = -2x^2y^2$

14. $0 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 3xy & 4x^2y & 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 \end{pmatrix}$

$= -4x^3y^2 - 3x^3y^3 + 4x^3y^2 - 4x^3y^3 - 4y^3 = -7x^3y^3 - 4y^3 + 0.x^3y^2$

15. $-4x^4y^2 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 4x^2y & 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 & 0 \end{pmatrix}$

$= -4x^4y^2 - 4x^4y^3 + 4x^4y^2 + 8xy^3 - 2x^2y^3 = 0.x^4y^2 - 4x^4y^3 + 8xy^3 - 2x^2y^3$

$\therefore 0x^4y^2 + 3 = 0x^4y^2$

16 $0\,y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & y^2 & -2xy^2 & -2x^2y^2 & 0 & 0 \end{pmatrix}$

$= 0.y^3 + 8\,x^2y^3 - y^4 + 4x^3y^3 \qquad \therefore 0.y^1 + 3 = 0.y^3$

17 $0\,xy^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & y^2 & -2xy & -2x^2y^2 & 0 & 0 & 0 \end{pmatrix}$

$= 0\,xy^3 - xy^4 + 2xy^4 + 4x^4y^3 = 0\,xy^3 + xy^4 + 4x^4y^1 \quad \therefore 0\,xy^3 + 3 = 0\,xy^3$

18 $0.x^2y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ y^2 & -2xy^2 & -2x^2y^2 & 0 & 0 & 0 & 0 \end{pmatrix}$

$= 0.x^2y^3 - x^2y^4 + 2x^2y^4 + 2x^2y^4 = 0.x^2y^3 + 3x^2y^4 \qquad \therefore 0.x^2y^3 + 3 = 0.x^2y$

19 $0.x^3y^3 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ -2xy^2 & -2x^2y^2 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$= 0.x^3y^3 + 2x^3y^4 \qquad \therefore 0\,x^3y^1 + 3 = 0\,x^3y^3$

20 $0.y^4 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y^2 & y^2 & xy^2 & x^2y^2 \\ -2x^2y^2 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$= 0.y^4 + 2x^4y^4 \qquad . 0.y^4 + 3 = 0\,y^4$

21 $0.xy^4 - \begin{pmatrix} -2x & 2x^2 & 4y & 2x^2y & y^2 & xy^2 & x^2y^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$= 0.xy^4 - 0 \qquad \therefore 0.xy^4 + 3 = 0\,xy$

The remaining Quotients are Zeros

Final Quotient $= 1 + 2x + x^2 + 3xy + 4x^2y + y^2 - 2xy^2 - 2x^2y^2$

Final Remainder $= 0$

(d) Straight Division Method for three Variables

Problem:4 Divide $(5+2x+3y+4z+2xy+3xz+4yz+5x^2+6y^2+7z^2+2x^2y+3x^2z+4y^2x+9y^2z+5z^2x+4z^2y+6x^3+8y^3+3z^3+5x^2y^2+3y^2z^2+4x^2z^2)$
by $(5+7x+4y+2z)$

$7x+4y+2z$	5 +	$2x$ +	$3y$ +	$4z$ +	$2xy$ +	$3xz$ +	$4yz$ +	$5x^2$ +	$6y^2$ +	$7z^2$ 4 +
			$-4y$ $-y$	$-2z$ $2z$	$4xy$ $\frac{7}{5}xy$ $\frac{37}{5}xy$	$2xz$ $\frac{-14}{5}xz$ $\frac{11}{5}xz$	$2yz/5$ $\frac{-8}{5}yz$ $\frac{14}{5}yz$	$7x^2$ $12x^2$	$\frac{4}{5}y^2$ $\frac{34}{5}y^2$	$\frac{4}{5}z^2$ $\frac{31}{5}z^2$
5		0	0	0	0	0	0	0	0	0
Quotient line	1 (Q_1)	$-x$ (Q_2)	$-\frac{y}{5}$ (Q_3)	$+\frac{2z}{5}$ (Q_4)	$+\frac{37}{25}xy$ (Q_5)	$+\frac{11}{25}xz$ (Q_6)	$+\frac{14}{25}yz$ (Q_7)	$+\frac{12}{5}x^2$ (Q_8)	$+\frac{34}{25}y^2$ (Q_9)	$+\frac{31}{25}z^2$ (Q_{10})
Remainder line								$\frac{-74}{25}xyz$ (R_1) $\frac{-98}{25}xyz$ (R_2) $\frac{-44}{25}xyz$ (R_3)		

Continuation	$2x^2y$ +	$3x^2z$ +	$4y^2x$ +	$9y^2z$ +	$5z^2x$ +	$4z^2y$ +
	$\frac{-259}{25}x^2y$ $\frac{-48}{5}x^2y$ $\frac{-449}{25}x^2y$	$\frac{-77}{25}x^2z$ $\frac{-24}{5}x^2z$ $\frac{-122}{25}x^2z$	$\frac{-148}{25}xy^2$ $\frac{-238}{25}xy^2$ $\frac{-286}{25}xy^2$	$\frac{-56}{25}y^2z$ $\frac{-68}{25}y^2z$ $\frac{101}{25}y^2z$	$\frac{-22}{25}xz^2$ $\frac{-217}{25}xz^2$ $\frac{-114}{25}xz^2$	$\frac{-28}{25}yz^2$ $\frac{-124}{25}yz^2$ $\frac{-52}{25}yz^2$
	0	0	0	0	0	0
Continuation of Quotient Line	$-\frac{449}{125}x^2y$ (Q_{11})	$-\frac{122}{125}x^2z$ (Q_{12})	$-\frac{286}{125}xy^2$ (Q_{13})	$+\frac{101}{125}y^2z$ (Q_{14})	$-\frac{114}{125}xz^2$ (Q_{15})	$-\frac{52}{125}yz^2$ (Q_{16})
		$\frac{3143}{125}x^3y$ (R_4)	$+\frac{854}{125}x^3z$ (R_5)	$+\frac{898}{125}zx^2y$ (R_6) $+\frac{483}{125}xyz^2$ (R_7)	$-\frac{707}{125}xzy^2$ (R_8) $+\frac{1144}{125}y^3x$ (R_9)	$+\frac{272}{125}zy^2x$ (R_{10}) $-\frac{404}{125}y^3z$ (R_{11})

Continuation	$6x^3$ +	$8y^3$ +	$3z^3$ +	$5x^2y^2$ +	$3y^2z^2$ +	$4x^2z^2$		
	$-\frac{84}{5}x^3$	$-\frac{136}{25}y^3$	$-\frac{62}{25}z^3$	$\frac{1796}{125}x^2y^2$	$-\frac{202}{125}y^2z^2$	$\frac{244}{125}x^2z^2$		
	$\frac{-54}{5}x^3$	$\frac{64}{25}y^3$	$\frac{13}{25}z^3$	$\frac{2002}{125}x^2y^2$	$\frac{208}{125}y^2z^2$	$\frac{798}{125}x^2z^2$		
				$\frac{4423}{125}x^2y^2$	$\frac{381}{125}y^2z^2$			
	0	0	0	0	0	0		
Continuation of Quotient Line	$-\frac{54}{25}x^3$ (Q_{17})	$+\frac{64}{125}y^3$ (Q_{18})	$+\frac{13}{125}z^3$ (Q_{19})	$+\frac{4423}{625}x^2y^2$ (Q_{20})	$+\frac{381}{625}y^2z^2$ (Q_{21})	$-\frac{1542}{625}x^2z^2$ (Q_{22})		
	$-\frac{364}{125}xz^2y$ (R_{12})	$-\frac{228}{125}xz^3$ (R_{14})	$\frac{104}{125}z^3y$ (R_{16})	$\frac{216}{25}yx^3$ (R_{18})	$-\frac{91}{125}xz^3$ (R_{20})	$-\frac{30961}{625}x^3y^2$ (R_{22})	$-\frac{52}{125}z^3y$ (R_{24})	$-\frac{2667}{625}xy^2$ (R_{26})
	$+\frac{456}{125}yxz^2$ (R_{13})	$\frac{378}{25}x^4$ (R_{15})	$-\frac{448}{25}xy^3$ (R_{17})	$\frac{108}{25}x^3z$ (R_{19})	$-\frac{256}{125}y^4$ (R_{21})	$-\frac{128}{125}xy^3$ (R_{23})	$-\frac{26}{125}z^4$ (R_{25})	$-\frac{17692}{625}x^2$ (R_{27})

Excess Remainders

$-\frac{8846}{125}x^2y^2z$ (R_{28}) $\quad -\frac{1524}{625}y^3z^2$ (R_{29})

$-\frac{10794}{125}x^3z^2$ (R_{30}) $\quad -\frac{762}{625}y^2z^3$ (R_{31}) $\quad -\frac{6168}{625}x^2z^2y$ (R_{32})

$-\frac{3084}{625}x^2z^3$ (R_{33})

Verification : Dividend = (Quotient) (Divisor) + Remainder

$$Q = 1 \quad - x \quad - \frac{y}{5} \quad \frac{2z}{5} + \frac{37}{25}xy \quad \frac{11}{25}zx \quad \frac{14}{25}yz + \frac{12}{5}x^2 + \frac{34}{25}y^2 + \frac{31}{25}z^2 - \frac{449}{125}x^2y - \frac{122}{125}x^2z - \frac{286}{125}xy^2 + \frac{101}{125}y^2z - \frac{114}{125}xz^2$$

Div = 5 - 7x + 4y + 2z

$$5 \quad + 7x + 4y + 2z + \frac{37}{5}xz + \frac{14}{5}zx + \frac{8}{5}yz + 12x^2 + \frac{34}{5}y^2 \quad \frac{31}{5}z^2 \quad - \frac{449}{25}yx^2 - \frac{122}{5}zx^2 + \frac{238}{25}xy^2 + \frac{56}{25}y^2z - \frac{114}{25}xz^2$$

$$-5x \quad - y \quad + 2z \quad - \frac{7}{5}xy \quad - 2zx \quad - \frac{2}{5}yz \quad - 7x^2 \quad - \frac{4}{5}y^2 \quad \frac{4}{5}z^2 \quad \frac{48}{5}yx^2 \quad \frac{24}{5}x^2z \quad - \frac{286}{25}xy^2 + \frac{101}{25}zy^2 + \frac{22}{25}xz^2$$

$$5 \quad 2x \quad 3y \quad 4z \quad \underline{-4xy \quad + \frac{11}{5}zx \quad + \frac{14}{5}yz} \quad 5x^2 \quad 6y^2 \quad 7z^2 \quad \frac{\frac{259}{25}yx^2}{2x^2y} \quad \frac{\frac{77}{25}x^2z}{3x^2z} \quad \frac{\frac{148}{25}xy^2}{4xy^2} \quad \frac{+\frac{68}{25}zy^2}{9zy^2} \quad \frac{\frac{217}{25}xz^2}{5xz^2}$$

$$2xy \quad 3zx \quad 4yz$$

Excess terms

$$(1)\ \frac{74}{25}xyz \quad (2)\ \frac{44}{25}xyz \quad (3)\ \frac{98}{25}xyz \quad (4)\ -\frac{3143}{125}x^3y \quad (5) - \frac{898}{125}zx^2y \quad (6)\cdot\frac{854}{125}zx^3 \quad (7) - \frac{488}{125}xyz^2 \quad (8) + \frac{707}{125}zxy^3 \quad (9) - \frac{1144}{125}xy^3$$

$$(10) - \frac{572}{125}xy^2z\ (11) + \frac{404}{125}y^3z\ (12) - \frac{364}{125}xz^2y\ (13) - \frac{456}{125}yxz^3\ (14) - \frac{228}{125}xz^3 \quad (15) - \frac{378}{25}x^4 \quad (16) - \frac{104}{125}x^3y \quad (17) + \frac{448}{125}xy^3 \quad (18) - \frac{216}{25}yx^3$$

$$(19) - \frac{108}{25}x^3z \quad (20) + \frac{91}{125}xz^3 \quad (21) + \frac{256}{125}y^4$$

$$(22) + \frac{30961}{625}x^3y^2 \quad (23)\ \frac{128}{125}xy^3 \quad (24) + \frac{52}{125}x^3y$$

$$(25)\ \frac{26}{125}z^4 \quad (26) + \frac{2607}{625}xy^2z^2 \quad (27)\ \frac{17692}{625}x^2y^3$$

$$\frac{-52}{125}yz^2 \quad - \frac{54}{25}x^3 \quad + \frac{64}{125}y^3 \quad + \frac{13}{125}z^3 \quad + \frac{4423}{625}x^2y^2 \quad + \frac{381}{625}y^2z^2 \quad + \frac{1542}{625}x^2z^2$$

$$5 + 7x + 4y + 2z$$

$$(28)\ \frac{8846}{125}x^2y^2z \quad (29)\cdot\frac{1524}{625}y^3z^2 \quad (30)\ \frac{10794}{625}x^3z^2$$

$$(31)\ \frac{762}{625}y^2z^3 \quad (32)\ \frac{6168}{625}x^2zy \quad (33)\ \frac{3084}{625}x^2z^3$$

$$\frac{28}{25}yz^2 \quad + \frac{84}{25}x^3 \quad + \frac{136}{25}y^3 \quad + \frac{62}{25}z^3 \quad - \frac{1796}{125}x^2y^2 \quad + \frac{202}{125}y^2z^3 \quad - \frac{244}{125}x^2z^3$$

$$+ \frac{124}{25}yz^2 \quad - \frac{54}{5}x^3 \quad + \frac{64}{25}y^3 \quad + \frac{13}{25}z^3 \quad - \frac{2022}{125}x^2y^2 \quad - \frac{208}{125}y^2z^2 \quad - \frac{798}{125}x^2z^2$$

$$\frac{+\frac{52}{25}yz^2}{4yz^2} \quad \overline{6x^3} \quad \overline{8y^3} \quad \overline{3z^3} \quad \frac{+\frac{4423}{125}x^2y^2}{5x^2y^2} \quad \frac{+\frac{381}{125}y^2z^2}{3y^2z^2} \quad \frac{+\frac{1542}{125}x^2z^2}{4x^2z^2}$$

Working details of problem . 4 Page No

(1) $\frac{5}{5} = 1 \quad Q_1$

(2) $$2x - \begin{pmatrix} 7x \\ 1 \end{pmatrix} = -\frac{5x}{5} = -x \quad (Q_2)$$

(3) $$3y - \begin{pmatrix} 7x & 4y \\ 1 & -x \end{pmatrix} = 3y + 7x^2 - 4y = -y + 7x^2, \quad -y + 5 = \frac{-y}{5} \qquad (Q_3)$$

(4) $$4z - \begin{pmatrix} 7x & 4y & 2z \\ 1 & -x & \frac{-y}{5} \end{pmatrix} = 4z - 2z + \frac{7xy}{5} + 4xy = 2z + \frac{27}{5}xy, 2z + 5 = \frac{2z}{5} \ (Q_4)$$

(5) $$\left(\overset{*}{\frac{27xy}{5}} + \overset{\text{Div}}{2xy}\right) = \frac{37xy}{5} - \begin{pmatrix} 7x & 4y & 2z \\ -x & \frac{-y}{5} & \frac{2z}{5} \end{pmatrix} = \frac{37xy}{5} + \overset{*}{2xz} - \overset{*}{\frac{14xz}{5}} + \frac{4y^2}{5}; \quad \frac{37xy}{25} \quad (Q_5)$$

(6) $$\left(\overset{\text{Div}}{3xz} + \overset{*}{2xz} - \overset{*}{\frac{14xz}{5}}\right) = \frac{11xz}{5} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{-y}{5} & \frac{2z}{5} & \frac{37xy}{25} \end{pmatrix} = \frac{11xz}{5} - \overset{*}{\frac{259x^2y}{25}} + \overset{*}{\frac{2yz}{5}} - \overset{*}{\frac{8yz}{5}};$$

$$= \frac{11xz}{5} + 5 = \frac{11}{25}xz \quad (Q_6)$$

(7) $$\left(\overset{\text{Div}}{4yz} + \overset{*}{\frac{2yz}{5}} - \overset{*}{\frac{8yz}{5}}\right) = \frac{14yz}{5} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{2z}{5} & \frac{37xy}{25} & \frac{11xz}{25} \end{pmatrix} = \frac{14yz}{5} - \frac{77x^2y}{25} - \frac{4z^2}{5} - \frac{148xy^2}{25},$$

$$\therefore \frac{14yz}{5} + 5 = \frac{14yz}{25} \quad (Q_7)$$

* Terms carried over to their following proper terms,
Div – Original Dividend term

$$(8)\qquad (\overset{*}{5x^2}+\overset{\text{Div}}{7x^2})=12x^2-\begin{pmatrix}7x & 4y & 2z\\ \\ \frac{37xy}{25} & \frac{11xz}{25} & \frac{14yz}{25}\end{pmatrix}=12x^2-\underset{(R_2)}{\frac{98xyz}{25}}-\underset{(R_1)}{\frac{74xyz}{25}}-\underset{(R_3)}{\frac{44xyz}{25}};\ \frac{12x^2}{5}\ (Q_8)$$

$$(9)\qquad \left(\overset{\text{Div}}{6y^2}+\overset{*}{\frac{4y^2}{5}}\right)=\frac{34y^2}{5}-\begin{pmatrix}7x & 4y & 2z\\ \\ \frac{11xz}{25} & \frac{14yz}{25} & \frac{12x^2}{5}\end{pmatrix}=\frac{34y^2}{5}-\overset{*}{\frac{84x^3}{5}}-\overset{*}{\frac{22xz^2}{25}}-\overset{*}{\frac{56y^2z}{25}};\frac{34y^2}{25}\,(Q_9)$$

$$(10)\qquad (\overset{\text{Div}}{7z^2}+\overset{*}{\frac{4z^2}{5}})=\frac{31z^2}{5}\cdot\begin{pmatrix}7x & 4y & 2z\\ \\ \frac{14yz}{25} & \frac{12x^2}{5} & \frac{34y^2}{25}\end{pmatrix}=\frac{31z^2}{5}-\overset{*}{\frac{238xy^2}{25}}-\overset{*}{\frac{28yz^2}{25}}-\overset{*}{\frac{48x^2y}{5}};\frac{31z^2}{25}\ (Q_{10})$$

$$(11)\qquad \left(\overset{\text{Div}}{2x^2y}-\overset{*}{\frac{259x^2y}{25}}-\overset{*}{\frac{48x^2y}{5}}\right)-\begin{pmatrix}7x & 4y & 2z\\ \\ \frac{12x^2}{5} & \frac{34y^2}{25} & \frac{31z^2}{25}\end{pmatrix}=\overset{*}{\frac{-449x^2y}{25}}-\overset{*}{\frac{217xz^2}{25}}-\overset{*}{\frac{24x^2z}{5}}-\frac{136y^3}{25}$$

$$\therefore\frac{-449x^2y}{125}\quad (Q_{11})$$

$$(12)\qquad \left(\overset{\text{Div}}{3x^2z}-\overset{*}{\frac{77x^2y}{25}}-\overset{*}{\frac{24x^2y}{5}}\right)=\frac{-122x^2z}{25}-\begin{pmatrix}7x & 4y & 2z\\ \\ \frac{34y^2}{25} & \frac{31z^2}{25} & \frac{-449x^2y}{125}\end{pmatrix}=\frac{-122x^3z}{25}+\underset{(R_4)}{\frac{3143x^3y}{125}}-\overset{*}{\frac{68y^2z}{25}}-\overset{*}{\frac{124yz^2}{25}}$$

$$\therefore\frac{-122x^3z}{125}\quad (Q_{12})$$

$$(13)\quad \left(\overset{\text{Div}}{4y^2x} - \overset{*}{\frac{148xy^2}{25}} - \overset{*}{\frac{238xy^2}{25}}\right) = \frac{-286xy^2}{25} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{31z^2}{25} & \frac{-449x^2y}{125} & \frac{-122x^2z}{125} \end{pmatrix}$$

$$\frac{-286xy^2}{25} - \frac{62z^2}{25} + \frac{854x^3z}{125} + \frac{1796x^2y^2}{25} \;;\; \frac{-286xy^2}{125}\,(Q_{13})$$

(R_5)

$$(14)\quad \left(\overset{\text{Div}}{9y^2z} - \overset{*}{\frac{56y^2z}{25}} - \overset{*}{\frac{68y^2z}{25}}\right) = \frac{101y^2z}{25} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{-449x^2y}{125} & \frac{-122x^2z}{125} & -\frac{286xy^2}{125} \end{pmatrix}$$

$$\frac{101y^2z}{25} + \frac{2002x^2z^2}{125} + \frac{898x^2yz}{125} + \frac{488x^2yz}{125} \;,\; \frac{101y^2z}{125} \quad (Q_{14})$$

R_6 R_7

$$(15)\quad \left(\overset{\text{Div}}{5z^2x} - \overset{*}{\frac{22z^2x}{25}} - \overset{*}{\frac{217z^2x}{25}}\right) = \frac{-114z^2x}{25} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{-122x^2z}{125} & \frac{-286xy^2}{125} & -\frac{101y^2z}{125} \end{pmatrix}$$

$$\frac{-114yz^2}{25} - \frac{707xy^2z}{125} + \frac{244x^2z^2}{125} + \frac{1144xy^3}{125} \;,\; \frac{-114xz^2}{125} \quad (Q_{15}),\ 0$$

(R_8) (R_9)

$$(16)\quad \left(\overset{\text{Div}}{4yz^2} - \overset{*}{\frac{28yz^2}{25}} - \overset{*}{\frac{124yz^2}{25}}\right) = -\frac{52yz^2}{25} - \begin{pmatrix} 7x & 4y & 2z \\ \frac{-286xy^2}{125} & \frac{101y^2z}{125} & -\frac{114xz^2}{125} \end{pmatrix}$$

$$-\frac{52yz^2}{25} - \frac{798x^2z^2}{125} + \frac{572xy^2z}{125} - \frac{404y^3z}{125} \;;\; \therefore -\frac{52yz^2}{125} \quad (Q_{16}),\ 0$$

(R_{10}) (R_{11})

Div •

$$(17)\quad \left(6x^3-\frac{84x^3}{5}\right)=\frac{-54x^3}{25}-\begin{pmatrix}7x & 4y & 2z\\ \frac{101y^2z}{125} & -\frac{114xz^2}{125} & -\frac{52yz^2}{125}\end{pmatrix}$$

$$\frac{-54x^3}{5}+\frac{364xyz^2}{125}-\frac{202y^2z^2}{125}+\frac{456xyz^2}{125}\qquad \therefore \frac{-54x^3}{25}\quad (Q_{17})$$

(R_{12}) (R_{13})

Div •

$$(18)\quad \left(8y^3-\frac{136y^3}{25}\right)=\frac{64y^3}{25}-\begin{pmatrix}7x & 4y & 2z\\ \frac{-114xz^2}{125} & \frac{-52yz^2}{125} & -\frac{54x^3}{25}\end{pmatrix}$$

$$\frac{64y^3}{25}+\frac{378x^4}{25}+\frac{228xz^3}{125}+\frac{208y^2z^2}{125}\ ,\qquad \frac{64y^3}{125}\quad (Q_{18})$$

(R_{15}) (R_{14})

Div •

$$(19)\quad \left(3z^3-\frac{62z^3}{25}\right)=\frac{13z^3}{25}-\begin{pmatrix}7x & 4y & 2z\\ \frac{-52yz^2}{125} & \frac{-54x^3}{25} & \frac{64y^3}{125}\end{pmatrix}$$

$$\frac{13z^3}{25}-\frac{448xy^3}{125}+\frac{104yz^3}{125}+\frac{216yx^3}{25}\ ;\qquad \frac{13z^3}{125}\quad (Q_{19})$$

(R_{17}) (R_{16}) (R_{18})

Div • •

$$(20)\quad \left(5x^2y^2+\frac{1796x^2y^2}{125}+\frac{2002x^2y^2}{125}\right)=\frac{4423x^2y^2}{125}-\begin{pmatrix}7x & 4y & 2z\\ \frac{-54x^3}{25} & \frac{64y^3}{125} & +\frac{13z^3}{125}\end{pmatrix}$$

$$\frac{4423x^2y^2}{125}+\frac{108x^3z}{25}-\frac{91xz^3}{125}-\frac{256y^4}{125}\ ;\ \frac{4423x^2y^2}{625}\quad (Q_{20})$$

(R_{19}) (R_{20}) (R_{21})

Div

(21) $$\left(3y^2z^2 - \overset{*}{\frac{202y^2z^2}{125}} + \overset{*}{\frac{208y^2z^2}{125}}\right) = \frac{381y^2z^2}{125} - \left(\begin{array}{ccc} 7x & 4y & 2z \\ \frac{64y^3}{125} & \frac{13z^3}{125} & \frac{4423x^2y^2}{625} \end{array}\right)$$

$$\frac{381y^2z^2}{125} - \underset{(R_{22})}{\frac{30961x^3y^2}{625}} - \underset{(R_{23})}{\frac{128y^3z}{125}} - \underset{(R_{24})}{\frac{52yz^3}{125}} \;;\; \therefore \frac{381y^2z^2}{625} \quad (Q_{21})$$

Div

22) $$\left(4x^2z^2 + \overset{*}{\frac{244x^2z^2}{125}} + \overset{*}{\frac{798x^2z^2}{125}}\right) = \frac{1542x^2z^2}{125} - \left(\begin{array}{ccc} 7x & 4y & 2z \\ \frac{13z^3}{125} & \frac{4423x^2y^2}{625} & \frac{381y^2z^2}{625} \end{array}\right)$$

$$\frac{1542x^2z^2}{125} - \underset{(R_{26})}{\frac{2667xy^2z^2}{625}} - \underset{(R_{25})}{\frac{26z^4}{125}} - \underset{(R_{27})}{\frac{17692x^2y^3}{625}} \quad , \therefore + \frac{1542x^2z^2}{625} \quad (Q_{22})$$

(23) $$0 - \left(\begin{array}{ccc} 7x & 4y & 2z \\ \frac{4423x^2y^2}{625} & \frac{381y^2z^2}{625} & \frac{1542x^2z^2}{625} \end{array}\right)$$

$$-\underset{(R_{10})}{\frac{10794x^3z^2}{625}} - \underset{(R_{28})}{\frac{8846x^2y^2z}{625}} - \underset{(R_{29})}{\frac{1524y^3z^2}{625}}$$

(24) $$0 - \left(\begin{array}{cc} 4y & 2z \\ \frac{381y^2z^2}{625} & \frac{1542x^2z^2}{625} \end{array}\right) = -\underset{(R_{31})}{\frac{762y^2z^3}{625}} - \underset{(R_{32})}{\frac{6168x^2yz^2}{625}}$$

(25) $$0 - \left(\begin{array}{c} 2z \\ \frac{1542x^2z^2}{625} \end{array}\right) = -\underset{(R_{33})}{\frac{3084}{625}}x^2z^3$$

(e) Argumental Division applied to Bipolynomials :

An attempt is made to describe the methods of division of Bipolynomials by Bipolynomial using the array display of the terms of both dividend and divisor separately. The result is explained in the array display but the choice of the terms is at our disposal.

The simple relation between the dividend, divisor and the quotient is taken as (in the form of arrays)

Dividend Divisor Quotient (Designated)

$$\begin{array}{c|cccc} & 1 & x & x^2 & x^3 \ldots \\ \hline 1 & . & . & & \ldots\ldots \\ y & . & . & \ldots & . \\ y^2 & . & . & & \\ y^3 & & & . & \ldots \end{array} \quad \begin{array}{c|cccc} & 1 & x & x^2 & x^3 \ldots\ldots \\ \hline 1 & . & . & & \ldots\ldots \\ y & & & & \\ + y^2 & & & & \\ y^3 & & & & \\ & & . & & .. \\ & & & . & \end{array} \quad = \quad \begin{array}{c|cccc} & 1 & x & x^2 & x^3 \ldots\ldots \\ \hline 1 & a_0 & a_1 & a_2 & a_3 \ldots\ldots \\ y & b_0 & b_1 & b_2 & b_3 \ldots\ldots \\ y^2 & c_0 & c_1 & c_2 & c_3 \ldots\ldots \\ y^3 & d_0 & d_1 & d_2 & d_3 \ldots \\ . & & & . & \ldots \\ & . & & & \ldots \end{array}$$

Divisor x Quotient = Dividend

Quotient consists of terms derived from remainders also

We have adopted for the term remainder concept to any term that doesn't belong to the given dividend form.

Now the procedure is to collect the absolute term, co-efficient of $x, x^2, x^3 \ldots$ e.t.c $y, xy, x^2y, x^3y \ldots$ e.t.c, $y^2, y^2x, y^2x^2\, y^2x^3 \ldots$ e.t.c. Similarly the other powers.

In the actual multiplication of the given divisor and the designated quotient one has to equate the absolute term with the absolute term of the dividend, and co efficient s of the similar powers of the product terms with those of the dividend terms. Thus the designated terms $a_0, a_1, \ldots$ could be evaluated.

1 represents only absolute term

An example is worked out and the full details are given below

Dividend
$5 + 2x + 4x^2 + 5x^3 + 3y$
$+7xy + 8x^2y + 5y^2 + 8xy^2 + 6y^1$

Divisor
$5 + 7x + 4y$

Quotient

$$\begin{array}{c|ccccc} & 1 & x & x^2 & x^3 & x^4 \\ \hline 1 & 5 & 2 & 4 & 5 & 0 \\ y & 3 & 7 & 8 & 0 & 0 \\ y^2 & 5 & 8 & 0 & 0 & 0 \\ y^3 & 6 & 0 & 0 & 0 & 0 \end{array} \div \begin{array}{c|ccccc} & 1 & x & x^2 & x^1 & x^4 \\ \hline 1 & 5 & 7 & 0 & 0 & 0 \\ y & 4 & 0 & 0 & 0 & 0 \\ y^2 & 0 & 0 & 0 & 0 & 0 \\ y^3 & 0 & 0 & 0 & 0 & 0 \end{array} = \begin{array}{c|cccc} & 1 & x & x^2 & x^3 \\ \hline 1 & a_0 & a_1 & a_2 & a_3 \\ y & b_0 & b_1 & b_2 & b_3 \\ y^2 & c_0 & c_1 & c_2 & a_3 \\ y^3 & d_0 & d_1 & d_2 & d_3 \end{array}$$

(1) <u>Constant</u>

$5a_0 = 5 \Rightarrow \boxed{a_0 = 1}$

(2) <u>Coeff of x (a_1) :</u>
$x = k\,x,\ x.k$

$5a_1 + 7a_0 = 2$
or $5a_1 + 7 = 2 \Rightarrow \boxed{a_1 = -1}$

(3) <u>Coeff of x^2 (a_2):</u>
$x^2 = x.x,\ k.x^2$ Vice-versa

$5a_2 + 7a_1 = 4$

$\Rightarrow 5a_2 - 7 = 4 \quad \therefore \quad \boxed{a_2 = \frac{11}{5}}$

(4) <u>Coeff of x^3 (a_3)</u>
$x^3 = x.x^2,\ k.x^2$

$7a_2 + 5a_3 = 5$

$\frac{77}{5} + 5a_3 = 5$

$5a_3 = 5 - \frac{77}{5} = \frac{25 - 77}{5}$

$= -\frac{52}{5} \quad \therefore \quad \boxed{a_3 = -\frac{52}{25}}$

(5) <u>Coeff of y (b_0)</u>

$y = k.y, y.k$

$$5b_0 + 4a_0 = 3$$

$$\text{or} \quad \underline{5b_0 + 4} = 3$$

(6) <u>Coeff of xy (b_1)</u>

$xy = k.xy, x.y$ vice-versa

$$5b_1 + 4a_1 + 7b_0 = 7$$

$$5b_1 . 4 \quad - \frac{7}{5} \quad = 7$$

$$5b_1 - \frac{27}{5} \quad = 7$$

$$5b_1 = 7 + \frac{27}{-} = \frac{62}{-}$$

$$\therefore \boxed{b_1 = \frac{62}{5}}$$

(7) <u>Coeff of x^2y (b_2)</u>

$x^2y = x.xy, x^2y, k.x^2y$ vice-versa

$$4a_2 + 7b_1 + 0.b_0 + 5b_2 = 8$$

$$\frac{44}{5} + \frac{434}{25} + \quad 5b_2 = 8$$

$$\genfrac{}{}{0pt}{}{654}{25} - + 5b_2 \quad = 8$$

$$5b_2 = 8 - \frac{654}{25}$$

$$\genfrac{}{}{0pt}{}{200 - 654}{25} \quad \genfrac{}{}{0pt}{}{-454}{125} \quad \therefore \boxed{b_2 = \frac{-454}{125}}$$

(8) Coeff $x^3y(b_3)$

$x^3y = x.x^2y, x^2.xy, x^3 y, \ k x^3y,$ vice-versa

$x^3y = x.x^2y, x^2 xy, x^3 y \ldots$

$y. x^3 , k. x^3y$

$\therefore \ 5b_3 + 7b_2 + 0 + 0 + 4a_3 \qquad = 0$

$$5b_3 + 7\left(-\frac{454}{125}\right) + 4\left(-\frac{52}{25}\right) = 0$$

$$5b_3 = \frac{3178}{125} + \frac{208}{25}$$

$$5b_3 = \frac{3178 + 1040}{625} = \frac{4218}{625}$$

$$\therefore \quad \boxed{b_3 = \frac{4218}{625}}$$

(9) Coeff. $y^2 (c_0)$:

$y^2 = k.y^2, y.y$ vice-versa

$$5c_0 + 4b_0 = 5 \Rightarrow 5c_0 - \frac{4}{5} = 5$$

$$5c_0 = 5 + \frac{4}{5} \Rightarrow \boxed{\therefore c_0 = \frac{29}{25}}$$

(10) Coeff of $xy^2(c_1)$

$xy^2 = k.xy^2, x y^2, xy y$ vice-versa

$5c_1 + 7c_0 + 4b_1 \qquad = 8$

$$5c_1 + 7\left(\frac{29}{25}\right) + 4\left(\frac{62}{25}\right) = 8$$

$$5c_1 = 8 - \frac{451}{25} = \frac{200 - 451}{25} = -\frac{251}{25}$$

$$\boxed{\therefore c_1 = -\frac{251}{125}}$$

(11) Coeff. of $x^2y^2 (c_2)$.

$x^2y^2 = k x^2y^2, x^2 y^2, x.xy^2, xy xy, x^2y y$ vice-versa

$5c_2 + 7c_1 - 4b_2 = 0$

$$5c_2 - \frac{1757}{125} - \frac{1816}{125} = 0$$

$5c_2 = +\frac{3573}{125}$

$$\boxed{\therefore c_2 = +\frac{3573}{625}}$$

(12) coeff of $x^3y^2(c_3)$

$x^3y^2 = x\,(x^2y^2), x^2(xy^2), x^1(y^2), k(x^1y^2)$ vice-versa

$7c_2 + 4b_3 + 5c_3 = 0$

$+\frac{25011}{625} + \frac{16872}{625} + 5c_3 = 0$

$5c_3 = -\frac{41883}{625}$

$$\boxed{\therefore c_3 = -\frac{41883}{3125}}$$

(13) Coeff. $y^3\ (d_0)$:

$y^3 = y.y^2, y^2y$

$5d_0 + 4c \quad = 6$

$5d_0 + \frac{116}{25} = 6$

$5d_0 = \ 6 - \frac{116}{25} = \frac{150-116}{25} = \frac{34}{25}$

$$\boxed{\therefore d_0 = \frac{34}{125}}$$

(14) Coeff.of $xy^3\ (d_1)$.

$xy^3 = k(xy^3), x(y^3), xy(y^2), xy^2(y)$ vice-versa

$5d_1 + 7d_0 + 4c_1 = 0$

$5d_1 + \frac{238}{125} - \frac{1004}{125} = 0$

$5d_1 = +\frac{766}{125}$

$$\boxed{\therefore d_1 = +\frac{766}{625}}$$

(15) **Coeff. of x^2y^3 (d_2)**

$x^2y^3 = k.(x^2y^3), x^2(y^3), x(xy^3), x^2y(y^3), xy(xy^2), x^2y^2(y)$ vice-versa

$5d_2 + 0 + 0 + 0 + 7(d_1) + 0 + 0 + 0 + 0 + 0 + 0 + 4c_2 = 0$

$5d_2 + 7d_1 + 4c_2 = 0$

$$5d_2 + \frac{7(766)}{625} + \frac{4(3573)}{625} = 0$$

$$5d_2 = -\frac{19654}{625}$$

$$\boxed{d_2 = -\frac{19654}{3125}}$$

(16) **Coeff. of x^3y^2(d_3) :**

$x^3y^2 = k.(x^3y^2), x^3(y^2), x^2(xy^2), x(x^2y^2), x^3y^2(y), x^3y(y^2)$ vice-versa

$5d_3 + 0 + 0 + 0 + 0 + 0 + 7d_2 + 0 + 0 + 4c_3 + 0 + 0 = 0$

$5d_3 + 7d_2 + 4c_3 = 0$

$$5d_3 + 7\left(-\frac{19654}{3125}\right) + 4\left(-\frac{41883}{3125}\right)$$

$$5d_3 - \frac{305110}{15625} = 0$$

$$\boxed{d_3 = \frac{305110}{15625}}$$

Since $\frac{\textit{Divdend}}{\textit{Divisor}} = \textit{Quotient}$

⇒ Dividend = Quotient × Divisor the given problem by using D = Q x Divisor is written as follows

$$\text{Quotient} = \begin{array}{c|cccc} & 1 & x & x^2 & x^3 \\ \hline 1 & 1 & -1 & \frac{11}{5} & -\frac{52}{25} \\ y & -\frac{1}{5} & \frac{62}{5} & -\frac{454}{125} & \frac{4218}{625} \\ y^2 & \frac{29}{25} & -\frac{251}{125} & \frac{3573}{625} & -\frac{41883}{3125} \\ y^3 & \frac{34}{125} & \frac{766}{625} & -\frac{19654}{3125} & \frac{305110}{15625} \end{array} d$$

The same problem is worked out by the authors, using the general straight division method by writing down the problem in linear form and also considering partition, part divisor and Dhwajanka. A comparison of this with the values obtained by the Argumental division method is as follows.

Authors Straight Division	**Argumental Division Method**
Quotients:	**Quotients:**
$Q_1 = 1$	$Q_1 = 1$
$Q_2 = -x$	$Q_2 = -x$
$Q_3 = \frac{11}{5}x^2$	$Q_3 = \frac{11}{5}x^2$
$Q_4 = -\frac{52}{25}x^3$	$Q_4 = -\frac{52}{25}x^3$
$Q_5 = -\frac{y}{5}$	$Q_5 = -\frac{y}{5}$
$Q_6 = \frac{62}{25}xy$	$Q_6 = \frac{62}{25}xy$
$Q_7 = -\frac{454}{125}x^2y$	$Q_7 = -\frac{454}{125}x^2y$
$Q_8 = \frac{29}{25}y2$	$Q_8 = \frac{29}{25}y2$
$Q_9 = -\frac{251}{125}xy^2$	$Q_9 = -\frac{251}{125}xy^2$
$Q_{10} = \frac{34}{125}y^3$	$Q_{10} = \frac{34}{125}y^3$
1) $R_1 = \frac{364}{25}x^4$	Not extended to that power (x^4)
2) $R_2 + R_3 = \frac{4218}{125}x^3y$ (This becomes quotient (Q_{11}) when divided by P.D.5)	$Q_{11} = \frac{4218}{625}x^3y$
3) $R_4 + R_5 = \frac{3573}{125}x^2y^2$ (But when this is divided by P D 5 one gets the quotient (Q_{12}))	$Q_{12} = \frac{3573}{625}x^2y^2$
4) One will get this value of it is still extended to get quotient term x^3y^2	$Q_{13} = \frac{4883}{3125}x^3y^2$
5) $R_6 + R_7 = \frac{766}{125}xy^3$ (This when divided by 5 gives the quotient term Q 14.)	$Q_{14} = \frac{766}{625}xy^3$
6) One has to still extend to get quotient terms x^2y^3 and x^3y^3	$Q_{15} = -\frac{19654}{3125}x^2y^3$ $Q_{16} = -\frac{3051103}{15625}x^3y^3$
7) $R_8 = -\frac{136}{125}y^4$	Not extended to that power (y^4)

A crucial difference between two is observed as follows.

In the I Method, (i.e Straight Division) Verification can be carried at every stage of the division by considering the general rule, divisor x quotient + remainder = dividend.

Whereas in the II Method (i.e., Argumental Method, procedure given by British authors) describing the arrangement of dividend, divisor and the result in the form of arrays where in the remainder concept has not been included. It is found that the above verification rule is not directly applicable unless one has the idea of remainders and also one extends the calculations, to explain the excess terms in the process of verification.

(f) Successive division of Remainders :

A comparison of the results of the two methods (viz , Straight Division and Argumental method) leading to some interesting results of few extensions in quotients and remainders. In order to study such difference we extended the division of the remainders successively treating the set of remainders as new dividends by the divisor to obtain the new quotients and remainders using St Division method

It is interesting to note that division by the part divisor of any remainder gives the corresponding quotient and such successive divisions are attempted to on observe the new quotients and remainders

This process of division is unending and one can stop the working at a particular choice of the powers of polynomials The successive division is also compute programmed for problem.

Considering the remainders as the dividend one can continue the division this results in quotients and remainders. The process is further continued with the new set of remainders to get the author set of reminders one may continue This process to get the required quotient or remainders at the choice of the individual. Here the authors have done for three sets of remainders for the further division. (I, II, III)

I The remainders, which are obtained by St division by authors

R_1 $\frac{364}{25}x^4$

R_2 $\frac{208x^3y}{25}+\frac{3178x^3y}{125}=\frac{1040}{125}+\frac{3178}{125}=\frac{4218x^3y}{125}$

R_3 $\frac{1816x^2y^2}{125}+\frac{1757x^2y^2}{125}=\frac{3573x^2y^2}{125}$

R_4 $\frac{1004xy^3}{125}-\frac{238xy^3}{125}=\frac{766xy^3}{125}$

R_5 $-\frac{136y^4}{125}$

$$\begin{array}{l|ccccc|cccc}
 & R_1 & & R_2 & & R_3 & R_4 & & R_5 \\
7x+4y & \frac{364x^4}{25} & + & \frac{4218x^3y}{125} & + & \frac{3573x^2y^2}{125} & +\frac{766xy^3}{125} & - & \frac{136y^4}{125} \\
 & & & / & & / & / & & / \\
5 & & & 0 & & 0 & 0 & & 0 \\
\hline
 & \frac{364x^4}{125} & + & \frac{4218x^3y}{625} & + & \frac{3573x^2y^2}{625} & +\frac{766xy^3}{625} & - & \frac{136y^4}{625} \\
 & Q_1' & & Q_2' & & Q_3' & Q_4' & & Q_5'
\end{array}$$

(1) $$\frac{364x^4}{25}+5=\frac{364x^4}{125}(Q_1')$$

(2) $$\frac{4218x^3y}{125}-\left|\begin{array}{c}7x\\ \uparrow\\ \frac{364x^4}{125}\end{array}\right|=\frac{4218x^3y}{125}-\frac{2548x^5}{125}$$

$$\frac{4218x^3y}{125}+5=\frac{4218x^3y}{625}(Q'_2)\ \&\ -\frac{2548x^5}{125}(R_1')$$

(3) $$\frac{3573x^2y^2}{125}-\left|\begin{array}{cc}7x & 4y\\ \frac{364x^4}{125} & \frac{4218x^3y}{625}\end{array}\right|=\frac{3573x^2y^2}{125}-\frac{29526x^4y}{625}-\frac{1456x^4y}{125}$$

$$=\frac{3573x^2y^2}{125}-\frac{36806x^4y}{625}$$

$$\frac{3573x^2y^2}{125}+5=\frac{3573x^2y^2}{625}(Q_3'),\ -\frac{36806x^4y}{625}(R_2')$$

(4) $$\frac{766xy^3}{125}-\left|\begin{array}{cc}7x & 4y\\ \frac{4218x^3y}{625} & +\frac{3573x^2y^2}{625}\end{array}\right|$$

$$=\frac{766xy^3}{125}-\frac{25011x^3y^2}{625}-\frac{16872x^3y^2}{625}=\frac{766xy^3}{125}-\frac{41883x^3y^2}{625}$$

$$\cdot\ \frac{766xy^3}{625}(Q_4')\ \&\ -\frac{41883x^3y^2}{625}(R_1')$$

(5)
$$-\frac{136y^4}{125}\quad \begin{array}{cc|} 7x & 4y \\ \frac{3573x^2y^2}{625} & \frac{766xy^3}{625} \end{array} = -\frac{136y^4}{125}\quad \frac{5362x^2y^1}{625}\quad \frac{14292x^2y^1}{625}$$

$$-\frac{136y^4}{125}\quad \frac{19654x^2y^3}{625}$$

$$\therefore \frac{-136y^4}{625}(Q_4') \,\&\, -\frac{19654x^2y^3}{625}(R_4')$$

(6)
$$0\cdot\left|\begin{array}{cc} 7x & 4y \\ \frac{766xy^3}{625} & \frac{-136}{625}y^4 \end{array}\right| \qquad \begin{array}{c} 4y \\ \\ \frac{136y^4}{625}\,) \end{array}$$

$$0+\frac{952}{625}xy^4-\frac{3064}{625}xy^4+\frac{544}{625}y^5$$

$$\underset{R_5'}{\frac{-2112xy^4}{625}}+\underset{R_6'}{\frac{544}{625}y^5}$$

$$\therefore\ Q' = Q_1'+Q_2'+Q_3'+Q_4'+Q_5' - \frac{364x^4}{125}+\frac{4218x^3y}{625}+\frac{3573x^2y^2}{625}+\frac{766xy^3}{625}-\frac{136y^4}{625}$$

$$R' = R_1'+R_2'+R_3'+R_4'+R_5'+R_6' = -\frac{2548x^5}{125}-\frac{36806x^4y}{625}-\frac{41883x^3y^2}{625}-\frac{19654x^2y^3}{625}-\frac{2112}{625}xy^4+\frac{544}{625},$$

II In order to obtain further quotients, R_1, R_2, Are to the dividend with the divisor which is shown as under

$$\begin{array}{l|cccc|cc} 7x+4y & -\frac{2548x^5}{125} & -\frac{36806x^4y}{625} & -\frac{41883x^3y^2}{625} & -\frac{19654x^2y^3}{625} & \frac{-2112xy^4}{625} & +\frac{544}{625}y^5 \\ & & / & / & / & / & / \\ 5 & & 0 & 0 & 0 & 0 & 0 \\ \hline & \frac{-2548x^5}{625} & -\frac{36806x^4y}{3125} & -\frac{41883x^3y^2}{3125} & -\frac{19654x^2y^3}{3125} & \frac{-2112xy^4}{3125} & +\frac{544}{3125}y^5 \\ & Q_1'' & Q_2'' & Q_3'' & Q_4'' & Q_5' & Q_6'' \end{array}$$

(1)
$$\frac{-2548x^5}{125}\div 5 = -\frac{-2548x^5}{625}(Q_1\ )$$

(2) $$-\frac{36806x^4y}{625} - \begin{array}{c}7x\\ -2548x^5\\ 625\end{array} = -\frac{36806x^4y}{625} + \frac{17836x^6}{625}$$

$$\therefore -\frac{36806}{625}x^4y \div 5 = \frac{-36806x^4y}{3125}(Q_2''), and \frac{17836x^6}{625}(R_1'')$$

(3) $$-\frac{41883}{625}x^3y^2 - \begin{array}{c}7x\\ -\dfrac{2548x^5}{625}\end{array} \quad \begin{array}{c}4y\\ -\dfrac{36806}{3125}x^4\end{array}$$

$$-\frac{41883x^3y^2}{625} - \frac{-257642x^5y}{3125} - 10192x^5y \Big| = \frac{-41883x^3y^2}{625} + \frac{308602x^5y}{3125}$$

$$-\frac{41883}{625}x^3y^2 \div 5 = -\frac{41883}{3125}x^3y^2$$

$$\therefore \frac{-41883x^3y^2}{3125}(Q_3'')and + \frac{308602x^5y}{3125}(R_2'')$$

(4) $$\frac{-19654x^2y^3}{625} \quad \begin{array}{c}7x\\ \dfrac{-36806x^4y}{3125}\end{array} \quad \begin{array}{c}4y\\ -\dfrac{41883x^3y^2}{3125}\end{array}$$

$$= \frac{-19654x^2y^3}{625} + \frac{293181x^4y^2}{3125} + \frac{147224x^4y^2}{3125} = \frac{-19654x^2y^3}{625} + \frac{440405x^4y^2}{3125}$$

$$-\frac{19654}{625}x^2y^3 + 5 = \frac{-19654}{3125}x^2y^3$$

$$\therefore \frac{-19654x^2y^3}{3125}(Q_4'')and \frac{440405x^4y^2}{3125}(R_1'')$$

(5) $$\frac{-2112}{625}xy^4 - \left| \begin{array}{c}7x\\ -\dfrac{41883}{3125}x^3y^2\end{array} \quad \begin{array}{c}4y\\ -\dfrac{19654}{3125}x^2y^3\end{array} \right|$$

$$-\frac{2112}{}xy^4 + \frac{137578}{}x^3y^3 + \frac{167532}{}x^3y^3$$

$$-\frac{2112}{625}xy^4 + 5 = -\frac{2112}{3125}xy^4(Q'_5) + \frac{305110}{3125}x^3y^3(R'_4)$$

(6)
$$\frac{544}{625}y^5 - \overset{7x}{\frac{19654}{3125}x^2y^3} \quad \overset{4y}{-\frac{2112}{3125}xy^4}$$

$$= \frac{544}{625}y^5 + \frac{14784}{3125}x^2y^4 + \frac{78616}{3125}x^2y^4$$

$$\frac{544}{625}y^5 + 5 = \frac{544}{3125}y^5(Q_6'') + \frac{93400}{3125}x^2y^4(R\ _5)$$

(7) Remainders

$$0 - \overset{7x}{\frac{-2112}{3125}xy^4} \quad \overset{4y}{\frac{544}{3125}y^5} \quad \begin{array}{|l} 4y \\ \\ \frac{544}{3125}y^5 \end{array}$$

$$= -\frac{3808}{3125}xy^5 + \frac{8448}{3125}xy^5 - \frac{2176}{3125}y^6$$

$$= \frac{4640}{3125}xy^5(R''_6) - \frac{2176}{3125}y^6(R''_7)$$

$$\therefore Q \quad Q_1' + Q_2' + Q_3' + Q_4' + Q_5'' + Q_6'' = -\frac{2548x^5}{625} - \frac{36806x^4y}{3125} + \frac{41883x^3y^2}{3125} - \frac{19654x^2y^3}{3125} - \frac{2112}{3125}xy^4 + \frac{544}{3125}y^5$$

$$R = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 = +\frac{17836x^6}{625} + \frac{308602x^5y}{3125} + \frac{440405x^4y^2}{3125} + \frac{305110x^3y^3}{3125}$$
$$+ \frac{93400}{3125}x^2y^4 + \frac{4640}{3125}xy^5 - \frac{2176}{3125}y^6$$

III

$$\begin{array}{l|lllll|ll} 7x+4y & \frac{17836x^6}{625} & +\ \frac{308602x^5y}{3125} & +\ \frac{440405}{3125}x^4y^2 & +\ \frac{305110}{3125}x^3y^3 & +\frac{93400}{3125}x^2y^4 & +\frac{4640}{3125}xy^5 & -\frac{2176}{3125} \\ & & \quad / & \quad / & \quad / & \quad / & \quad / & \quad / \\ 5 & & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\ & & & & & & & \\ \hline & \frac{17836x^6}{3125} & +\ \frac{308602x^5y}{15625} & +\ \frac{440405}{15625}x^4y^2 & +\ \frac{305110}{15625}x^3y^3 & +\frac{93400}{15625}x^2y^4 & +\frac{4640}{15625}xy^5 & -\frac{2176}{1562} \\ & \quad Q_1 & \quad Q_2 & \quad Q_3 & \quad Q_4 & \quad\quad Q_5 & \quad Q_6 & \quad Q_7 \end{array}$$

(1) $$\frac{17836x^6}{625}+5=\frac{17836}{3125}x^6\ (Q_1''')$$

(2) $$\frac{308602}{3125}x^5y-\left|\begin{array}{l}7x\\ \frac{17836}{3125}x^6\end{array}\right|$$

$$=\qquad \frac{308602}{3125}x^5y-\frac{124852}{3125}x^7,\ \therefore \frac{308602}{3125}x^5y+5=\frac{308602}{15625}x^5y\ (Q_2'''),\ -\frac{124852}{3125}x^7\,(R_1''')$$

(3) $$\frac{440405}{3125}x^4y^2-\left|\begin{array}{cc}7x & 4y\\ \frac{17836}{3125}x^6 & \frac{308602}{15625}x^5y\end{array}\right|$$

$$=\frac{440405}{3125}x^4y^2-\frac{2160214}{15625}\ x^6y-\frac{71344}{3125}x^6y$$

$$\therefore \frac{440405}{15625}x^4y^2+5=\frac{440405}{15625}x^4y^2\,(Q'''_3),\ -\frac{2516934}{3125}x^6y\,(R'''_2)$$

(4) $$\frac{305110}{3125}x^3y^3-\left|\begin{array}{cc}7x & 4y\\ \frac{308602}{15625}x^5y & \frac{440405}{15625}x^4y^2\end{array}\right|$$

$$=\frac{305110}{3125}x^3y^3-\frac{3082835}{15625}x^5y^2-\frac{1234408}{15625}x^5y^2$$

$$\therefore \frac{305110}{3125}x^3y^3+5=\frac{305110}{15625}x^3y^3\,(Q'''_4),\ \ -\frac{4317243}{15625}x^5y^2\,(R'''_3)$$

(5) $$\frac{93400}{3125}x^2y^4-\left|\begin{array}{cc}7x & 4y\\ \frac{440405}{15625}x^4y^2 & \frac{305110}{15625}x^3y^3\end{array}\right|$$

$$=\frac{93400}{3125}x^2y^4-\ \frac{2135770}{15625}x^4y^3\quad \frac{1761620}{15625}x^4y^3$$

$$\therefore \frac{93400}{3125}x^2y^4+5=\frac{93400}{15625}x^2y^4\,(Q'''_5)-\frac{3897390}{15625}x^4y^3\,(R'''_4)$$

(6) $$\frac{4640}{3125}xy^5-\left|\begin{array}{cc}7x & 4y\\ \frac{305110}{15625}x^3y^3 & \frac{93400}{15625}x^2y^4\end{array}\right|$$

$$=\frac{4640}{3125}xy^5-\frac{653800}{15625}x^3y^4-\frac{1220440}{15625}x^3y^4$$

$$\therefore \frac{4640}{3125}xy^5 + 5 = \frac{4640}{15625}xy^5 (Q'''_6) - \frac{1874240}{15625}x^3y^4 (R'''_5)$$

(7)

$$-\frac{2176}{3125}y^6 - \left| \begin{array}{cc} 7x & 4y \\ \frac{93400}{15625}x^2y^4 & \frac{4640}{15625}xy^5 \end{array} \right|$$

$$= \frac{-2176}{3125}y^6 - \frac{32480}{15625}x^2y^5 - \frac{373600}{15625}x^2y^5$$

$$\therefore \frac{-2176}{3125}y^6 + 5 = \frac{-2176}{15625}y^6 (Q'''_7) - \frac{406080}{15625}x^2y^5 (R'''_6)$$

Remainders

$$0 - \left. \begin{array}{cc} 7x & 4y \\ \frac{4640}{15625}xy^5 & -\frac{2176}{15625}y^6 \end{array} \right| - \left| \begin{array}{c} 4y \\ \\ -\frac{2176}{15625}y^6 \end{array} \right|$$

$$= + \frac{15232}{15625}xy^6 - \frac{18560}{15625}xy^5 + \frac{8704}{15625}y^7$$

$$= \frac{-3328}{15625}xy^6 (R'''_7) + \frac{8704}{15625}y^7 \ (R'''_8)$$

$$Q''' = Q_1''' + Q_2''' + Q_3''' + Q_4'''' + Q_5''' + Q_6''' + Q_7'$$

$$\frac{17836}{3125}x^6 + \frac{308602}{15625}x^5y + \frac{440405}{15625}x^4y^2$$
$$+ \frac{305110}{15625}x^3y^3 + \frac{93400}{15625}x^2y^4 + \frac{4640}{15625}xy^5 - \frac{21766}{15625}y^6$$

$$R''' = R_1''' + R_2''' + R_3''' + R_4''' + R_5''' + R_6'' + R_7''' + R_8''$$

$$R''' = -\frac{124852}{3125}x^7 - \frac{251934}{3125}x^6y - \frac{4317243}{15625}x^5y^2$$
$$- \frac{3897390}{15625}x^4y^3 - \frac{1874240}{15625}x^3y^4 - \frac{4060802}{15625}xy^5$$
$$- \frac{3328}{15625}xy^6 + \frac{8704}{15625}y^7$$

(g) The straight division method as explained for Bipolynomials by British authors is as follows:

Consider dividend and divisor as sets of arrays. The terms which are beyond the dividend are treated as remainders. The working details are deducing the quotients of each row, similarly the remainder terms of each row. The following are the working details attempted by the authors for an example given in this book.

Consider $(5 + 2x + 4x^2 + 5x^3 + 3y + 7xy + 8x^2y + 5y^2 + 8xy^2 + 6y^3) + (5 + 7x + 4y)$

The dividend is given below.

$$\begin{array}{c|cccc} & 1 & x & x^2 & x^3 \\ \hline 1 & 5 & 2 & 4 & 5 \\ y & 3 & 7 & 8 & 0 \\ y^2 & 5 & 8 & 0 & 0 \\ y^3 & 6 & 0 & 0 & 0 \end{array}$$

The divisor is

$$\begin{array}{c|cc} & 1 & x \\ \hline 1 & 5 & 7 \\ y & 4 & 0 \end{array}$$

Applying straight division method, the part divisor (PD) is considered as 5 and the two Dhwaajankas are $7x$ and $4y$
D_1 D_2

The working details are given below in the form of steps for rows of the dividend

Row1: (Const.,x, x^2, x^3) The four elements are represented as (1) (2) (3) and (4) by a diagram, exclusively for each digit

Row 1 now consists of

(1)	(2)	(3)	(4)
5 ...	.2 ..	..4.	...5

The working details of division as follows

Step 1 Consider (1) and divide it by 5 (PD)

$\boxed{5\ldots} \div 5 = \boxed{1\ldots}$ (Q_1)

Step 2 Consider (2) The working is to subtract the product of first Dhwajanka D_1 with Q_1 from (2) and the result is divided by 5(P D)

$\boxed{.2..} - \boxed{\begin{matrix}0 & \overset{D_1}{7}\\ 0 & \end{matrix}} \underset{Q_1}{\boxed{1\ldots}} \div 5$ -1x 9 (Q.) and is represented as $\boxed{\ 1\ .\ .}$ (Q_2)

Step 3 Consider (3). Then subtract the (x^2) cross multiplication represented by D_1, D_2, Q_1 , Q_2

$\boxed{.\ .4\ .} - \boxed{.\ 7} \underset{Q_1\ Q_2}{\boxed{1\ -1\ .\ .}} \div 5$

$[4 \quad (\ 7)] \div 5 = \frac{11}{5} x^2$ and is represented as $\boxed{.\ .\ \frac{11}{5}\ .}$ (Q_3)

Step 4 Consider (4)

$\boxed{.\ .\ .\ 5} - \boxed{.\ 7}\ \boxed{1 - 1\ \frac{11}{5}\ .} \div 5$

$= \frac{1}{5}(5 \quad \frac{77}{5}) = -\frac{52}{5} \div 5 = -\frac{52}{25} x^1$ and is represented as $\boxed{.\ .\ .\ -\frac{52}{25}}$ (Q_4)

For the remainders in the first row the same procedure is continued but is not to be divided by the part divisor.

Step5 · Remainder in x^4

$\boxed{.\ .\ .\ .\ 0} - \overset{x}{\boxed{.\ 7}}\ \overset{x^3}{\boxed{.\ .\ .\ -\frac{52}{25}}} = \frac{364}{25}$ represented as $\boxed{.\ .\ .\ .\ -\frac{364}{25}}$ R_1

Row 2 (y, xy, x^2y), x^3y = 0 similar is the procedure for other rows second row consists of three elements and they are

(1) $\boxed{3}$ (2) $\boxed{7}$ (3) $\boxed{8}$

Step : 1 (y) **Consider (1)**

$$\begin{array}{|c|}\hline \\ 3 \end{array} - \overset{D}{\begin{array}{|cc|}\hline & 7 \\ 4 & . \end{array}} \quad \underset{Q_1}{\overset{Q}{\begin{array}{|cccc|}\hline 1 & . & . & . \end{array}}} + 5$$

$(3-4)+5 = -1 + 5 = \frac{-1y}{5}$ represented as $= \begin{array}{|c|}\hline . \\ -\frac{1}{5} \end{array}$ Q_5

Step 2: (xy)

Consider (2)

$$\begin{array}{|cc|}\hline & \\ . & 7 \end{array} - \overset{D}{\begin{array}{|cc|}\hline . & 7 \\ 4 & \end{array}} \quad \underset{Q_5}{\overset{Q}{\begin{array}{|cccc|}\hline & & \underset{Q_2}{-1} & \\ \frac{1}{5} & . & . & . \end{array}}} + 5$$

$7 - \left\{(4)(-1) + (7)(-\frac{1}{5})\right\} + 5 = \{ 7 + 4 + \frac{7}{5} \} + 5 = \frac{62}{25}$ xy and is represented as $\begin{array}{|cc|}\hline . & . \\ . & \frac{62}{25} \end{array}$ Q

Step 3· (x^2y) Consider (3)

$$\begin{array}{|cccc|}\hline . & . & . & . \\ & & & 8 \end{array} - \overset{D}{\begin{array}{|cc|}\hline & 7 \\ 4 & . \end{array}} \quad \overset{Q}{\begin{array}{|ccc|}\hline . & . & \underset{Q_3}{\frac{11}{5}} \\ . & \underset{Q_6}{\frac{62}{25}} & . \end{array}} + 5 = \{8 - [(4)(\frac{11}{5}) + (7)(\frac{62}{25})]\} + 5 = -\frac{454}{125} x^2y \text{ and is}$$

represented as $= \begin{array}{|ccc|}\hline . & . & . \\ & & -\frac{454}{125} \end{array}$ Q_7

For the remainders in the second row the procedure is as follows

Step 4: (x^3y)

$$\begin{array}{|ccccc|}\hline & & . & & . \\ . & . & . & . & 0 \end{array} - \overset{D}{\begin{array}{|cc|}\hline & 7 \\ 4 & . \end{array}} \quad \overset{Q}{\begin{array}{|ccccc|}\hline . & . & . & . & \underset{Q_7}{-\frac{52}{25}} \\ . & & & \underset{Q_9}{-\frac{454}{125}} & \end{array}}$$

$$= -\left[(7)\left(\frac{-454}{125}\right)+(4)\left(\frac{-52}{25}\right)\right]$$

$= [\frac{3178}{125}+\frac{208}{25}] = \frac{4218}{125}$ x^3y and is represented as $\boxed{\frac{4218}{125}}$ R_2

Row 3 . The elements in this row are $\overset{(1)}{\boxed{5}}$, $\overset{(2)}{\boxed{8}}$

Step 1 (y^2) Consider (1)

$$\boxed{5} - \overset{D}{\boxed{4\ {}^{7}}} \quad \underset{Q_5}{\overset{Q}{\boxed{-\frac{1}{5}}}} \div 5$$

$[5-(4)(-\frac{1}{5})] \div 5 = \frac{29}{25}y^2$ and is represented as $\boxed{\frac{29}{25}}$ Q_8

Step 2 (xy^2) Consider (2)

$$\boxed{8} - \boxed{4\ {}^{7}} \quad \boxed{\begin{matrix} & \frac{62}{25} \\ & Q_6 \\ \frac{29}{25} & \end{matrix}}_{Q_8} \div 5$$

$[8-\{(4)(\frac{62}{25})+(7)(\frac{29}{25})\}] \div 5 = \frac{1}{5}[8 - \frac{248}{25} - \frac{203}{25}] = -\frac{251}{125}$ x^2y and is Represented as $\boxed{-\frac{251}{125}}$

For the remainders in the third row the procedure is as follows

Step 3· (x^2y^2)

$$\left|\begin{array}{c} \\ \\ 0 \end{array}\right| - \left|\begin{array}{cc} & 7 \\ 4 & . \end{array}\right| \left|\begin{array}{ccc} & & -\frac{454}{125} \\ & & Q_7 \\ . & \underset{Q_9}{-\frac{251}{125}} & \end{array}\right|$$

$0-(7)(-\frac{251}{125}) - (4)(-\frac{454}{125}) = \frac{1757}{125} + \frac{1816}{125} = \frac{3573}{125}\, x^2y^2$ and is represented as $\left|\begin{array}{cc} . & . \\ . & \dot{\frac{3573}{125}} \\ . & . \end{array}\right| \quad R_3'$

Row 4: the only element in row 4 is $\overset{(1)}{\left|\begin{array}{c} \\ . \\ 6 \end{array}\right|}$

Step 1 (y^3)

$$\left|\begin{array}{c} . \\ \\ 6 \end{array}\right| - \left|\begin{array}{cc} . & 7 \\ 4 & \end{array}\right| \underset{Q_8}{\left|\begin{array}{c} \\ \dot{\frac{29}{25}} \end{array}\right|} + 5 \;\; (6-4\times\frac{29}{25})+5 = \frac{34}{125}y^3 \text{ represented as } \left|\begin{array}{c} . \\ . \\ \frac{34}{125} \end{array}\right| Q_{10}$$

Fourth row remainder is worked out as follows

Step 2 (xy^3)

$$\left|\begin{array}{cc} . & . \\ & . \\ . & . \\ & 0 \end{array}\right| - \left|\begin{array}{cc} & 7 \\ 4 & \end{array}\right| \left|\begin{array}{cc} . & \\ . & -\dot{\frac{251}{125}} \\ . & Q_9 \\ \underset{Q_{10}}{\frac{34}{125}} & . \end{array}\right|$$

$=0-(7)(\frac{34}{125})-(4)(-\frac{251}{125}) = -\frac{238}{125} + \frac{1004}{125} = \frac{766}{125}\, y^3x$ and is represented as $\left|\begin{array}{cc} . & \\ . & . \\ . & \dot{\frac{766}{125}} \\ . & \end{array}\right| \quad R_4$

The remainder y^4 is evaluated from row 5 as follows

Row 5: Consider the element in row5 as zero

Step 1: (y^4)

$$\begin{array}{|c|} \cdot \\ \\ 0 \end{array} - \begin{array}{|cc|} . & 7 \\ 4 & \end{array} \begin{array}{|c|} \cdot \\ \frac{34}{125} \end{array} = 0-(4)(\frac{34}{125}) = -\frac{136}{125}y^4 \text{ and is represented as } \begin{array}{|c|} \\ -\frac{136}{125} \end{array} R_5$$

$$Q_{10}$$

For all the other remainder the procedure is similar.
The features of all the above steps are represented as below

	D		Dividend						
	.	7	5	2	4	5			
	4	.	3	7	8	0			
			5	8	0	0			
			6	0	0	0			
(PD)	5		1	-1	$\frac{11}{5}$	$-\frac{52}{25}$	$\frac{364}{25}$	x^4	R_1
			Q_1	Q_2	Q_3	Q_4			
			$-\frac{1}{5}$	$\frac{62}{25}$	$-\frac{454}{125}$	—	$\frac{4218}{125}$	x^3y	R_2
			Q_5	Q_6	Q_7				
			$\frac{29}{25}$	$-\frac{251}{125}$	—	—	$\frac{3573}{125}$	x^2y^2	R_3
			Q_8	Q_9					
			$\frac{34}{125}$	—	—	—	$\frac{766}{125}$	xy^3	R_4
			Q_{10}						
			$-\frac{136}{125}y^4$						
			R_5						

CHAPTER – VII

COMPUTER PROGRAMMING RESULTS

NIKHILAM

```
Division by Nikhilam Method


Enter the Dividend: 223

Enter the Divisor: 78

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0

The Nikhilam Divisor: 22
Quotient Part is: 2,
Remainder Part is: 2, 3,
Results of Multiplication: 4, 4,

The Nikhilam Quotient is: 2,
The Nikhilam Remainder is: 6, 7,
Quotient in Ordinary form: 2
Remainder in Ordinary form: 67

Quotient: 2
Remainder: 67

Do you want to continue with another division(y\n): n
```

```
Division by Nikhilam Method


Enter the Dividend: 897356

Enter the Divisor: 721

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0
        The Nikhilam Quotient is: 9,
The Nikhilam Remainder is: 26, 63, 86,
Quotient in Ordinary form: 1240
Remainder in Ordinary form: 3316
Quotient Part is: 3,
Remainder Part is: 3, 1, 6,
Results of Multiplication: 6, 21, 27,

The Nikhilam Quotient is: 3,
The Nikhilam Remainder is: 9, 22, 33,
Quotient in Ordinary form: 1243
Remainder in Ordinary form: 1153
Quotient Part is: 1,
Remainder Part is: 1, 5, 3,
Results of Multiplication: 2, 7, 9,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 3, 12, 12,
Quotient in Ordinary form: 1244
Remainder in Ordinary form: 432

Quotient: 1244
Remainder: 432

Do you want to continue with another division(y\n): n
```

```
    Division by Nikhilam Method


    Enter the Dividend: 45679

    Enter the Divisor: 99

    Enter 0 if to ignore the decimal part
    Enter the number of digits in decimal part: 0

    The Nikhilam Divisor: 1
    Quotient Part is: 4, 5, 6, 7,
    Remainder Part is: 9,
    Results of Multiplication: 4,
9,
15,
22,

    The Nikhilam Quotient is: 4, 9, 15, 22,
    The Nikhilam Remainder is: 31,
    Quotient in Ordinary form: 5072
    Remainder in Ordinary form: 31

    Quotient: 5072
    Remainder: 31

    Do you want to continue with another division(y\n): n


    Division by Nikhilam Method


    Enter the Dividend: 31589

    Enter the Divisor: 7

    Enter 0 if to ignore the decimal part
```

```
Enter the number of digits in decimal part: 0

        The Nikhilam Quotient is: 4,
The Nikhilam Remainder is: 19,
Quotient in Ordinary form: 4510
Remainder in Ordinary form: 19
Quotient Part is: 1,
Remainder Part is: 9,
Results of Multiplication: 3,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 12,
Quotient in Ordinary form: 4511
Remainder in Ordinary form: 12
Quotient Part is: 1,
Remainder Part is: 2,
Results of Multiplication: 3,

The Nikhilam Quotient is: 1,
The Nikhilam Remainder is: 5,
Quotient in Ordinary form: 4512
Remainder in Ordinary form: 5

Quotient: 4512
Remainder: 5

Do you want to continue with another division(y\n): n

Division by Nikhilam Method


Enter the Dividend: 42567

Enter the Divisor: 8

Enter 0 if to ignore the decimal part
Enter the number of digits in decimal part: 0
```

```
        The Nikhilam Remainder is: 119,
    Quotient in Ordinary form: 5306
    Remainder in Ordinary form: 119
    Quotient Part is: 1, 1,
    Remainder Part is: 9,
    Results of Multiplication: 2,
6,

    The Nikhilam Quotient is: 1, 3,
    The Nikhilam Remainder is: 15,
    Quotient in Ordinary form: 5319
    Remainder in Ordinary form: 15
    Quotient Part is: 1,
    Remainder Part is: 5,
    Results of Multiplication: 2,

    The Nikhilam Quotient is: 1,
    The Nikhilam Remainder is: 7,
    Quotient in Ordinary form: 5320
    Remainder in Ordinary form: 7

    Quotient: 5320
    Remainder: 7

    Do you want to continue with another division(y\n): n
```

REDUCTION

```
   DIVISION PROCESS
   ************************

 The first number(DIVIDEND) is: 0.1 2 4

 The second number(DIVISOR) is: 2 1 2 2

 The intermediate remainders are: 0 0 1 2 3 4 4
-----------------------------------------------------------------

 The final result is : 0.0 0 0 0 5 8 4 3 5

 Remainder = 13


Do you want to have another calculation (Y/N)n



   DIVISION PROCESS


 The first number(DIVIDEND) is: 1 2 4

 The second number(DIVISOR) is: 2 1 2 .2

 The intermediate remainders are: 0 1 2 3 4 4
--------------------------------------------------------------

 The final result is : 0 .5 8 4 3 5

 Remainder = 1240


Do you want to have another calculation (Y/N)n
```

```
     DIVISION PROCESS


    The first number(DIVIDEND) is: 1 2 4

    The second number(DIVISOR) is: 2 1 .2 2

    The intermediate remainders are: 0 1 2 3 4 4
------------------------------------------------------------------

    The final result is : 0 5 .8 4 3 5

    Remainder = 1790


Do you want to have another calculation (Y/N)n



     DIVISION PROCESS
     -------------------------

    The first number(DIVIDEND) is: 8 9 7 3 5 6

    The second number(DIVISOR) is: 7 2 1

    The intermediate remainders are: 0 1 3 4 5 8 9 4 6
------------------------------------------------------------------

    The final result is : 1 2 4 4 .5 9 9 1 6

    Remainder = 432


Do you want to have another calculation (Y/N)n
```

```
    DIVISION PROCESS
    ----------------------------

  The first number(DIVIDEND) is: 7 8

  The second number(DIVISOR) is: 2 1 3 4 5

  The intermediate remainders are: 0 1 3 5 7


  The final result is : 0.0 0 3 6 5 4 2

  Remainder = 7800


Do you want to have another calculation (Y/N)n



    DIVISION PROCESS


  The first number(DIVIDEND) is: 0.8 9 2 7 1 2 4

  The second number(DIVISOR) is: 9 6 2 1 8 7 3 4

  The intermediate remainders are: 0 0 8 6 10 14 23 21 24 31 32


  The final result is : 0.0 0 0 0 0 0 0 0 9 2 7 7 9 4 7 8 9

  Remainder = 892713


Do you want to have another calculation (Y/N)n
```

```
        DIVISION PROCESS


     The first number(DIVIDEND) is: 9 8 7 6 5

     The second number(DIVISOR) is: 1 3 2 1

     The intermediate remainders are: 0 2 3 4 4 4 3 3
----------------------------------------------------------------------

     The final result is . 7 4 .7 6 5 3 2 9

     Remainder = 1011


Do you want to have another calculation (Y/N)n
```

VINCULUM

```
      DIVISION PROCESS


   The first number(DIVIDEND) is: 1 5 6 2 8

   The second number(DIVISOR) is: 2 3 .4

   The intermediate reminders are: 0 1 1 -1 0 1 0 0 1 -1 -1 0 0 -1
------------------------------------------------------------------

   The final result is : 0 6 6 7 .8 6 3 2 4 7 8 6 8 4


do you want to have another calculation (Y/N)N


      DIVISION PROCESS


   The first number(DIVIDEND) is: 0 1 1

   The second number(DIVISOR) is: 1 1 1

   The intermediate reminders are: 0 0 0 0 0 0 0 0 0 0 0 0
------------------------------------------------------------------

'  The final result is : 0 .0 9 9 0 9 9 0 9 9 1 0


do you want to have another calculation (Y/N)N
```

```
        DIVISION PROCESS
       ----------------------

     The first number(DIVIDEND) is: 0 0 0 .4 6 1 3 9 7

     The second number(DIVISOR) is: 1 2 3 .4

     The intermediate reminders are: 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
-------------------------------------------------------------

     The final result is : 0 .0 0 3 7 3 9 0 3 5 6 5 6 3 9 9 6 4


do you want to have another calculaton (Y/N)N
```

PARVARTYA POLYNOMIALS

```
            DIVISION


            The Dividend is: 6 * x^3 -12 * x^2 +3 * x^1 -10
            The Divisor is: 2 * x^1 -5


            The Paravartya form: 5,


Intermediate Multiplicants in the Quotient part are: 30/2 * x^2, 30/4 * x^1,


            Quotient:  3 * x^2  + 3/2 * x^1  + 21/4


Intermediate Multiplicants in the Remainder part are  105/4 * x^0,


            Remainder:  +65/4


            Do you want to continue with another calculation(y/n)n



            DIVISION


            The Dividend is. 6 * x^5 +2 * x^4 +5 * x^3 ++1
            The Divisor is· 3 * x^2 -2 * x^1 +1


            The Paravartya form: 2, -1,


Intermediate Multiplicants in the Quotient part are: 12/3 * x^4, -6/3 * x^3, 36/
9 * x^3, -18/9 * x^2, 378/81 * x^2, -189/81 * x^1,


            Quotient.  2 * x^3  + 2 * x^2  + 7/3 * x^1  + 8/9


Intermediate Multiplicants in the Remainder part are. 16/9 * x^1, -8/9 * x^0,


            Remainder: -5/9 * x^1 + 1/9
```

```
          Do you want to continue with another calculation(y/n)n


          DIVISION
          ------------
          The Dividend is: 1 * x^3 -6 * x^2 +11 * x^1 -6
          The Divisor is: 2 * x^1 -1

        . The Paravartya form: 1,

Intermediate Multiplicants in the Quotient part are: 1/2 * x^2, -11/4 * x^1,

          Quotient:  1/2 * x^2  -11/4 * x^1  + 33/8

Intermediate Multiplicants in the Remainder part are: 33/8 * x^0,

          Remainder:  -15/8

          Do you want to continue with another calculation(y/n)n
```

PARAVARTYA NUMERALS

```
          Division by Paravartya Method



          Enter the Dividend: 29429
          Enter the Divisor: 1463

          Enter 0 if to ignore the decimal part
          Enter the number of digits in decimal part: 0

          Paravartya form is: -4, -6, -3,
          Quotient part is: 2, 9,
          Remainder part is: 4, 2, 9,
Results of Multiplication with final quotient digits: -8, -12, -6,
-4, -6, -3,

          Quotient in Vinculum form: 2, 1,
          Remainder in Vinculum form: -12, -10, 6,
          Quotient in Ordinary form: 21
          Remainder in Vinculum form: -1294

          Quotient: 20
          Remainder: 169

          Do you want to continue with another division(y\n): n


          Division by Paravartya Method



          Enter the Dividend: 25935
          Enter the Divisor: 829

          Enter 0 if to ignore the decimal part
          Enter the number of digits in decimal part: 0

          Paravartya form is: 2, -3, 1,
```

```
          Quotient part is: 2, 5,
          Remainder part is: 9, 3, 5,
Results of Multiplication with final quotient digits: 4, -6, 2,
18, -27, 9,

          Quotient in Vinculum form: 2, 9,
          Remainder in Vinculum form: 21, -22, 14,
          Quotient in Ordinary form: 29
          Remainder in Vinculum form: 1894

          Quotient: 31
          Remainder: 236

          Do you want to continue with another division(y\n): n



          Division by Paravartya Method


          Enter the Dividend: 101100
          Enter the Divisor: 486

          Enter 0 if to ignore the decimal part
          Enter the number of digits in decimal part: 0

          Paravartya form is: 0, 3, -2,
          Quotient part is: 1, 0, 1,
          Remainder part is: 1, 0, 0,
Results of Multiplication with final quotient digits. 0, 3, -2,
0, 0, 0,
0, 12, -8,

          Quotient in Vinculum form: 1, 0, 4,
          Remainder in Vinculum form: -1, 12, -8,
          Quotient: 208
          Remainder: 12
```

```
        Do you want to continue with another division(y\n). n


      Division by Paravartya Method


        Enter the Dividend. 897356
        Enter the Divisor: 721

        Enter 0 if to ignore the decimal part
        Enter the number of digits in decimal part  0

        Paravartya form is: 3, -2, -1,
        Quotient part is: 8, 9, 7,
        Remainder part is· 3, 5, 6,
Results of Multiplication with final quotient digits  24, -16, -8,
99, -66, -33,
270, -180, -90,

        Quotient in Vinculum form. 8, 33, 90,
        Remainder in Vinculum form: 199, -208, -84,
        Quotient in Ordinary form: 1220
        Remainder in Vinculum form: 17736

        Quotient: 1244
        Remainder: 432

        Do you want to continue with another division(y\n): n
```

ARGUMENTAL POLYNOMIALS

```
          ARGUMENTAL DIVISION

      The Dividend is: 24 * x^4 +50 * x^3 +35 * x^2 +10 * x^1 +13
      The Divisor is: 4 * x^1 +1

The coefficient of the power of x=(maxpower of dividend-maxpower of divisor= 3)
, 6
    The rest of the coefficients are in decreasing order.
    11, 6, 1,

    Quotient:  + 6 * x^3  + 11 * x^2  + 6 * x^1  + 1

      The individual remainders are: 0, 0, 0, 0, 12,

    Remainder:  12

      Do you want to continue with another calculation(y/n)n

          ARGUMENTAL DIVISION

      The Dividend is: 10 * x^4 +17 * x^3 +20 * x^2 +6 * x^1 +3
      The Divisor is: 2 * x^2 +3 * x^1 +3

The coefficient of the power of x=(maxpower of dividend-maxpower of divisor= 2)
, 5
    The rest of the coefficients are in decreasing order.
    1, 1,

    Quotient:  + 5 * x^2  + 1 * x^1  + 1

      The individual remainders are: 0, 0, 0, 0, 0,

    Remainder: 0

      Do you want to continue with another calculation(y/n)n
```

```
                ARGUMENTAL DIVISION

            The Dividend is: 2 * x^10 +4 * x^9 +9 * x^8 +14 * x^7 +17 * x^6
+20 * x^5 +15 * x^4 +16 * x^3 +16 * x^2 +8 * x^1 +10
            The Divisor is: 2 * x^5 +2 * x^4 +3 * x^3 +1 * x^2 +2 * x^1 +3

 The coefficient of the power of x=(maxpower of dividend-maxpower of divisor= 5)
, 1
      The rest of the coefficients are in decreasing order.
      1, 2, 3, 1, 1,

      Quotient. + 1 * x^5  + 1 * x^4  + 2 * x^3  + 3 * x^2  + 1 * x^1  + 1

            The Individual remainders are: 0, 0, 0, 0, 0, 0, 0, 0, 4, 3, 7,


      Remainder  4 * x^2 +3 * x^1 + 7

            Do you want to continue with another calculation(y/n)n
```

ARGUMENTAL NUMERALS

```
        Division by Argument Method


        Enter the Dividend: 109876548

        Enter the Divisor: 6783

        Intermediate Remainders are: 1, 4, 0, -1, -6, -5, -14, -128, -12
69,
        Intermediate Quotients are: 0, 1, 7, -8, 0, -1,

        Quotient: 16198
        Remainder: 5514

        Do you want to continue with another division(y\n): N



        Division by Argument Method


        Enter the Dividend: 89765

        Enter the Divisor: 321

        Intermediate Remainders are: 2, 1, -1, -12, -115,
        Intermediate Quotients are: 2, 8, 0,

        Quotient: 279
        Remainder: 206

        Do you want to continue with another division(y\n)· N
```

```
Division by Argument Method


Enter the Dividend: 134289

Enter the Divisor: 2760

Intermediate Remainders are: 1, 1, 0, 64, 732, 7329,
Intermediate Quotients are: 0, 0, 6, -14,

Quotient: 48
Remainder: 1809

Do you want to continue with another division(y\n): N
```

STRAIGHT DIVISION 1 VARIABLE

```
      STRAIGHT DIVISION

    The Dividend is: 5 * x^4 +3 * x^3 +2 * x^2 +1 * x^1 +2
    The Divisor is: 3 * x^2 +1 * x^1 +4

    The part divisor: 3 * x^2
    The Dhwajanka part: +1 * x^1 +4
.
    In the Quotient Region....
R 1 = 0, ID 1 = 0 + 3 * x^3
R 2 = 0, ID 2 = 0 + 2 * x^2
R 3 = 0, ID 3 = 0 + 2 * x^2


    Quotient:  5/3 * x^2  + 4/9 * x^1  -46/27
    In Remainder Region....R 4 = 0, ID 4 = 0 + 1 * x^1
R 5 = 0, ID 5 = 0 + 1 * x^1
R 6 = 0, ID 6 = 0 + 2 * x^0


    Remainder: 25/27 * x^1 +238/27

    Do you want to continue with another calculation(y/n)N
```

STRAIGHT DIVISION

```
        The Dividend is: 8 * x^5 +9 * x^4 +7 * x^3 +3 * x^2 +5 * x^1 +6
        The Divisor is: 7 * x^2 +2 * x^1 +1

        The part divisor: 7 * x^2
        The Dhwajanka part: +2 * x^1 +1

        In the Quotient Region.. .
R 1 = 0, ID 1 = 0 + 9 * x^4
R 2 = 0, ID 2 = 0 + 7 * x^3
R 3 = 0, ID 3 = 0 + 7 * x^3
R 4 = 0, ID 4 = 0 + 3 * x^2
R 5 = 0, ID 5 = 0 + 3 * x^2


        Quotient.  8/7 * x^3  + 47/49 * x^2  + 193/343 * x^1  + 314/2401

        In Remainder Region....R 6 = 0, ID 6 = 0 + 5 * x^1
R 7 = 0, ID 7 = 0 + 5 * x^1
R 8 = 0, ID 8 = 0 + 6 * x^0


        Remainder  10026/2401 * x^1 +14092/2401

        Do you want to continue with another calculation(y/n)N
```

```
         STRAIGHT DIVISION
        ___________

      The Dividend is: 8 * x^5 -9 * x^4 +7 * x^3 -3 * x^2 +5 * x^1 +2
      The Divisor is: 7 * x^2 +2 * x^1 +1

      The part divisor: 7 * x^2
      The Dhwajanka part: +2 * x^1 +1

     . In the Quotient Region....
R 1 = 0, ID 1 = 0 + -9 * x^4
R 2 = 0, ID 2 = 0 + 7 * x^3
R 3 = 0, ID 3 = 0 + 7 * x^3
R 4 = 0, ID 4 = 0 + -3 * x^2
R 5 = 0, ID 5 = 0 + -3 * x^2


      Quotient:  8/7 * x^3  -79/49 * x^2  -445/343 * x^1  -1368/2401
      In Remainder Region....R 6 = 0, ID 6 = 0 + 5 * x^1
R 7 = 0, ID 7 = 0 + 5 * x^1
R 8 = 0, ID 8 = 0 + 2 * x^0


      Remainder: 17852/2401 * x^1 +6168/2401

      Do you want to continue with another calculation(y/n)N
```

```
        STRAIGHT DIVISION


     The Dividend is: 7 * X^10 + 26 * X^9 + 53 * X^8 + 56 * X^7 + 43 * X^6 + 40 * x^5 + 41 *
X^4 + 38 * X^3 + 19 * X^2 + 8 * X^1 + 5
     The Divisor is: 1 * X^5 + 3 * X^4 + 5 * X^3 + 3 * X^2 + 1 * X^1 + 1


    R 15 = 0, ID 15 = 0 + 40 * x^5



     Quotient:  7 * x^5  + 5 * x^4  + 3 * x^3  + 1 * x^2  + 3 * x^1
+ 5
     In Remainder Region.   R 16 = 0, ID 16 = 0 + 41 * x^4
  R 17 = 0, ID 17 = 0 + 41 * x^4
  R 18 = 0, ID 18 = 0 + 41 * x^4
  R 19 = 0, ID 19 = 0 + 41 * x^4
  R 20 = 0, ID 20 = 0 + 41 * x^4
  R 21 = 0, ID 21 = 0 + 38 * x^3
  R 22 = 0, ID 22 = 0 + 38 * x^3
  R 23 = 0, ID 23 = 0 + 38 * x^3
  R 24 = 0, ID 24 = 0 + 38 * x^3
  R 25 = 0, ID 25 = 0 + 19 * x^2
  R 26 = 0, ID 26 = 0 + 19 * x^2
  R 27 = 0, ID 27 = 0 + 19 * x^2
  R 28 = 0, ID 28 = 0 + 8 * x^1
  R 29 = 0, ID 29 = 0 + 8 * x^1
  R 30 = 0, ID 30 = 0 + 5 * x^0


     Remainder  0

     Do you want to continue with another calculation(y/n)N
```

STARAIGHT DIVISION 2 VARIABLES

```
          STRAIGHT DIVISION

          The Dividend is: 3 +4 * x^1 +1 * x^2 +2 * x^3 +2 * x^4 +4 * y^1
+17 * x^1 * y^1 +12 * x^2 * y^1 +2 * x^3 * y^1 +10 * x^4 * y^1 +4 * y^2 +7 * x^1
 * y^2 +20 * x^2 * y^2 +9 * x^3 * y^2 +5 * x^4 * y^2 +4 * y^3 -5 * x^1 * y^3 +1
* x^2 * y^3 +3 * x^3 * y^3 +1 * y^4 -1 * x^1 * y^4 -3 * x^2 * y^4 -4 * x^3 * y^4
 -2 * x^4 * y^4
          The Divisor is: 3 +-2 * x^1 +2 * x^2 +4 * y^1 +2 * x^2 * y^1 +1
* y^2 +1 * x^1 * y^2 +1 * x^2 * y^2


          Quotient:  + 1  + 2 * x^1  + 1 * x^2  + 3 * x^1 * y^1  + 4 * x^2
 * y^1  + 1 * y^2  -2 * x^1 * y^2  -2 * x^2 * y^2


          Remainder:


          Do you want to continue with another calculation(y/n)n




          STRAIGHT DIVISION


          The Dividend is: 2 +4 * x^1 +6 * x^2 +4 * y^1 +8 * x^1 * y^1 +1
* x^2 * y^1
          The Divisor is: 2 +1 * x^1 +1 * x^2 +2 * y^1 +3 * x^1 * y^1


          Quotient:  + 1  + 3/2 * x^1  + 7/4 * x^2  + 1 * y^1  + 1/2 * x^1
 * y^1  -17/4 * x^2 * y^1


          Remainder:  -13/4 * x^3  -7/4 * x^4  -3/2 * x^3 * y^1  -2 * y^2
 + 17/4 * x^4 * y^1  -4 * x^1 * y^2  + 7 * x^2 * y^2  + 51/4 * x^3 * y^2


          Do you want to continue with another calculation(y/n)n
```

```
        STRAIGHT DIVISION


        The Dividend is: 5 +2 * x^1 +4 * x^2 +5 * x^3 +3 * y^1 +7 * x^1
* y^1 +5 * y^2 +8 * x^1 * y^2 +6 * y^3
        The Divisor is: 5 +7 * x^1 +4 * y^1


        Quotient:  + 1  -1 * x^1  + 11/5 * x^2  -52/25 * x^3  -1/5 * y^1
 + 62/25 * x^1 * y^1  + 19/25 * y^2  -381/125 * x^1 * y^2  + 74/125 * y^3


        Remainder.  + 364/25 * x^4  -654/25 * x^2 * y^1  + 208/25 * x^3
* y^1  + 30667/125 * x^2 * y^2  + 1006/125 * x^1 * y^3  -296/125 * y^4


        Do you want to continue with another calculation(y/n)n


        STRAIGHT DIVISION


        The Dividend is: 8 +18 * x^1 +9 * x^2 +8 * y^1 +19 * x^1 * y^1 +
12 * x^2 * y^1 +2 * y^2 +5 * x^1 * y^2 +3 * x^2 * y^2
        The Divisor is: 4 +3 * x^1 +2 * y^1 +3 * x^1 * y^1


        Quotient:  + 2  + 3 * x^1  + 1 * y^1  + 1 * x^1 * y^1


        Remainder:


        Do you want to continue with another calculation(y/n)n


        STRAIGHT DIVISION


        The Dividend is: 3 +-4 * x^1 -4 * x^2 -5 * y^1 -1 * x^1 * y^1 +6
* x^2 * y^1 -12 * y^2 +13 * x^1 * y^2 +4 * x^2 * y^2
        The Divisor is: 1 +-2 * x^1 -3 * y^1 +4 * x^1 * y^1


        Quotient.  + 3  + 2 * x^1  + 4 * y^1  + 1 * x^1 * y^1


        Remainder


        Do you want to continue with another calculation(y/n)n
```

```
            STRAIGHT DIVISION

            The Dividend is: 3 +-4 * x^1 -4 * x^2 -5 * y^1 -1 * x^1 * y^1 +6
 * x^2 * y^1 -12 * y^2 +13 * x^1 * y^2 +4 * x^2 * y^2
            The Divisor is: 1 +-2 * x^1 -3 * y^1 +4 * x^1 * y^1

            Quotient:  + 3  + 2 * x^1  + 4 * y^1  + 1 * x^1 * y^1

            Remainder:
           .
             Do you want to continue with another calculation(y/n)n

            STRAIGHT DIVISION

            The Dividend is: 8 +22 * x^1 +12 * x^2 +10 * y^1 -7 * x^1 * y^1
+4 * x^2 * y^1 +6 * x^3 * y^1 +1 * y^2 +5 * x^1 * y^2 -16 * x^2 * y^2 +4 * x^3 *
 y^2 -1 * y^3 +4 * x^1 * y^3 -4 * x^2 * y^3
            The Divisor is: 4 +3 * x^1 -1 * y^1 +2 * x^1 * y^1

            Quotient:  + 2  + 4 * x^1  + 3 * y^1  -4 * x^1 * y^1  + 2 * x^2
* y^1  + 1 * y^2  -2 * x^1 * y^2

            Remainder:

            Do you want to continue with another calculation(y/n)n

            STRAIGHT DIVISION
            ----------------
            The Dividend is: 8 +22 * x^1 +12 * x^2 +10 * y^1 -7 * x^1 * y^1
+4 * x^2 * y^1 +6 * x^3 * y^1 +1 * y^2 +5 * x^1 * y^2 -16 * x^2 * y^2 +4 * x^3 *
 y^2 -1 * y^3 +4 * x^1 * y^3 -4 * x^2 * y^3
            The Divisor is: 4 +3 * x^1 -1 * y^1 +2 * x^1 * y^1

            Quotient:  + 2  + 4 * x^1  + 3 * y^1  -4 * x^1 * y^1  + 2 * x^2
* y^1  + 1 * y^2  -2 * x^1 * y^2

            Remainder:
Do you want to continue with another calculation(y/n)n
```

```
          STRAIGHT DIVISION

          The Dividend is: 3 +4 * x^1 +1 * x^2 +2 * x^3 +2 * x^4 +4 * y^1
+17 * x^1 * y^1 +12 * x^2 * y^1 +2 * x^3 * y^1 +10 * x^4 * y^1 +4 * y^2 +7 * x^1
 * y^2 +20 * x^2 * y^2 +9 * x^3 * y^2 +5 * x^4 * y^2 +4 * y^3 -5 * x^1 * y^3 +1
* x^2 * y^3 +3 * x^3 * y^3 +1 * y^4 -1 * x^1 * y^4 -3 * x^2 * y^4 -4 * x^3 * y^4
 -2 * x^4 * y^4
          The Divisor is: 3 +-2 * x^1 +2 * x^2 +4 * y^1 +2 * x^2 * y^1 +1
* y^2 +1 * x^1 * y^2 +1 * x^2 * y^2

          Quotient:  + 1  + 2 * x^1  + 1 * x^2  + 3 * x^1 * y^1  + 4 * x^2
 * y^1  + 1 * y^2  -2 * x^1 * y^2  -2 * x^2 * y^2

          Remainder:

          Do you want to continue with another calculation(y/n)n
```

(REMAINDER DIVISION)

```
          STRAIGHT DIVISION

          The Dividend is. 5 +2 * x^1 +4 * x^2 +5 * x^3 +3 * y^1 +7 * x^1
* y^1 +8 * x^2 * y^1 +5 * y^2 +8 * x^1 * y^2 +6 * y^3
          The Divisor is: 5 +7 * x^1 +4 * y^1

          Quotient:  + 1  -1 * x^1  + 11/5 * x^2  -52/25 * x^3  -1/5 * y^1
  + 62/25 * x^1 * y^1  -454/125 * x^2 * y^1  + 29/25 * y^2  -251/125 * x^1 * y^2
  + 34/125 * y^3

          Remainder:  + 364/25 * x^4  + 4218/125 * x^3 * y^1  + 3573/125 *
 x^2 * y^2  + 766/125 * x^1 * y^3  -136/125 * y^4

          Do you want to continue with another calculation(y/n)n
```

```
        STRAIGHT DIVISION            (COMMON LCM 125)

        The Dividend is: 1820 * x^4 +4218 * x^3 * y^1 +3573 * x^2 * y^2
+766 * x^1 * y^3 -136 * y^4
        The Divisor is: 5 +7 * x^1 +4 * y^1

        Quotient:  + 364 * x^4  + 4218/5 * x^3 * y^1  + 3573/5 * x^2 * y
^2  + 766/5 * x^1 * y^3  -136/5 * y^4

        Remainder:  -2548 * x^5  -36806/5 * x^4 * y^1  -41883/5 * x^3 *
y^2  -19654/5 * x^2 * y^3  -2112/5 * x^1 * y^4  + 544/5 * y^5

        Do you want to continue with another calculation(y/n)n

        STRAIGHT DIVISION                       (COMMON LCM 625)

        The Dividend is: -12740 * x^5 -36806 * x^4 * y^1 -41883 * x^3 *
y^2 -19654 * x^2 * y^3 -2112 * x^1 * y^4 +544 * y^5
        The Divisor is: 5 +7 * x^1 +4 * y^1

        Quotient:  -2548 * x^5  -36806/5 * x^4 * y^1  -41883/5 * x^3 * y
^2  -19654/5 * x^2 * y^3  -2112/5 * x^1 * y^4  + 544/5 * y^5

        Remainder:  + 17836 * x^6  + 308602/5 * x^5 * y^1  + 88081 * x^4
 * y^2  + 61022 * x^3 * y^3  + 18680 * x^2 * y^4  + 928 * x^1 * y^5  -2176/5 * y
^6

        Do you want to continue with another calculation(y/n)n
```

```
            STRAIGHT DIVISION
             -----------
            The Dividend is: 89180 * x^6 +308602 * x^5 * y^1 +440405 * x^4 *
 y^2 +305110 * x^3 * y^3 +93400 * x^2 * y^4 +4640 * x^1 * y^5 -2176 * y^6
            The Divisor is: 5 +7 * x^1 +4 * y^1

            Quotient.  + 17836 * x^6  + 308602/5 * x^5 * y^1  + 88081 * x^4
* y^2  + 61022 * x^3 * y^3  + 18680 * x^2 * y^4  + 928 * x^1 * y^5  -2176/5 * y^
6

            Remainder:  -124852 * x^7  -2516934/5 * x^6 * y^1  -4317243/5 *
x^5 * y^2  -779478 * x^4 * y^3  -374848 * x^3 * y^4  -81216 * x^2 * y^5  -3328/5
 * x^1 * y^6  + 8704/5 * y^7

            Do you want to continue with another calculation(y/n)n
```

STRAIGHT DIVISION 3VARIABLES

```
        STRAIGHT DIVISION

        The Dividend is: 5 +2 * x^1 +3 * y^1 +4 * z^1 +2 * x^1 * y^1 +3
* x^1 * z^1 +4 * y^1 * z^1 +5 * x^2 +6 * y^2 +7 * z^2 +2 * x^2 * y^1 +3 * x^2 *
z^1 +4 * x^1 * y^2 +8 * y^2 * z^1 +5 * x^1 * z^2 +4 * y^1 * z^2 +6 * x^3 +8 * y^
3 +3 * z^3 +5 * x^2 * y^2 +3 * y^2 * z^2 +4 * x^2 * z^2
        The Divisor is:  5 +7 * x^1 +4 * y^1 +2 * z^1

        Quotient:  + 1  -1 * x^1  -1/5 * y^1  + 2/5 * z^1  + 37/25 * x^1
 * y^1  + 11/25 * x^1 * z^1  + 14/25 * y^1 * z^1  + 12/5 * x^2  + 34/25 * y^2  +
 31/25 * z^2  -449/125 * x^2 * y^1  -122/125 * x^2 * z^1  -286/125 * x^1 * y^2
+ 101/125 * y^2 * z^1  -114/125 * x^1 * z^2  -52/125 * y^1 * z^2  -54/25 * x^3
+ 64/125 * y^3  + 13/125 * z^3  + 4423/625 * x^2 * y^2  + 381/625 * y^2 * z^2  +
 1542/625 * x^2 * z^2

        Remainder:  -216/25 * x^1 * y^1 * z^1  + 4223/125 * x^3 * y^1  +
 1394/125 * x^3 * z^1  + 1386/125 * x^2 * y^1 * z^1  -27/25 * x^1 * y^2 * z^1  +
 696/125 * x^1 * y^3  -532/125 * y^3 * z^1  + 164/25 * x^1 * y^1 * z^2  + 378/25
 * x^4  + 137/125 * x^1 * z^3  + 52/125 * y^1 * z^3  -256/125 * y^4  -30961/625
* x^3 * y^2  -2667/625 * x^1 * y^2 * z^2  -17692/625 * x^2 * y^3  -26/125 * z^4
 -10794/625 * x^3 * z^2  -1524/625 * y^3 * z^2  -8846/625 * x^2 * y^2 * z^1  -61
68/625 * x^2 * y^1 * z^2  -762/625 * y^2 * z^3  -3084/625 * x^2 * z^3

        Do you want to continue with another calculation(y/n)N
```